Mixed Revision Questions – Pack 1

1.

$$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

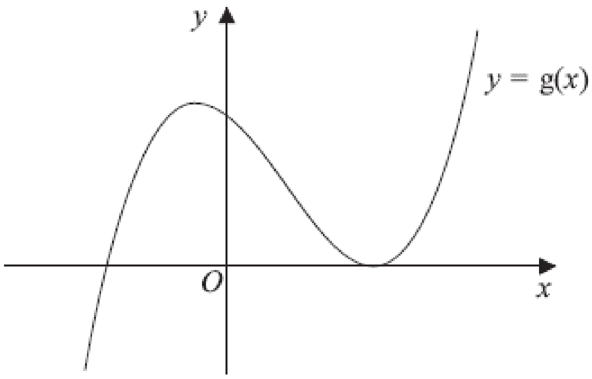


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leqslant 0$

(ii) $g(2x) = 0$

(3)

2.

(a) Sketch the curve $y = -\frac{1}{x^2}$. **[1]**

(b) The curve $y = -\frac{1}{x^2}$ is translated by 2 units in the positive x-direction.

State the equation of the curve after it has been translated. **[2]**

(c) The curve $y = -\frac{1}{x^2}$ is stretched parallel to the y-axis with scale factor $\frac{1}{2}$ and, as a result, the point $\left(\frac{1}{2}, -4\right)$ on the curve is transformed to the point P.

State the coordinates of P. **[2]**

3.

(i) The curve $y = \frac{2}{3+x}$ is translated by four units in the positive x-direction. State the equation of the curve after it has been translated. **[2]**

(ii) Describe fully the single transformation that transforms the curve $y = \frac{2}{3+x}$ to $y = \frac{5}{3+x}$. **[2]**

4.

$$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

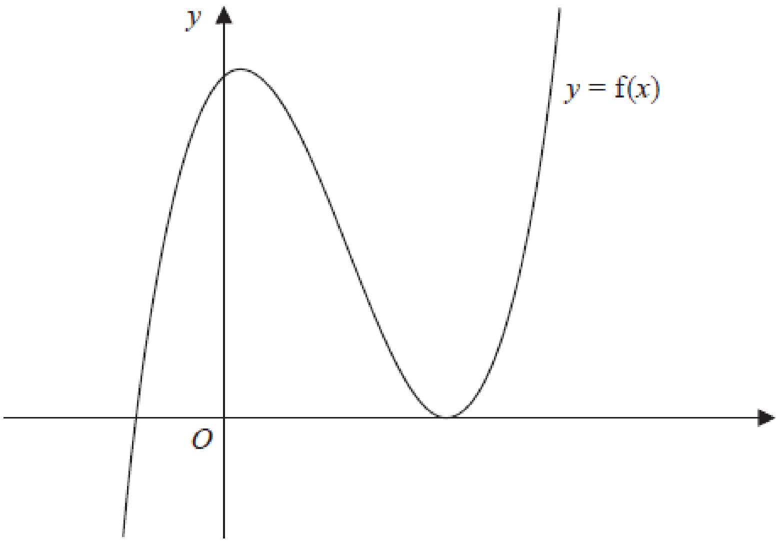


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k.

(2)

5.

The diagram in the Printed Answer Booklet shows part of the graph of $y = x^2 - 4x + 3$.

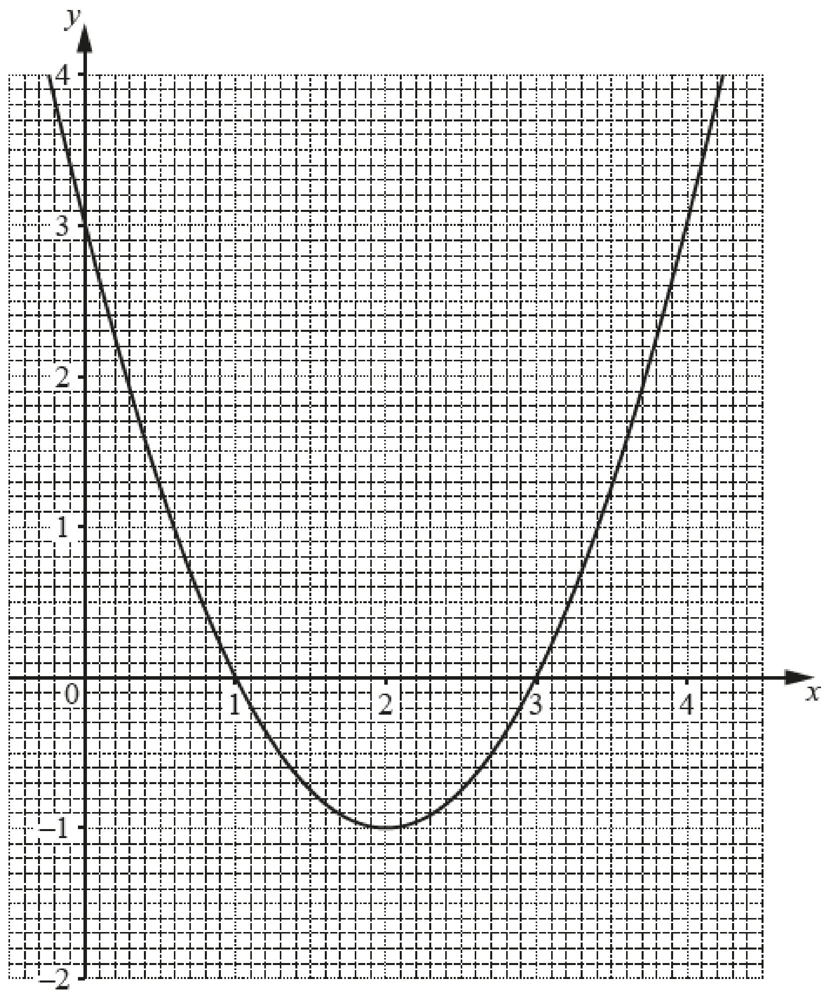
(a) It is required to solve the equation $x^2 - 3x + 1 = 0$ graphically by drawing a straight line with equation $y = mx + c$ on the diagram, where m and c are constants.

Find the values of m and c. **[2]**

(b) Use the graph to find approximate values of the roots of the equation $x^2 - 3x + 1 = 0$. **[2]**

(c) By shading, or otherwise, indicate clearly the regions where **all** of the following inequalities are satisfied. You should use the values of m and c found in part **(a)**.

$x \geqslant 0$ $\quad$ $x \leqslant 4$ $\quad$ $y \leqslant x^2 - 4x + 3$ $\quad$ $y \geqslant mx + c$ **[3]**



6.

The circle $x^2 + y^2 - 4x + ky + 12 = 0$ has radius 1.

Find the two possible values of the constant k. **[4]**

7.

The lines $y = \frac{1}{2}x$ and $y = -\frac{1}{2}x$ are tangents to a circle at (2, 1) and (−2, 1) respectively. Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants. **[6]**

8.

In a triangle ABC, side AB has length 10 cm, side AC has length 5 cm, and angle $BAC = \theta$ where θ is measured in degrees. The area of triangle ABC is 15 cm²

(a) Find the two possible values of $\cos\theta$

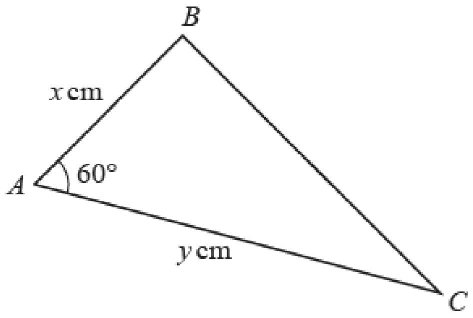
(4)

Given that BC is the longest side of the triangle,

(b) find the exact length of BC.

(2)

9.



The diagram shows triangle ABC, with $AB = x$ cm, $AC = y$ cm and angle $BAC = 60°$. It is given that the area of the triangle is $(x+y)\sqrt{3}$ cm^2.

(a) Show that $4x + 4y = xy$. **[2]**

When the vertices of the triangle are placed on the circumference of a circle, AC is a diameter of the circle.

(b) Determine the value of x and the value of y. **[4]**

10.

(a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} \equiv 4 - 5\cos\theta$$

(4)

(b) Hence, or otherwise, solve, for $0 \leqslant x < 360°$, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 3\sin x$$

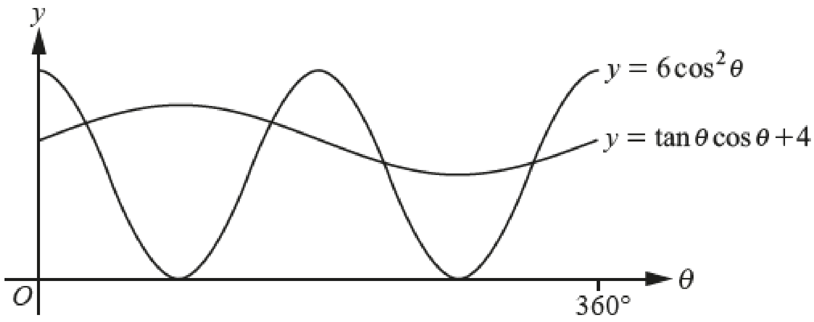
(3)

11.

(a) Show that the equation $6\cos^2\theta = \tan\theta\cos\theta + 4$

can be expressed in the form $6\sin^2\theta + \sin\theta - 2 = 0$. **[2]**

(b)



The diagram shows parts of the curves $y = 6\cos^2\theta$ and $y = \tan\theta\cos\theta + 4$, where θ is in degrees.

Solve the inequality $6\cos^2\theta > \tan\theta\cos\theta + 4$ for $0° < \theta < 360°$. **[5]**

12.

(a) Show that the equation

$$4\cos\theta - 1 = 2\sin\theta\tan\theta$$

can be written in the form

$$6\cos^2\theta - \cos\theta - 2 = 0$$

(4)

(b) Hence solve, for $0 \leqslant x < 90°$

$$4\cos 3x - 1 = 2\sin 3x\tan 3x$$

giving your answers, where appropriate, to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

13.

(a) Show that the equation $2\log_2 x = \log_2(kx - 1) + 3$, where k is a constant, can be expressed in the form $x^2 - 8kx + 8 = 0$. **[4]**

(b) Given that the equation $2\log_2 x = \log_2(kx - 1) + 3$ has only one real root, find the value of this root. **[4]**

14.

(a) Find the positive value of x such that

$$\log_x 64 = 2$$

(2)

(b) Solve for x

$$\log_2(11 - 6x) = 2\log_2(x - 1) + 3$$

(6)

15.

During some research the size, P, of a population of insects, at time t months after the start of the research, is modelled by the following formula.

$P = 100\,e^t$

(a) Use this model to answer the following.

(i) Find the value of P when $t = 4$. **[1]**

(ii) Find the value of t when the population is 9000. **[2]**

(b) It is suspected that a more appropriate model would be the following formula.

$P = ka^t$ where k and a are constants.

(i) Show that, using this model, the graph of $\log_{10} P$ against t would be a straight line. **[2]**

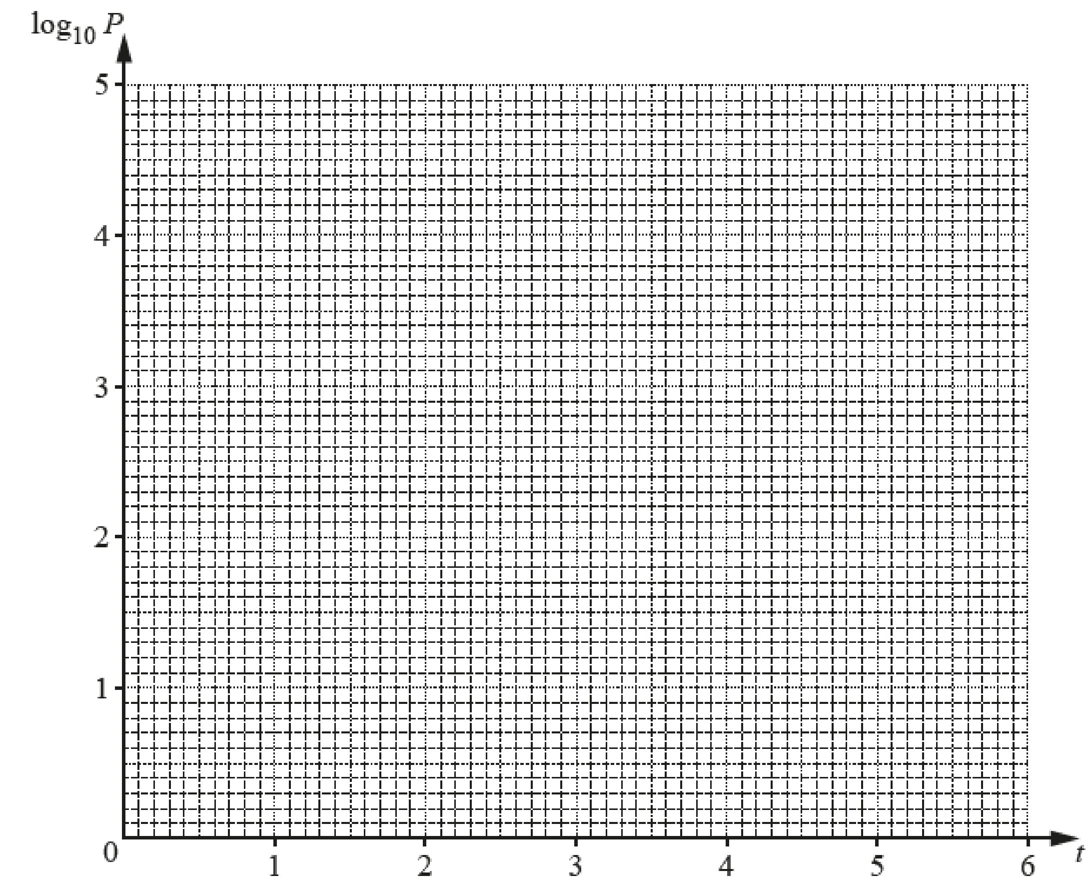
Some observations of t and P gave the following results.

t	1	2	3	4	5
P	100	500	1800	7000	19 000
$\log_{10} P$	2.00	2.70	3.26	3.85	4.28

(ii) On the grid in the Printed Answer Booklet, draw a line of best fit for the data points $(t, \log_{10} P)$ given in the table. **[2]**

(iii) Hence estimate the values of k and a. **[4]**

t	1	2	3	4	5
P	100	500	1800	7000	19000
$\log_{10}P$	2.00	2.70	3.26	3.85	4.28



16.

(a) Find the coordinates of the stationary points on the curve $y = x^3 - 6x^2 + 9x$. **[4]**

(b) The equation $x^3 - 6x^2 + 9x + k = 0$ has exactly one real root.

Using your answers from part **(a)** or otherwise, find the range of possible values of k. **[2]**

17.
A curve has equation

$$y = 3x^2 + \frac{24}{x} + 2 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

(b) Hence find the exact range of values of x for which the curve is increasing.

(2)

18.

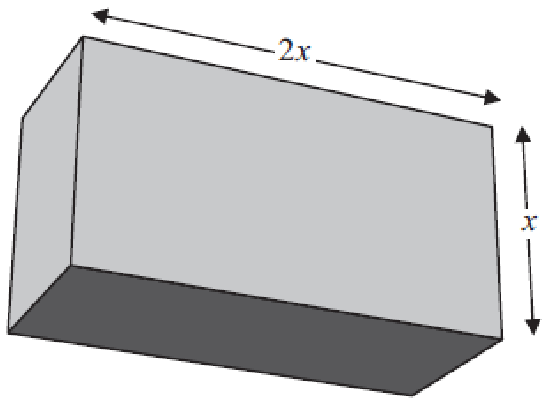


Figure 2

A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm, as shown in Figure 2.
The volume of the cuboid is 81 cubic centimetres.

(a) Show that the total length, L cm, of the twelve edges of the cuboid is given by

$$L = 12x + \frac{162}{x^2}$$

(3)

(b) Use calculus to find the minimum value of L.

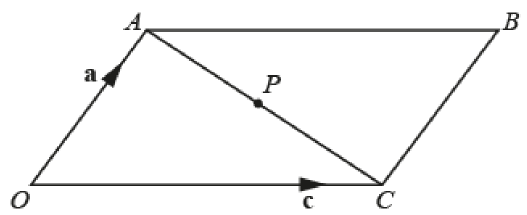
(6)

(c) Justify, by further differentiation, that the value of L that you have found is a minimum.

(2)

19.

$OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. P is the midpoint of AC.



(i) Find the following in terms of **a** and **c**, simplifying your answers.

(a) $\overrightarrow{AC}$ **[1]**

(b) $\overrightarrow{OP}$ **[2]**

(ii) Hence prove that the diagonals of a parallelogram bisect one another. **[4]**

20.

Points A, B, C and D have position vectors $\mathbf{a} = \begin{pmatrix}1\\2\end{pmatrix}$, $\mathbf{b} = \begin{pmatrix}3\\5\end{pmatrix}$, $\mathbf{c} = \begin{pmatrix}7\\4\end{pmatrix}$ and $\mathbf{d} = \begin{pmatrix}4\\k\end{pmatrix}$.

(a) Find the value of k for which D is the midpoint of AC. **[1]**

(b) Find the two values of k for which $|\overrightarrow{AD}| = \sqrt{13}$. **[3]**

(c) Find one value of k for which the four points form a trapezium. **[2]**

21.

The curve C has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote.

(3)

The line l has equation $y = -2x + 5$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of k for which l is a tangent to C.

(3)

22.

A tree was planted in the ground.
Its height, H metres, was measured t years after planting.

Exactly 3 years after planting, the height of the tree was 2.35 metres.
Exactly 6 years after planting, the height of the tree was 3.28 metres.

Using a linear model,

(a) find an equation linking H with t.

(3)

The height of the tree was approximately 140 cm when it was planted.

(b) Explain whether or not this fact supports the use of the linear model in part (a).

(2)

23.

The distance a particular car can travel in a journey starting with a full tank of fuel was investigated.

- From a full tank of fuel, 40 litres remained in the car's fuel tank after the car had travelled 80 km
- From a full tank of fuel, 25 litres remained in the car's fuel tank after the car had travelled 200 km

Using a **linear model**, with V litres being the volume of fuel remaining in the car's fuel tank and d km being the distance the car had travelled,

(a) find an equation linking V with d.

(4)

Given that, on a particular journey

- the fuel tank of the car was initially full
- the car continued until it ran out of fuel

find, according to the model,

(b) (i) the initial volume of fuel that was in the fuel tank of the car,

(ii) the distance that the car travelled on this journey.

(3)

In fact the car travelled 320 km on this journey.

(c) Evaluate the model in light of this information.

(1)

24.

(i) Show that $x^2 - 8x + 17 > 0$ for all real values of x

(3)

(ii) "If I add 3 to a number and square the sum, the result is greater than the square of the original number."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

25.

(i) Express $4x^2 - 12x + 11$ in the form $a(x+b)^2 + c$. **[3]**

(ii) State the number of real roots of the equation $4x^2 - 12x + 11 = 0$. **[1]**

(iii) Explain fully how the value of r is related to the number of real roots of the equation $p(x+q)^2 + r = 0$ where p, q and r are real constants and $p > 0$. **[2]**

26.

A company started mining tin in Riverdale on 1st January 2019.

A model to find the total mass of tin that will be mined by the company in Riverdale is given by the equation

$$T = 1200 - 3(n - 20)^2$$

where T tonnes is the total mass of tin mined in the n years after the start of mining.

Using this model,

(a) calculate the mass of tin that will be mined up to 1st January 2020,

(1)

(b) deduce the maximum total mass of tin that could be mined,

(1)

(c) calculate the mass of tin that will be mined in 2023.

(2)

(d) State, giving reasons, the limitation on the values of n.

(2)