Mixed Exercise 1

1. AQA/Jan 2006/MPC1/Q4)

The quadratic equation $x^2 + (m + 4)x + (4m + 1) = 0$, where m is a constant, has equal roots.

(a) Show that $m^2 - 8m + 12 = 0$. *(3 marks)*

(b) Hence find the possible values of m. *(2 marks)*

2.

The quadratic equation $(k + 1)x^2 + 12x + (k - 4) = 0$ has real roots.

Find the possible value of k.

(4 marks)

3.

The quadratic equation

$$(2k - 3)x^2 + 2x + (k - 1) = 0$$

where k is a constant, has real roots.

(a) Show that $2k^2 - 5k + 2 \leqslant 0$. *(3 marks)*

(b) (i) Factorise $2k^2 - 5k + 2$. *(1 mark)*

(ii) Hence, or otherwise, solve the quadratic inequality

$$2k^2 - 5k + 2 \leqslant 0$$ *(3 marks)*

4.

Find the set of values of x for which

(*a*) $3(2x + 1) > 5 - 2x$,

(2)

(*b*) $2x^2 - 7x + 3 > 0$,

(4)

(*c*) **both** $3(2x + 1) > 5 - 2x$ **and** $2x^2 - 7x + 3 > 0$.

(2)

5 .

(a) (i) Express $x^2 - 4x + 9$ in the form $(x-p)^2 + q$, where p and q are integers.
(2 marks)

(ii) Hence, or otherwise, state the coordinates of the minimum point of the curve with equation $y = x^2 - 4x + 9$. *(2 marks)*

(b) The line L has equation $y + 2x = 12$ and the curve C has equation $y = x^2 - 4x + 9$.

(i) Show that the x-coordinates of the points of intersection of L and C satisfy the equation

$$x^2 - 2x - 3 = 0$$ *(1 mark)*

(ii) Hence find the coordinates of the points of intersection of L and C. *(4 marks)*

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6. (Edexcel/Jan 2005/C2)

(*a*) Use the factor theorem to show that $(x + 4)$ is a factor of $2x^3 + x^2 - 25x + 12$.
(2)

(*b*) Factorise $2x^3 + x^2 - 25x + 12$ completely.
(4)

7.

The polynomial $f(x)$ is given by $f(x) = x^3 + 4x - 5$.

(i) Use the Factor Theorem to show that $x - 1$ is a factor of $f(x)$. *(2 marks)*

(ii) Express $f(x)$ in the form $(x-1)(x^2 + px + q)$, where p and q are integers.
(2 marks)

(iii) Hence show that the equation $f(x) = 0$ has exactly one real root and state its value. *(3 marks)*

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8. (Edexcel/Jan 2005/C2)

(*a*) Write down the first three terms, in ascending powers of x, of the binomial expansion of $(1 + px)^{12}$, where p is a non-zero constant.
(2)

Given that, in the expansion of $(1 + px)^{12}$, the coefficient of x is $(-q)$ and the coefficient of x^2 is $11q$,

(*b*) find the value of p and the value of q.
(4)

9.

(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1-2x)^5$. Give each term in its simplest form.

(4)

(b) If x is small, so that x^2 and higher powers can be ignored, show that

$$(1+x)(1-2x)^5 \approx 1-9x.$$

(2)

10.

(a) Find the first 4 terms of the expansion of $\left(1+\frac{x}{2}\right)^{10}$ in ascending powers of x, giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places.

(3)

11.

(a) The expression $(1-2x)^4$ can be written in the form

$$1+px+qx^2-32x^3+16x^4$$

By using the binomial expansion, or otherwise, find the values of the integers p and q.

(3 marks)

(b) Find the coefficient of x in the expansion of $(2+x)^9$. *(2 marks)*

(c) Find the coefficient of x in the expansion of $(1-2x)^4(2+x)^9$. *(3 marks)*

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12. (Edexcel/Jan 2006/C2)

(*a*) Find all the values of θ, to 1 decimal place, in the interval $0° \leq \theta < 360°$ for which

$$5\sin(\theta + 30°) = 3.$$

(4)

(*b*) Find all the values of θ, to 1 decimal place, in the interval $0° \leq \theta < 360°$ for which

$$\tan^2\theta = 4.$$

(5)

13.

(a) Show that the equation

$$4\sin^2 x + 9\cos x - 6 = 0$$

can be written as

$$4\cos^2 x - 9\cos x + 2 = 0.$$

(2)

(b) Hence solve, for $0 \leqslant x < 720°$,

$$4\sin^2 x + 9\cos x - 6 = 0,$$

giving your answers to 1 decimal place.

(6)

14.

Solve, for $0 \leqslant x < 360°$,

(a) $\sin(x - 20°) = \dfrac{1}{\sqrt{2}}$

(4)

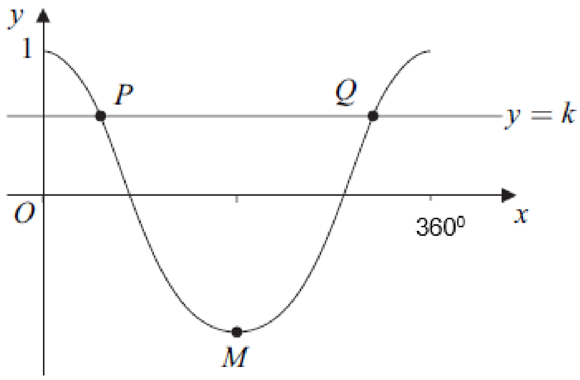
(b) $\cos 3x = -\dfrac{1}{2}$

(6)

15.

(a) Solve the equation $\cos x = 0.3$ in the interval $0 \leqslant x \leqslant 360^0$, giving your answers in degrees to three significant figures. *(3 marks)*

(b) The diagram shows the graph of $y = \cos x$ for $0 \leqslant x \leqslant 360^0$ and the line $y = k$.



The line $y = k$ intersects the curve $y = \cos x$, $0 \leqslant x \leqslant 360^0$, at the points P and Q. The point M is the minimum point of the curve.

(i) Write down the coordinates of the point M. *(2 marks)*

(ii) The x-coordinate of P is α.

Write down the x-coordinate of Q in terms of α. *(1 mark)*

16.

The point A has coordinates $(1, 7)$ and the point B has coordinates $(5, 1)$.

(a) (i) Find the gradient of the line AB. *(2 marks)*

(ii) Hence, or otherwise, show that the line AB has equation $3x + 2y = 17$. *(2 marks)*

(b) The line AB intersects the line with equation $x - 4y = 8$ at the point C. Find the coordinates of C. *(3 marks)*

(c) Find an equation of the line through A which is perpendicular to AB. *(3 marks)*

17.

The points A and B have coordinates $(6, -1)$ and $(2, 5)$ respectively.

(a) (i) Show that the gradient of AB is $-\frac{3}{2}$. *(2 marks)*

(ii) Hence find an equation of the line AB, giving your answer in the form $ax + by = c$, where a, b and c are integers. *(2 marks)*

(b) (i) Find an equation of the line which passes through B and which is perpendicular to the line AB. *(2 marks)*

(ii) The point C has coordinates $(k, 7)$ and angle ABC is a right angle.

Find the value of the constant k. *(2 marks)*

18.

A circle with centre C has equation $(x + 3)^2 + (y - 2)^2 = 25$.

(a) Write down:

(i) the coordinates of C; *(2 marks)*

(ii) the radius of the circle. *(1 mark)*

(b) (i) Verify that the point $N(0, -2)$ lies on the circle. *(1 mark)*

(ii) Sketch the circle. *(2 marks)*

(iii) Find an equation of the normal to the circle at the point N. *(3 marks)*

(c) The point P has coordinates $(2, 6)$.

(i) Find the distance PC, leaving your answer in surd form. *(2 marks)*

(ii) Find the length of a tangent drawn from P to the circle. *(3 marks)*

19.

A circle has equation $x^2 + y^2 - 4x - 14 = 0$.

(a) Find:

(i) the coordinates of the centre of the circle; *(3 marks)*

(ii) the radius of the circle in the form $p\sqrt{2}$, where p is an integer. *(3 marks)*

(b) A chord of the circle has length 8. Find the perpendicular distance from the centre of the circle to this chord. *(3 marks)*

(c) A line has equation $y = 2k - x$, where k is a constant.

(i) Show that the x-coordinate of any point of intersection of the line and the circle satisfies the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$ *(3 marks)*

(ii) Find the values of k for which the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$

has equal roots. *(4 marks)*

(iii) Describe the geometrical relationship between the line and the circle when k takes either of the values found in part (c)(ii). *(1 mark)*

20.

A circle with centre C has equation $x^2 + y^2 + 2x - 12y + 12 = 0$.

(a) By completing the square, express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(c) Show that the circle does **not** intersect the x-axis. *(2 marks)*

(d) The line with equation $x + y = 4$ intersects the circle at the points P and Q.

(i) Show that the x-coordinates of P and Q satisfy the equation

$$x^2 + 3x - 10 = 0$$ *(3 marks)*

(ii) Given that P has coordinates $(2, 2)$, find the coordinates of Q. *(2 marks)*

(iii) Hence find the coordinates of the midpoint of PQ. *(2 marks)*