Coordinate Geometry – Circles

Exercise A

1.

The line PQ is the diameter of the circle, where P and Q are $(5, 6)$ and $(-2, 2)$ respectively. Find the equation of the circle.

2.

The point $P(1, -2)$ lies on the circle centre $(4, 6)$.

a Find the equation of the circle.

b Find the equation of the tangent to the circle at P.

3.

The tangent to the circle $(x + 4)^2 + (y - 1)^2 = 242$ at $(7, -10)$ meets the y-axis at S and the x-axis at T.

a Find the coordinates of S and T.

b Hence, find the area of $\triangle OST$, where O is the origin.

Exercise B

1 Find where the circle $(x - 1)^2 + (y - 3)^2 = 45$ meets the x-axis.

2 Find where the circle $(x - 2)^2 + (y + 3)^2 = 29$ meets the y-axis.

3 The circle $(x - 3)^2 + (y + 3)^2 = 34$ meets the x-axis at $(a, 0)$ and the y-axis at $(0, b)$. Find the possible values of a and b.

4 The line $y = x + 4$ meets the circle $(x - 3)^2 + (y - 5)^2 = 34$ at A and B. Find the coordinates of A and B.

5 Find where the line $x + y + 5 = 0$ meets the circle $(x + 3)^2 + (y + 5)^2 = 65$.

6 Show that the line $y = x - 10$ does not meet the circle $(x - 2)^2 + y^2 = 25$.

7 Show that the line $x + y = 11$ is a tangent to the circle $x^2 + (y - 3)^2 = 32$.

Hint for question 7: Show that the line meets the circle at one point only.

8 Show that the line $3x - 4y + 25 = 0$ is a tangent to the circle $x^2 + y^2 = 25$.

9 The line $y = 2x - 2$ meets the circle $(x - 2)^2 + (y - 2)^2 = 20$ at A and B.

a Find the coordinates of A and B.

b Show that AB is a diameter of the circle.

10 The line $x + y = a$ meets the circle $(x - p)^2 + (y - 6)^2 = 20$ at $(3, 10)$, where a and p are constants.

a Work out the value of a.

b Work out the two possible values of p.

Exercise C

1.

The points A and B have coordinates $(5, -1)$ and $(13, 11)$ respectively.

(a) Find the coordinates of the mid-point of AB.

(2)

Given that AB is a diameter of the circle C,

(b) find an equation for C.

(4)

2.

The circle C, with centre at the point A, has equation $x^2 + y^2 - 10x + 9 = 0$.

Find

(a) the coordinates of A,

(2)

(b) the radius of C,

(2)

(c) the coordinates of the points at which C crosses the x-axis.

(2)

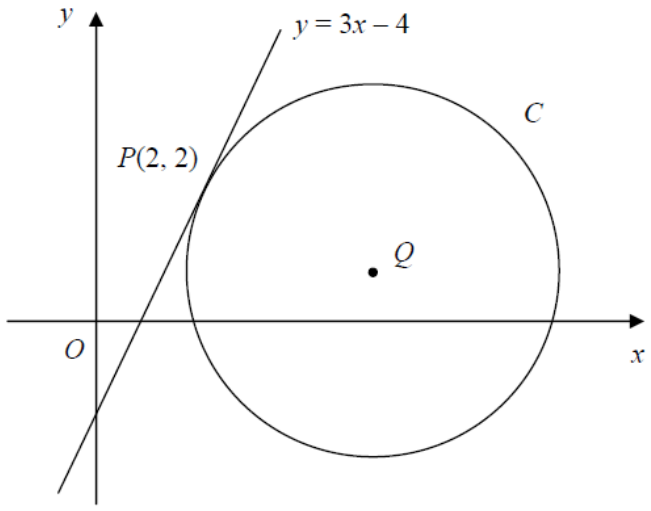
Given that the line l with gradient $\frac{7}{2}$ is a tangent to C, and that l touches C at the point T,

(d) find an equation of the line which passes through A and T.

(3)

3.

Figure 1



The line $y = 3x - 4$ is a tangent to the circle C, touching C at the point P(2, 2), as shown in Figure 1.

The point Q is the centre of C.

(a) Find an equation of the straight line through P and Q.

(3)

Given that Q lies on the line $y = 1$,

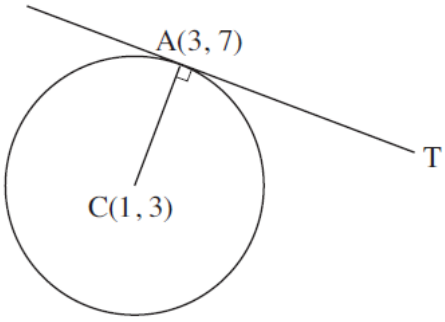
(b) show that the x-coordinate of Q is 5,

(1)

(c) find an equation for C.

(4)

4.



Not to scale

Fig. 11

A circle has centre C(1, 3) and passes through the point A(3, 7) as shown in Fig. 11.

(i) Show that the equation of the tangent at A is $x + 2y = 17$. [4]

(ii) The line with equation $y = 2x - 9$ intersects this tangent at the point T.

Find the coordinates of T. [3]

(iii) The equation of the circle is $(x - 1)^2 + (y - 3)^2 = 20$.

Show that the line with equation $y = 2x - 9$ is a tangent to the circle. Give the coordinates of the point where this tangent touches the circle. [5]

5.

A circle with centre C has equation $x^2 + y^2 - 10x + 12y + 41 = 0$. The point $A(3, -2)$ lies on the circle.

(a) Express the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = k$$

[3 marks]

(b) (i) Write down the coordinates of C.

[1 mark]

(ii) Show that the circle has radius $n\sqrt{5}$, where n is an integer.

[2 marks]

(c) Find the equation of the tangent to the circle at the point A, giving your answer in the form $x + py = q$, where p and q are integers.

[5 marks]

(d) The point B lies on the tangent to the circle at A and the length of BC is 6. Find the length of AB.

[3 marks]

6.

The points D, E and F have coordinates $(-2, 0)$, $(0, -1)$ and $(2, 3)$ respectively.

(i) Calculate the gradient of DE. [1]

(ii) Find the equation of the line through F, parallel to DE, giving your answer in the form $ax + by + c = 0$. [3]

(iii) By calculating the gradient of EF, show that DEF is a right-angled triangle. [2]

(iv) Calculate the length of DF. [2]

(v) Use the results of parts **(iii)** and **(iv)** to show that the circle which passes through D, E and F has equation $x^2 + y^2 - 3y - 4 = 0$. [5]
