Past Paper Questions – Set 1

1.

(i) Given that $y = 5x^3 + 7x + 3$, find

(*a*) $\frac{dy}{dx}$,

(3)

(*b*) $\frac{d^2y}{dx^2}$.

(1)

(ii) Find $\int \left(1 + 3\sqrt{x} - \frac{1}{x^2}\right) dx$.

(4)

2.

Given that $y = 6x - \frac{4}{x^2}$, $x \neq 0$,

(*a*) find $\frac{dy}{dx}$,

(2)

(*b*) find $\int y \, dx$.

(3)

3.

Given that $y = 2x^2 - \frac{6}{x^3}$, $x \neq 0$,

(*a*) find $\frac{dy}{dx}$,

(2)

(*b*) find $\int y \, dx$.

(3)

4.

Find $\int (6x^2 + 2 + x^{-\frac{1}{2}}) \, dx$, giving each term in its simplest form.

(4)

5.

Differentiate with respect to x

(*a*) $x^4 + 6\sqrt{x}$,

(3)

(*b*) $\dfrac{(x+4)^2}{x}$.

(4)

6.

Use calculus to find the exact value of $\displaystyle\int_1^2 \left(3x^2 + 5 + \frac{4}{x^2}\right) \mathrm{d}x$.

(5)

7.

The curve C has equation $y = 4x^2 + \dfrac{5-x}{x}$, $x \neq 0$. The point P on C has x-coordinate 1.

(*a*) Show that the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at P is 3.

(5)

(*b*) Find an equation of the tangent to C at P.

(3)

This tangent meets the x-axis at the point $(k, 0)$.

(*c*) Find the value of k.

(2)

8.

Figure 2

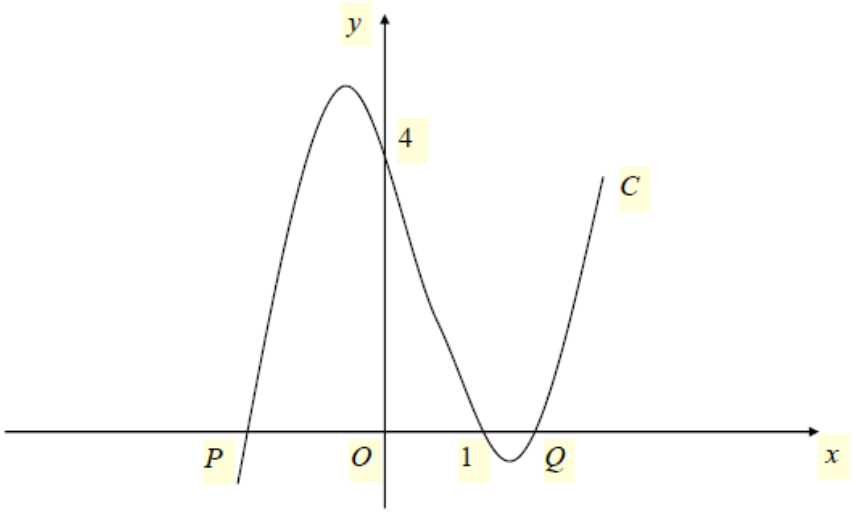

Figure 2 shows part of the curve C with equation

$$y = (x - 1)(x^2 - 4).$$

The curve cuts the x-axis at the points P, (1, 0) and Q, as shown in Figure 2.

(a) Write down the x-coordinate of P and the x-coordinate of Q.

(2)

(b) Show that $\frac{dy}{dx} = 3x^2 - 2x - 4$.

(3)

(c) Show that $y = x + 7$ is an equation of the tangent to C at the point (–1, 6).

(2)

The tangent to C at the point R is parallel to the tangent at the point (–1, 6).

(d) Find the exact coordinates of R.

(5)

9.

Find the coordinates of the stationary point on the curve with equation $y = 2x^2 - 12x$.

(4)

10.

The curve C has equation

$$y = 2x^3 - 5x^2 - 4x + 2.$$

(a) Find $\frac{dy}{dx}$.

(2)

(b) Using the result from part (a), find the coordinates of the turning points of C.

(4)

(c) Find $\frac{d^2y}{dx^2}$.

(2)

(d) Hence, or otherwise, determine the nature of the turning points of C.

(2)

11.

Find the set of values of x for which

$$x^2 - 7x - 18 > 0.$$

(4)

12.

Find the set of values of x for which

(*a*) $3(2x + 1) > 5 - 2x$,

(2)

(*b*) $2x^2 - 7x + 3 > 0$,

(4)

(*c*) **both** $3(2x + 1) > 5 - 2x$ **and** $2x^2 - 7x + 3 > 0$.

(2)

13.

$$x^2 - 8x - 29 \equiv (x + a)^2 + b,$$

where a and b are constants.

(*a*) Find the value of a and the value of b.

(3)

(*b*) Hence, or otherwise, show that the roots of

$$x^2 - 8x - 29 = 0$$

are $c \pm d\sqrt{5}$, where c and d are integers to be found.

(3)

14.

Given that

$$f(x) = x^2 - 6x + 18, \quad x \geq 0,$$

(*a*) express $f(x)$ in the form $(x - a)^2 + b$, where a and b are integers.

(3)

The curve C with equation $y = f(x)$, $x \geq 0$, meets the y-axis at P and has a minimum point at Q.

(*b*) Sketch the graph of C, showing the coordinates of P and Q.

(4)

The line $y = 41$ meets C at the point R.

(*c*) Find the x-coordinate of R, giving your answer in the form $p + q\sqrt{2}$, where p and q are integers.

(5)

15.

Given that the equation $kx^2 + 12x + k = 0$, where k is a positive constant, has equal roots, find the value of k.

(4)

16.

$$x^2 + 2x + 3 \equiv (x + a)^2 + b.$$

(*a*) Find the values of the constants a and b.

(2)

(*b*) Sketch the graph of $y = x^2 + 2x + 3$, indicating clearly the coordinates of any intersections with the coordinate axes.

(3)

(*c*) Find the value of the discriminant of $x^2 + 2x + 3$. Explain how the sign of the discriminant relates to your sketch in part (*b*).

(2)

The equation $x^2 + kx + 3 = 0$, where k is a constant, has no real roots.

(*d*) Find the set of possible values of k, giving your answer in surd form.

(4)

17.

The equation $x^2 + 2px + (3p + 4) = 0$, where p is a positive constant, has equal roots.

(*a*) Find the value of p.

(4)

(*b*) For this value of p, solve the equation $x^2 + 2px + (3p + 4) = 0$.

(2)
