Revision – Binomial Expansions (Year 12) - 1

1.

(a) The expression $(1-2x)^4$ can be written in the form

$$1 + px + qx^2 - 32x^3 + 16x^4$$

By using the binomial expansion, or otherwise, find the values of the integers p and q.

(3 marks)

- (b) Find the coefficient of x in the expansion of $(2+x)^9$. (2 marks)
- (c) Find the coefficient of x in the expansion of $(1-2x)^4(2+x)^9$. (3 marks)

2.

- (a) The first four terms of the binomial expansion of $(1+2x)^8$ in ascending powers of x are $1+ax+bx^2+cx^3$. Find the values of the integers a, b and c. (4 marks)
- (b) Hence find the coefficient of x^3 in the expansion of $\left(1 + \frac{1}{2}x\right)(1 + 2x)^8$. (3 marks)

3.

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(1+px)^9,$$

where p is a constant.

(2)

The first 3 terms are 1, 36x and qx^2 , where q is a constant.

(b) Find the value of p and the value of q.

(4)

4.

Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3-\frac{1}{3}x\right)^5$$

giving each term in its simplest form.

(4)

(a) Write down the first three terms, in ascending powers of x, of the binomial expansion of $(1 + px)^{12}$, where p is a non-zero constant.

(2)

Given that, in the expansion of $(1 + px)^{12}$, the coefficient of x is (-q) and the coefficient of x^2 is 11q,

(b) find the value of p and the value of q.

(4)

6.

(a) Find the first four terms, in ascending powers of x, in the binomial expansion of $(1+kx)^6$, where k is a non-zero constant.

(3)

Given that, in this expansion, the coefficients of x and x^2 are equal, find

(b) the value of k,

(2)

(c) the coefficient of x^3 .

(1)

7.

Given that $\binom{40}{4} = \frac{40!}{4!b!}$,

(a) write down the value of b.

(1)

In the binomial expansion of $(1+x)^{40}$, the coefficients of x^4 and x^5 are p and q respectively.

(b) Find the value of $\frac{q}{p}$.

(3)

8.

(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1-2x)^5$. Give each term in its simplest form.

(4)

(b) If x is small, so that x^2 and higher powers can be ignored, show that

$$(1+x)(1-2x)^5 \approx 1-9x$$
.

(2)

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2-9x)^4$$

giving each term in its simplest form.

(4)

$$f(x) = (1 + kx)(2 - 9x)^4$$
, where k is a constant

The expansion, in ascending powers of x, of f(x) up to and including the term in x^2 is

$$A - 232x + Bx^2$$

where A and B are constants.

(b) Write down the value of A.

(1)

(c) Find the value of k.

(2)

(d) Hence find the value of B.

(2)

10.

- (i) Find and simplify the first three terms in the binomial expansion of $(2 + ax)^6$ in ascending powers of x.
- (ii) In the expansion of $(3-5x)(2+ax)^6$, the coefficient of x is 64. Find the value of a. [3]

11.

- (i) Find and simplify the first three terms in the expansion of $(2 + 5x)^6$ in ascending powers of x. [4]
- (ii) In the expansion of $(3 + cx)^2(2 + 5x)^6$, the coefficient of x is 4416. Find the value of c. [3]

12.

(a) Find the first 4 terms of the expansion of $\left(1+\frac{x}{2}\right)^{10}$ in ascending powers of x, giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of (1.005)¹⁰, giving your answer to 5 decimal places.

(3)

(a) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$\left(1+\frac{x}{4}\right)^8$$

giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places.

(3)