Revision – Differentiation (Year 12)

1.

The curve C has equation $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$.

The point P has coordinates (3, 0).

(*a*) Show that P lies on C.

(1)

(*b*) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(5)

Another point Q also lies on C. The tangent to C at Q is parallel to the tangent to C at P.

(*c*) Find the coordinates of Q.

(5)

2.

The curve C has equation $y = 4x^2 + \dfrac{5-x}{x}$, $x \neq 0$. The point P on C has x-coordinate 1.

(*a*) Show that the value of $\dfrac{dy}{dx}$ at P is 3.

(5)

(*b*) Find an equation of the tangent to C at P.

(3)

This tangent meets the x-axis at the point $(k, 0)$.

(*c*) Find the value of k.

(2)

3.

(*a*) Show that $\dfrac{(3-\sqrt{x})^2}{\sqrt{x}}$ can be written as $9x^{-\frac{1}{2}} - 6 + x^{\frac{1}{2}}$.

(2)

Given that $\dfrac{dy}{dx} = \dfrac{(3-\sqrt{x})^2}{\sqrt{x}}$, $x > 0$, and that $y = \frac{2}{3}$ at $x = 1$,

(*b*) find y in terms of x.

(6)

4.

The gradient of the curve C is given by

$$\frac{dy}{dx} = (3x - 1)^2.$$

The point $P(1, 4)$ lies on C.

(*a*) Find an equation of the normal to C at P.

(4)

(*b*) Find an equation for the curve C in the form $y = f(x)$.

(5)

(*c*) Using $\frac{dy}{dx} = (3x - 1)^2$, show that there is no point on C at which the tangent is parallel to the line $y = 1 - 2x$.

(2)

5.

Figure 2

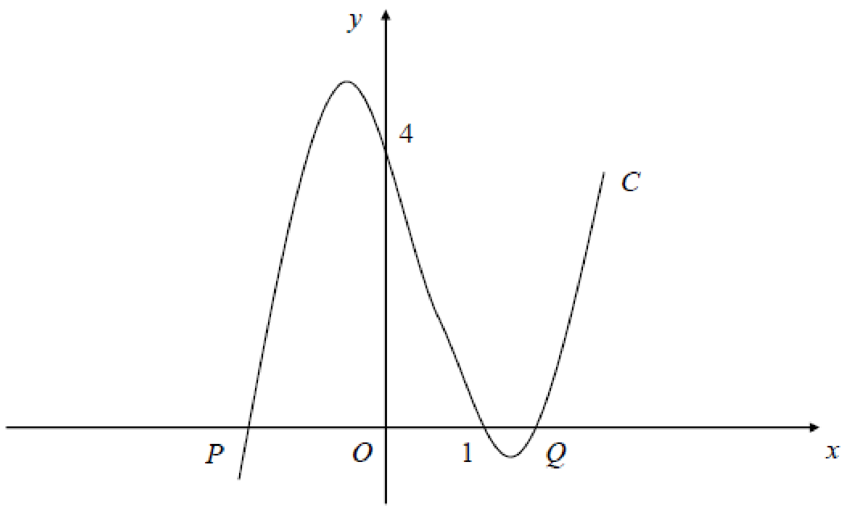

Figure 2 shows part of the curve C with equation

$$y = (x - 1)(x^2 - 4).$$

The curve cuts the x-axis at the points P, $(1, 0)$ and Q, as shown in Figure 2.

(*a*) Write down the x-coordinate of P and the x-coordinate of Q.

(2)

(*b*) Show that $\frac{dy}{dx} = 3x^2 - 2x - 4$.

(3)

(*c*) Show that $y = x + 7$ is an equation of the tangent to C at the point $(-1, 6)$.

(2)

The tangent to C at the point R is parallel to the tangent at the point $(-1, 6)$.

(*d*) Find the exact coordinates of R.

(5)

6.

The curve C has equation

$$y = 2x^3 - 5x^2 - 4x + 2.$$

(*a*) Find $\frac{dy}{dx}$.

(2)

(*b*) Using the result from part (*a*), find the coordinates of the turning points of C.

(4)

(*c*) Find $\frac{d^2y}{dx^2}$.

(2)

(*d*) Hence, or otherwise, determine the nature of the turning points of C.

(2)

7.

Figure 3

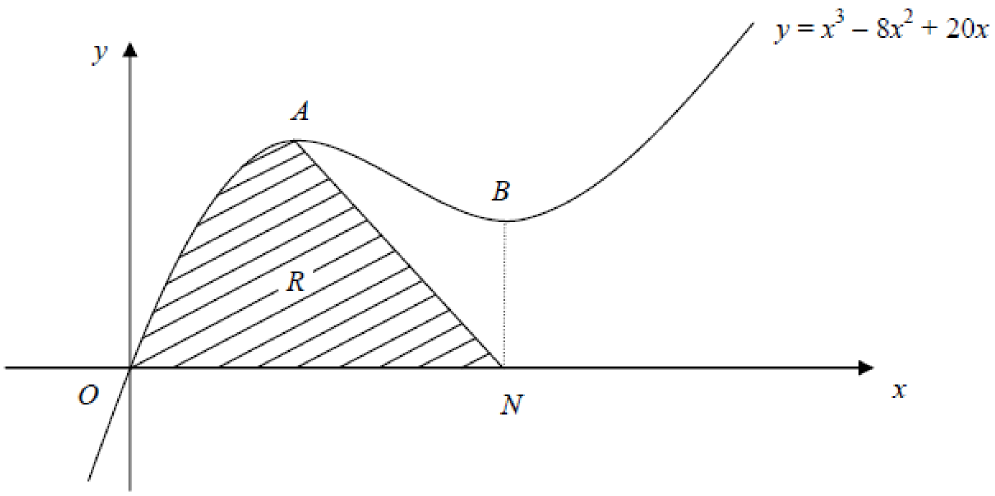

Figure 3 shows a sketch of part of the curve with equation $y = x^3 - 8x^2 + 20x$. The curve has stationary points A and B.

(*a*) Use calculus to find the x-coordinates of A and B.

(4)

(*b*) Find the value of $\frac{d^2y}{dx^2}$ at A, and hence verify that A is a maximum.

(2)

The line through B parallel to the y-axis meets the x-axis at the point N. The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line from A to N.

(*c*) Find $\int (x^3 - 8x^2 + 20x) \; dx$.

(3)

(*d*) Hence calculate the exact area of R.

(5)

8.

The curve C has equation $y = f(x)$, $x \neq 0$, and the point $P(2, 1)$ lies on C. Given that

$$f'(x) = 3x^2 - 6 - \frac{8}{x^2},$$

(a) find $f(x)$.

(5)

(b) Find an equation for the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are integers.

(4)

9.

The curve C with equation $y = f(x)$, $x \neq 0$, passes through the point $(3, 7\frac{1}{2})$.

Given that $f'(x) = 2x + \frac{3}{x^2}$,

(*a*) find $f(x)$.

(5)

(*b*) Verify that $f(-2) = 5$.

(1)

(*c*) Find an equation for the tangent to C at the point $(-2, 5)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

10.

Figure 3

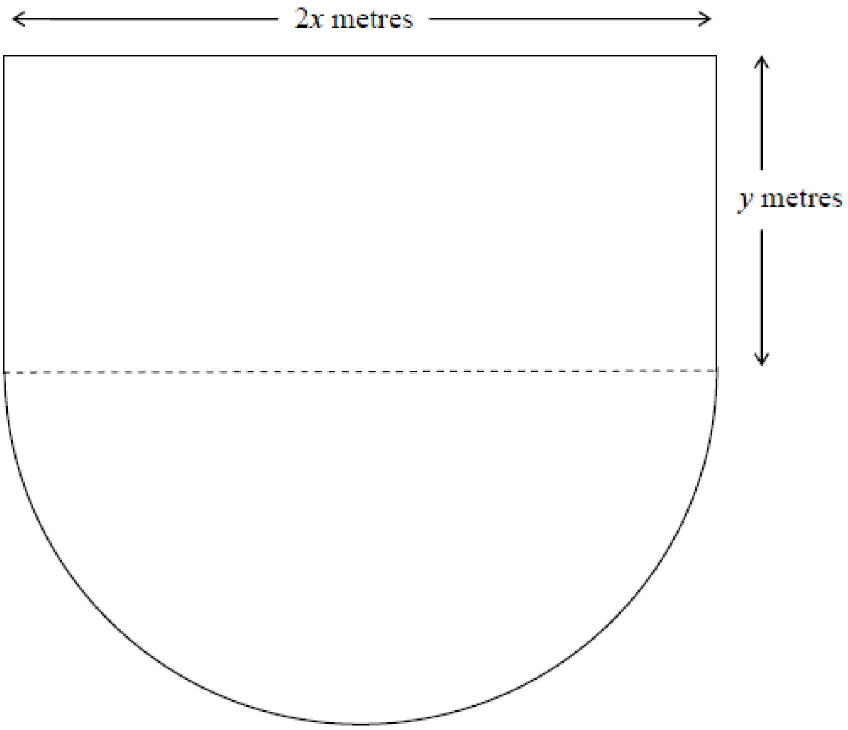

Figure 3 shows the plan of a stage in the shape of a rectangle joined to a semicircle. The length of the rectangular part is $2x$ metres and the width is y metres. The diameter of the semicircular part is $2x$ metres. The perimeter of the stage is 80 m.

(*a*) Show that the area, A m^2, of the stage is given by

$$A = 80x - \left(2 + \frac{\pi}{2}\right)x^2.$$

(4)

(*b*) Use calculus to find the value of x at which A has a stationary value.

(4)

(*c*) Prove that the value of x you found in part (*b*) gives the maximum value of A.

(2)

(*d*) Calculate, to the nearest m^2, the maximum area of the stage.

(2)

11.

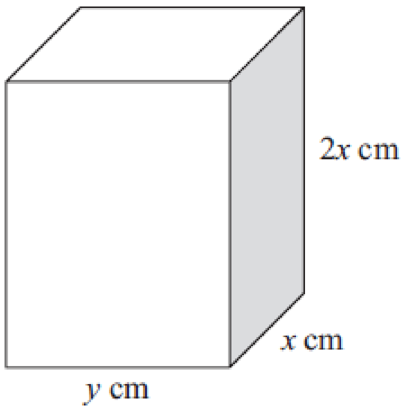

Figure 4

Figure 4 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2.

(a) Show that the volume, V cm^3, of the brick is given by

$$V = 200x - \frac{4x^3}{3}.$$

(4)

Given that x can vary,

(b) use calculus to find the maximum value of V, giving your answer to the nearest cm^3.
(5)

(c) Justify that the value of V you have found is a maximum.
(2)