Answers - Revision – Trigonometry (Year 12) - 1

1.

(i)	$\sin^2 x = 1 - \cos^2 x \Rightarrow 2\cos^2 x + \cos x - 1 = 0$	M1		For transforming to a quadratic in cos *x*
	Hence $(2\cos x - 1)(\cos x + 1) = 0$	M1		For solution of a quadratic in cos *x*
	$\cos x = \frac{1}{2} \Rightarrow x = 60^\circ$	A1		For correct answer 60°
	$\cos x = -1 \Rightarrow x = 180^\circ$	A1	**4**	For correct answer 180° [Max 3 out of 4 if any extra answers present in range, or in radians] **SR** answer only is B1, B1 justification – ie graph or substitution is B2, B2
(ii)	$\tan 2x = -1 \Rightarrow 2x = 135 \text{ or } 315$	M1 M1		For transforming to an equation of form tan2*x* = *k* For correct solution method, i.e. inverse tan followed by division by 2
	Hence $x = 67.5^\circ \text{ or } 157.5^\circ$	A1 A1	**4**	For correct value 67.5 For correct value 157.5
	OR $\sin^2 2x = \cos^2 2x$ $2\sin^2 2x = 1 \qquad 2\cos^2 2x = 1$ $\sin 2x = \pm\frac{1}{2}\sqrt{2} \qquad \cos 2x = \pm\frac{1}{2}\sqrt{2}$ Hence $x = 67.5^\circ \text{ or } 157.5^\circ$	M1 M1 A1 A1		Obtain linear equation in cos 2*x* or sin 2*x* Use correct solution method For correct value 67.5 For correct value 157.5 [Max 3 out of 4 if any extra answers present in range, or in radians] **SR** answer only is B1, B1 justification – ie graph or substitution is B2, B2

2.

(a) $(x+10=)$ 60 α		B1	
120	(M: $180-\alpha$ or $\pi-\alpha$)	M1	
$x=50$ $x=110$ (or 50.0 and 110.0)	(M: Subtract 10)	M1 A1	(4)
(b) $(2x=)$ 154.2 β	Allow a.w.r.t. 154 or a.w.r.t. 2.69 (radians)	B1	
205.8	(M: $360-\beta$ or $2\pi-\beta$)	M1	
$x=77.1$ $x=102.9$	(M: Divide by 2)	M1 A1	(4)

3.

(a) $\sin(\theta+30)=\frac{3}{5}$	($\frac{3}{5}$ on RHS)	B1	
$\theta+30=36.9$	(α = AWRT 37)	B1	
or = 143.1	$(180-\alpha)$	M1	
$\underline{\theta=6.9,\ 113.1}$		A1cao	(4)
(b) $\tan\theta=\pm2$ or $\sin\theta=\pm\frac{2}{\sqrt{5}}$ or $\cos\theta=\pm\frac{1}{\sqrt{5}}$		B1	
$(\tan\theta=2\Rightarrow)$ $\theta=\underline{63.4}$	(β = AWRT 63.4)	B1	
or $\underline{243.4}$	$(180+\beta)$	M1	
$(\tan\theta=-2\Rightarrow)$ $\theta=\underline{116.6}$	$(180-\beta)$	M1	
or $\underline{296.6}$	(180 + their 116.6)	M1	(5)

4.

(i)	$\sin\theta\tan\theta = \sin\theta \times \dfrac{\sin\theta}{\cos\theta} = \dfrac{1-\cos^2\theta}{\cos\theta}$	M1		For use of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
		M1		For use of $\cos^2\theta + \sin^2\theta = 1$
	Hence $1-\cos^2\theta = \cos\theta(\cos\theta+1)$,			
	i.e. $2\cos^2\theta + \cos\theta - 1 = 0$, or equiv	A1	**3**	For showing given equation correctly
(ii)	$(2\cos\theta - 1)(\cos\theta + 1) = 0$	M1		For solution of quadratic equation in $\cos\theta$
	Hence $\cos\theta = \frac{1}{2}$ or -1	A1		For both values of $\cos\theta$ correct
	So $\theta = 60°,\ 300°,\ 180°$	A1		For correct answer $60°$
		A1		For correct answer $180°$
		A1√	**5**	For a correct non-principal-value answer, following their value of $\cos\theta$ (excluding $\cos\theta = -1, 0, 1$) **and** no other values for θ.

5.

(a) $5(1-\sin^2 x) = 3(1+\sin x)$	M1	
$5 - 5\sin^2 x = 3 + 3\sin x$		
$\underline{\underline{0 = 5\sin^2 x + 3\sin x - 2}}$ *	A1 cso	(2)
(b) $0 = (5\sin x - 2)(\sin x + 1)$	M1	
$\sin x = \dfrac{2}{5}$, -1 (both)	A1 cso	
$\sin x = \dfrac{2}{5} \Rightarrow x = \underline{\underline{23.6}}$ ($\alpha = 23.6$ or 156.4)	B1	
, $\underline{\underline{156.4}}$ (180- α)	M1	
$\sin x = -1 \Rightarrow x = \underline{\underline{270}}$	B1	(5)
(ignore extra solutions <u>outside</u> the range)		(7)

6.

(i)		$\tan x\,(\sin x - \cos x) = 6\cos x$ $\tan x\,({}^{\sin x}/_{\cos x} - 1) = 6$ $\tan x\,(\tan x - 1) = 6$ $\tan^2 x - \tan x = 6$ $\tan^2 x - \tan x - 6 = 0$ **AG**	M1 A1 **[2]**	Use $\tan x = \frac{\sin x}{\cos x}$ correctly once Obtain $\tan^2 x - \tan x - 6 = 0$	Must be used clearly at least once - either explicitly or by writing eg 'divide by $\cos x$' at side of solution Allow M1 for any equiv eg $\sin x = \cos x \tan x$ Allow poor notation eg writing just tan rather than $\tan x$ Correct equation in given form, including $= 0$ Correct notation throughout so A0 if eg tan rather than $\tan x$ seen in solution
(ii)		$(\tan x - 3)(\tan x + 2) = 0$ $\tan x = 3$, $\tan x = -2$ $x = \tan^{-1}(3)$, $x = \tan^{-1}(-2)$ $x = 71.6°, 252°, 117°, 297°$	M1 M1 A1 A1 **[4]**	Attempt to solve quadratic in $\tan x$ Attempt to solve $\tan x = k$ at least once Obtain two correct solutions Obtain all 4 correct solutions, and no others in range	This M mark is just for solving a 3 term quadratic (see guidance sheet for acceptable methods) Condone any substitution used, inc $x = \tan x$ Attempt $\tan^{-1} k$ at least once Not dependent on previous mark so M0M1 possible If going straight from $\tan x = k$ to $x = \ldots$, then award M1 only if their angle is consistent with their k Allow 3sf or better Must come from a correct method to solve the quadratic (as far as correct factorisation or substitution into formula) Allow radian equivs ie 1.25 / 4.39 / 2.03 / 5.18 Must now all be in degrees Allow 3sf or better A0 if other incorrect solutions in range $0° - 360°$ (but ignore any outside this range) **SR** If no working shown then allow **B1** for each correct solution (max of **B3** if in radians, or if extra solns in range).

7.

(a)

$\tan\theta = 5$	B1	(1)

(b)

$\tan\theta = k$ $\quad\left(\theta = \tan^{-1} k\right)$	M1	
$\theta = 78.7, \quad 258.7$ (Accept awrt)	A1, A1ft	(3)

8.

(i)	substitution of $\tan x = \frac{\sin x}{\cos x}$ or $\sqrt{1-\sin^2 x} = \sqrt{\cos^2 x}$ or $\cos x$ in given LHS	**M1**	if no substitution, statements must follow a logical order and the argument must be clear; if one substitution made correctly, condone error in other part of LHS	condone omission of variable throughout for **M1** only , but allow recovery from omission of variable at end
	both substitutions seen and completion to $\sin x$ as final answer	**A1**	**NB AG**; answer must be stated allow consistent use of other variable eg θ for both marks	**M0** if first move is to square one or both sides Simply stating eg $\tan x = \frac{\sin x}{\cos x}$ is insufficient
		[2]		*Alternatively* **SC2** for complete argument eg $\tan x = \frac{\sin x}{\cos x}$ [$\tan x \times \cos x = \sin x$] $\sin^2 x + \cos^2 x = 1$ $\cos x = \sqrt{1-\sin^2 x}$ $\tan x = \frac{\sin x}{\sqrt{1-\sin^2 x}}$ $\tan x \times \sqrt{1-\sin^2 x} = \sin x$ oe

(ii)		0, 180, 360 14 or 14.47 to 14.5 166 or awrt 165.5	**B1** **B1** **B1** **[3]**	all 3 required radians: mark as scheme but deduct one from total 0, π, 2π; 0.25 or 0.253 or awrt 0.2527; 2.89 or 2.889 or awrt 2.8889	**NB** sin $y = 0$ or ¼ ignore extra values outside range if **B3**, deduct 1 mark for extra values within range

9.

(i)		sin kx $y = \sin 2x$	**M1** **A1** **[2]**	$k > 0$ and $k \neq 1$ must see "$y =$" at some stage for **A1**	condone use of other variable condone $f(x) = \sin 2x$
(ii)		sketch of sine curve with period 360° and amplitude 1 sine curve centred on $y = -3$ and starting at $(0, -3)$	**B1** **B1** **[2]**	for $0 \leq x \leq 450$; ignore curve outside this range; do not allow sketch of $y = \cos x$ or $y = -\sin x$ for either mark	amplitude, period and centring on $y = -3$ must be clear from correct numerical scale, numerical labelling or comment; strokes on axes insufficient to imply scale: mark intent allow full marks if $y = \sin x$ and $y = \sin x - 3$ seen on same diagram

10.

(i)	$\cos BCA = \frac{5^2+6^2-9^2}{2\times5\times6} = -\frac{1}{3}$	M1		For relevant use of the correct cosine formula
		M1		For attempt to rearrange correct formula
		A1		For obtaining the given value correctly
	So $\sin BCA = \frac{2}{3}\sqrt{2} \approx 0.9428 \ldots$	B1		For correct answer for sin *BCA* in any form
				OR
		M1		For substituting cos BCA = $-^1/_3$
		M1		For attempt at evaluation
		A1		For full verification
		B1	4	For correct answer for sin *BCA* in any form
(ii)	Angles *BCA* and *CAD* are equal	B1		For stating, using or implying the equal angles
	So $\sin ADC = \frac{5}{15}\sin CAD = \frac{1}{3}\times\frac{1}{3}\sqrt{8} = \frac{2}{9}\sqrt{2}$	M1		For correct use of the sine rule in Δ ADC (sides must be numerical, angles may still be in letters)
	$\Rightarrow ADC = 18.3^{\circ}$	A1√		For a correct equation from their value in (i)
		A1	4	For correct answer, from correct working
			8	

11.

(i) $\frac{LB}{\sin 65°} = \frac{200}{\sin 35°}$ OR $\frac{LA}{\sin 80°} = \frac{200}{\sin 35°}$		M1		For correct use of the sine rule in ΔLAB (could be in **ii**)
$\Rightarrow LB = 316.0198\ldots$	$\Rightarrow LA = 343.39\ldots$	A1		For correct value of (or explicit expression for) LB or LA
Hence $p = LB\sin 80° = 311\text{ m}$	$p = LA\sin 65 \;\; = 311\text{ m}$	M1		For calculation of perpendicular distance
		A1	**4**	For correct distance (rounding to) 311
(ii) $LC^2 = 200^2 + 316^2 - 2\times 200\times 316\times \cos 100°$		M1		For use of cosine rule in ΔLBC or LAC
$\left(or\ LC^2 = 400^2 + 343^2 - 2\times 400\times 343\times \cos 65°\right)$		A1√		For correct unsimplified numerical expression for LC^2 following their LA or LB
Hence $LC = 402$ m		A1	**3**	For correct distance (rounding to) 402

12.

(i)		area = $^1/_2 \times 8 \times 10 \times \sin 65°$	M1	Attempt area of triangle using $\frac{1}{2}ab\sin\theta$	Must be correct formula, including $\frac{1}{2}$ Allow if evaluated in radian mode (gives 33.1) If using $\frac{1}{2} \times b \times h$, then must be valid use of trig to find h
		= 36.3	A1 **[2]**	Obtain 36.3, or better	If > 3sf, allow answer rounding to 36.25 with no errors seen
(ii)		$BD^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 65°$	M1	Attempt use of correct cosine rule	Must be correct cosine rule Allow M1 if not square rooted, as long as BD^2 seen Allow if evaluated in radian mode (gives 15.9) Allow if correct formula is seen but is then evaluated incorrectly - using $(8^2 + 10^2 - 2 \times 8 \times 10) \times \cos 65°$ gives 1.30 Allow any equiv method, as long as valid use of trig
		$BD = 9.82$	A1 **[2]**	Obtain 9.82, or better	If > 3sf, allow answer rounding to 9.817 with no errors seen
(iii)		$\frac{BC}{\sin 65} = \frac{8}{\sin 30}$	M1	Attempt use of correct sine rule (or equiv)	Must get as far as attempting BC, not just quoting correct sine rule Allow any equiv method, as long as valid use of trig including attempt at any angles used If using their BD from part**(ii)** it must have been a valid attempt (eg M0 for $BD = 8\sin 65$, $BC = {}^{BD}/_{\sin 30} = 14.5$)
		$BC = 14.5$	A1 **[2]**	Obtain 14.5, or better	If >3sf, allow answer rounding to 14.5 with no errors in method seen In multi-step solutions (eg using 9.82) interim values may be slightly inaccurate – allow A1 if answer rounds to 14.5