Answers - Quadratics

Exercise A

1. ~~b~~ Since the equation has repeated real roots,

$$b^2 - 4ac = 0$$
$$(-6)^2 - 4(2)(k) = 0$$
$$36 = 8k$$
$$k = \underline{\underline{4.5}}$$

2.

(a) Discriminant $= b^2 - 4ac$

$$= (-4)^2 - 4(k)(k)$$
$$= \underline{\underline{16 - 4k^2}}$$

(b) Since there are equal roots, (repeated real roots),

$$b^2 - 4ac = 0$$
$$\therefore\ 16 - 4k^2 = 0$$
$$16 = 4k^2$$
$$k^2 = 4$$
$$\underline{\underline{k = \pm 2}}$$

3.

(a) Discriminant $= b^2 - 4ac$

$$= 7^2 - 4(-2)(3)$$
$$= 49 + 24$$
$$= 73$$

Since the discriminant is greater than 0, there are two distinct real roots.

4.(a) $a = k, \quad b = 4, \quad c = 5-k$

Since there are two distinct real roots,

$$b^2 - 4ac > 0$$

$$4^2 - 4k(5-k) > 0$$

$$16 - 20k + 4k^2 > 0$$

$(\div 4)$

$$4 - 5k + k^2 > 0$$

$$\therefore \quad k^2 - 5k + 4 > 0$$

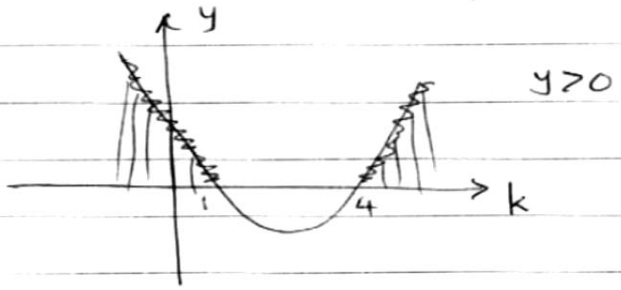
(b) $k^2 - 5k + 4 > 0$

To solve, sketch the graph of $y = k^2 - 5k + 4$.

Critical values: $k^2 - 5k + 4 = 0$

$$(k-4)(k-1) = 0$$

$$k = 4, \; 1$$



$$\therefore \quad k < 1 \quad \text{or} \quad k > 4$$

5. Since there are two distinct real roots,

$b^2 - 4ac > 0$

$6^2 - 4(2)(-k) > 0$

$36 + 8k > 0$

$\underline{\underline{k > -4.5}}$

6. Since there are no real solutions,

$b^2 - 4ac < 0$

$4^2 - 4(1)(-2m) < 0$

$16 + 8m < 0$

$8m < -16$

$\underline{\underline{m < -2}}$

7.

(a) Discriminant $= (-3)^2 - 4(p)(5)$

$= 9 - 20p$

(b) If there are no solutions,

$b^2 - 4ac < 0$

$9 - 20p < 0$

$9 < 20p$

$\underline{\underline{p > \frac{9}{20}}}$

8. If there are equal roots (repeated real roots),

$b^2 - 4ac = 0$

$(-6)^2 - 4(q)(q) = 0$

$36 - 4q^2 = 0$

$q^2 = 9$

$\therefore \underline{\underline{q = \pm 3}}$

Exercise B

You do not need to find the coordinates of the turning point unless you are asked to find them.

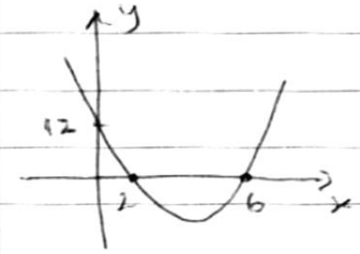
1. x-intercepts:

$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

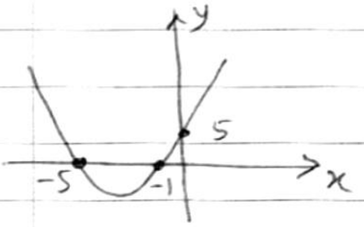
$x = 6, 2$

y-intercept $= 12$



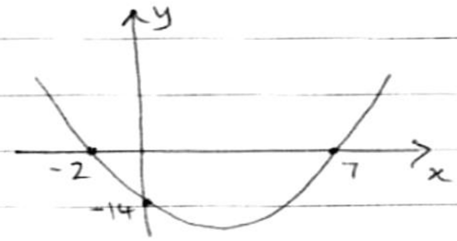
2. $(x+5)(x+1) = 0$

$x = -5, -1$



3. $(x-7)(x+2) = 0$

$x = 7, -2$

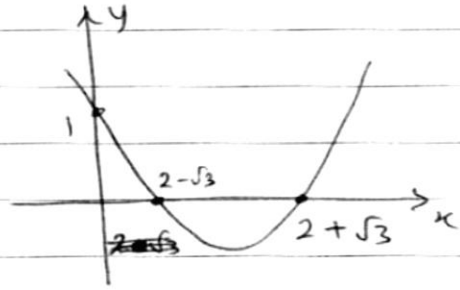


4. $x^2 - 4x + 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{16 - 4(1)(1)}}{2(1)}$$

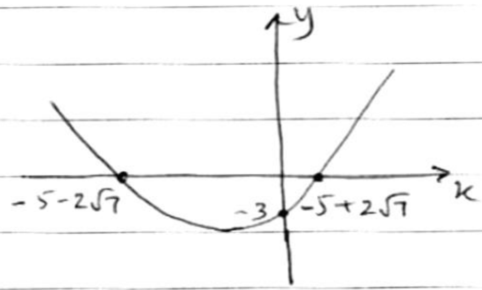
$x = 2 + \sqrt{3},\ 2 - \sqrt{3}$



5. $x^2 + 10x - 3 = 0$

$$x = \frac{-10 \pm \sqrt{100 - 4(1)(-3)}}{2(1)}$$

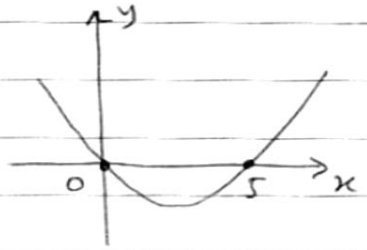
$x = -5 + 2\sqrt{7},\ -5 - 2\sqrt{7}$



6. $x^2 - 5x = 0$

$x(x-5) = 0$

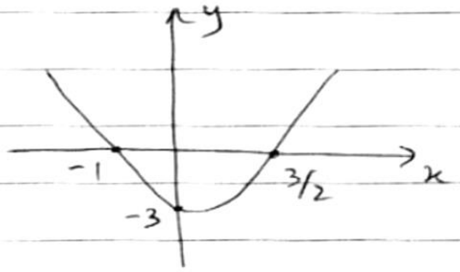
$x = 0, 5$



7. $2x^2 - x - 3 = 0$

$(2x-3)(x+1) = 0$

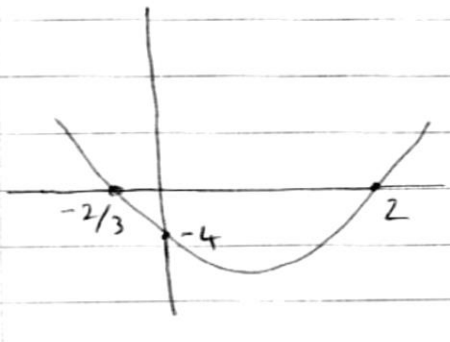
$x = \frac{3}{2}, -1$



8. $3x^2 - 4x - 4 = 0$

$(x-2)(3x+2) = 0$

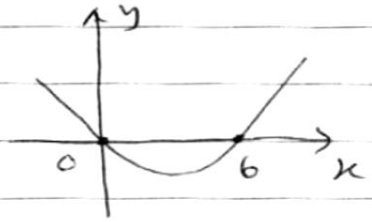
$x = 2, -2/3$



9. $2x^2 - 12x = 0$

$2x(x-6) = 0$

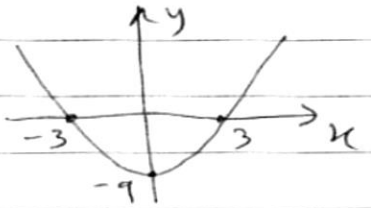
$x = 0, 6$



10. $x^2 - 9 = 0$

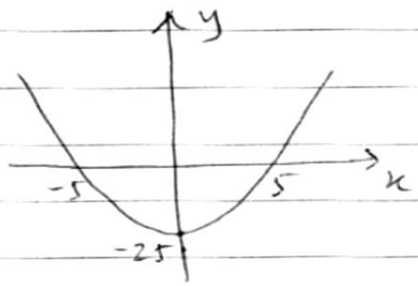
$x^2 = 9$

$x = \pm 3$



11. $x^2 - 25 = 0$

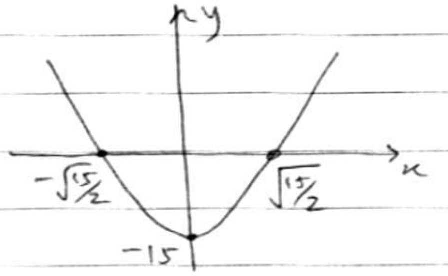
$x = \pm 5$



12. $2x^2 - 15 = 0$

$2x^2 = 15$

$x = \pm\sqrt{\frac{15}{2}}$



13. x - intercepts:

$x^2 + 2x + 10 = 0$

$x = \dfrac{-2 \pm \sqrt{4 - 4(1)(10)}}{2(1)}$

$x = \dfrac{-2 \pm \sqrt{-36}}{2}$

No real roots.

$\therefore$ No x-intercepts.

y-intercept $= 10$.

If there are no x-intercepts, you should find out the turning points to correctly sketch the graph, because it can tell you whether the turning point is to the left or right of the y-axis.

Turning point:

$y = x^2 + 2x + 10$

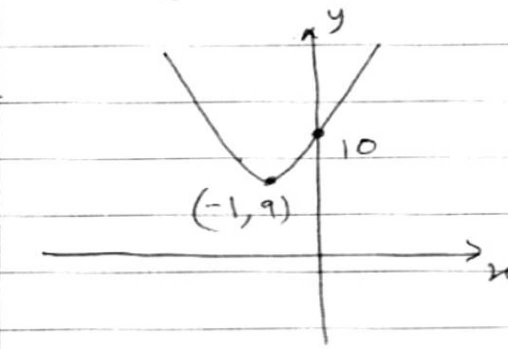
$= (x+1)^2 - 1 + 10$

$= (x+1)^2 + 9$

$x + 1 = 0$ $\qquad$ $y = +9$

$x = -1$

$(-1, 9)$



14.

$b^2 - 4ac = (-1)^2 - 4(1)(5)$

$= -19 < 0$

$\therefore x^2 - x + 5 = 0$ has no real roots. Hence the graph has no x-intercepts

y-intercept $= 5$

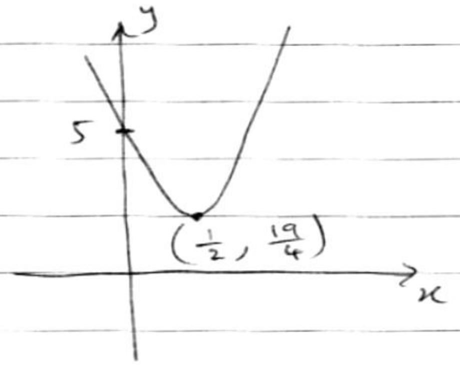
Turning point:

$y = x^2 - x + 5$

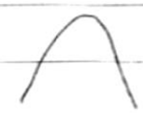
$= \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + 5$

$= \left(x - \frac{1}{2}\right)^2 + \frac{19}{4}$

$\left(\frac{1}{2}, +\frac{19}{4}\right)$



15. Shape :



$b^2 - 4ac = 4^2 - 4(-1)(-20)$

$= -64 < 0$

$\therefore$ No x-intercepts.

y-intercept $= -20$

Turning point :

$y = -x^2 + 4x - 20$

$= -1\left[x^2 - 4x\right] - 20$

$= -1\left[(x-2)^2 - 4\right] - 20$

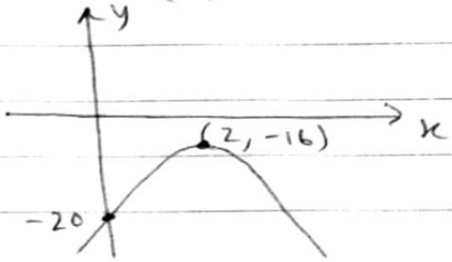
$= -(x-2)^2 + 4 - 20$

$= -(\underline{x-2})^2 - 16$

$x - 2 = 0$ $\quad y = -16$

$x = 2$

$(2, -16)$



16. $b^2 - 4ac = 3^2 - 4(-2)(-15)$

$= -111 < 0$

$\therefore$ No x intercepts.

y-intercept $= -15$

Shape :

Turning point :

$y = -2x^2 + 3x - 15$

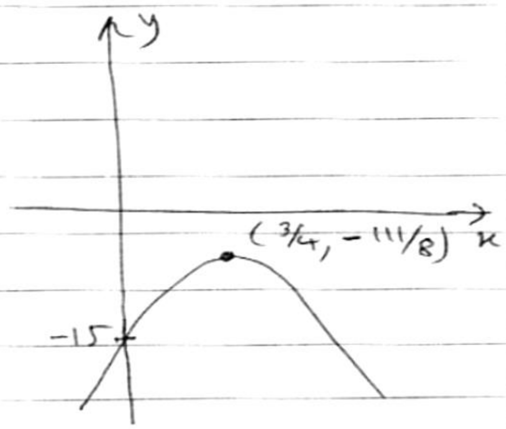
$= -2\left[x^2 - \frac{3}{2}x\right] - 15$

$= -2\left[\left(x - \frac{3}{4}\right)^2 - \frac{9}{16}\right] - 15$

$= -2\left(x - \frac{3}{4}\right)^2 + \frac{9}{8} - 15$

$= -2\left(x - \frac{3}{4}\right)^2 - \frac{111}{8}$

$\left(\frac{3}{4}, \frac{-111}{8}\right)$



Exercise C

1. Let $y = x^2$

Then, $y^2 = x^2 \times x^2 = x^4$

$\therefore y^2 - 5y + 4 = 0$

$(y-1)(y-4) = 0$

$y = 1$ or $y = 4$

$x^2 = 1$ or $x^2 = 4$

$x = \pm 1$ or $x = \pm 2$

2. Let $y = x^2$

$y^2 - 3y - 10 = 0$

$(y-5)(y+2) = 0$

$y = 5$ or $y = -2$

$x^2 = 5$ or $x^2 = -2$

$x = \pm 5$ No real roots.

3. Let $y = x^3$:

Then, $y^2 = x^3 \times x^3 = x^6$

$\therefore 2y^2 + 9y + 4 = 0$

$2y^2 + 8y + y + 4 = 0$

$2y(y+4) + 1(y+4) = 0$

$(y+4)(2y+1) = 0$

$y = -4$ or $y = -\frac{1}{2}$

$x^3 = -4$ or $x^3 = -\frac{1}{2}$

$x = \sqrt[3]{-4}$ or $x = \sqrt[3]{-\frac{1}{2}}$

4. Let $y = x^{1/4}$

Then $y^2 = x^{1/4} \times x^{1/4}$

$= x^{1/2}$

$\therefore 2y^2 - y - 3 = 0$

$2y^2 - 3y + 2y - 3 = 0$

$y(2y-3) + 1(2y-3) = 0$

$(2y-3)(y+1) = 0$

$y = \frac{3}{2}$ or $y = -1$

$x^{1/4} = \frac{3}{2}$ or $x^{1/4} = -1$

$x = \left(\frac{3}{2}\right)^4$ or $x = (-1)^4$

$x = \frac{81}{16}$ or $x = 1$

5. Let $y = x^{1/6}$

Then $y^2 = x^{1/6} \times x^{1/6}$

$= x^{1/3}$

$\therefore 3y^2 - 9y = 0$

$3y(y-3) = 0$

$y = 0$ or $y = 3$

$x^{1/3} = 0$ or $x^{1/3} = 3$

$x = 0^3$ or $x = 3^3$

$x = 0$ or $x = 27$