Answers - Coordinate Geometry - Straight Line Graphs

1.

Question number	Scheme Marks
.(a)	$m = \frac{8-2}{11+1} \quad (= \frac{1}{2})$ M1 A1 $y - 2 = \frac{1}{2}(x - -1)$ or $y - 8 = \frac{1}{2}(x - 11)$ o.e. M1 $y = \frac{1}{2}x + \frac{5}{2}$ accept exact equivalents
e.g. $\frac{6}{12}$	A1c.a.o. (4)
(b)	Gradient of $l_2 = -2$ M1 Equation of l_2: $y - 0 = -2(x - 10)$ $[y = -2x + 20]$ M1 $\frac{1}{2}x + \frac{5}{2} = -2x + 20$ M1 $\underline{x = 7 \text{ and } y = 6}$ depend on all 3
Ms	A1, A1 (5)
(c)	$RS^2 = (10-7)^2 + (0-6)^2 (= 3^2 + 6^2)$ M1 $RS = \sqrt{45} = 3\sqrt{5}$ (*) A1c.s.o. (2)
(d)	$PQ = \sqrt{12^2 + 6^2}, = 6\sqrt{5}$ or $\sqrt{180}$ or $PS = 4\sqrt{5}$ and $SQ = 2\sqrt{5}$ M1,A1 Area $= \frac{1}{2} PQ \times RS = \frac{1}{2} 6\sqrt{5} \times 3\sqrt{5}$ dM1

$\underline{= 45}$

A1 c.a.o. (4)

2.

(a)	$y-(-4)=\frac{1}{3}(x-9)$ or $\frac{y-(-4)}{x-9}=\frac{1}{3}$	M1 A1
	$3y-x+21=0$ (o.e.) (condone 3 terms with integer coefficients e.g. $3y+21=x$)	A1
		(3)
(b)	Equation of l_2 is: $y=-2x$ (o.e.)	B1
	Solving l_1 and l_2 : $-6x-x+21=0$	M1
	p is point where $x_p=3$, $y_p=-6$ x_p or y_p	A1
	y_p or x_p	A1f.t. $(-2x)$
		(4)
(c)	(l_1 is $y=\frac{1}{3}x-7$) C is (0, -7) or OC = 7	B1f.t.
	Area of $\Delta OCP=\frac{1}{2}OC\times x_p$, $=\frac{1}{2}\times 7\times 3=10.5$ or $\frac{21}{2}$	M1 A1c.a.o.
	By Integration: M1 for $\pm\int_0^{x_p}(l_1-l_2)\,dx$,	(3)
	B1 ft for correct integration (follow through their l_1) , then A1cao.	**(10)**

3.

(a) $y=-\frac{3}{2}x(+4)$ Gradient $=-\frac{3}{2}$	M1 A1	(2)
(b) $3x+2=-\frac{3}{2}x+4$ $x=\ldots,\ \underline{\frac{4}{9}}$	M1, A1	
$y=3\left(\frac{4}{9}\right)+2=\underline{\frac{10}{3}}\ \left(=3\frac{1}{3}\right)$	A1	(3)
(c) Where $y=1$, $l_1: x_A=-\frac{1}{3}$ $l_2: x_B=2$ M: Attempt one of these	M1 A1	
Area $=\frac{1}{2}(x_B-x_A)(y_P-1)$	M1	
$=\frac{1}{2}\times\frac{7}{3}\times\frac{7}{3}=\underline{\frac{49}{18}=2\frac{13}{18}}$ o.e.	A1	(4)

4.

(a)	$p = 15, \quad q = -3$	B1 B1	(2)
(b)	Grad. of line *ADC*: $m = -\frac{5}{7}$, Grad. of perp. line $= -\frac{1}{m} \left(= \frac{7}{5} \right)$	B1, M1	
	Equation of l: $y - 2 = \frac{7}{5}(x - 8)$	M1 A1ft	
	$7x - 5y - 46 = 0$ (Allow rearrangements, e.g. $5y = 7x - 46$)	A1	(5)
(c)	Substitute $y = 7$ into equation of l and find $x = \ldots$	M1	
	$\frac{81}{7}$ or $11\frac{4}{7}$ (or exact equiv.)	A1	(2)

5.

(i)	Gradient DE = $-\frac{1}{2}$	B1 1	$-\frac{1}{2}$ (any working seen must be correct)
(ii)	$y - 3 = -\frac{1}{2}(x - 2)$	M1	Correct equation for straight line, any gradient, passing through F
		A1	$y - 3 = -\frac{1}{2}(x - 2)$ aef
	$x + 2y - 8 = 0$	A1 3	$x + 2y - 8 = 0$ (this form but can have fractional coefficients e.g. $\frac{1}{2}x + y - 4 = 0$
(iii)	Gradient EF = $\frac{4}{2}$ =2	B1	Correct supporting working must be seen
	$-\frac{1}{2} \times 2 = -1$	B1 2	Attempt to show that product of their gradients = - 1 o.e.
(iv)	DF=$\sqrt{4^2 + 3^2} = 5$	M1	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ used
		A1 2	5

(v)	DF is a diameter as angle DEF is a right angle.	B1	Justification that DF is a diameter
	Mid-point of DF <u>or</u> centre of circle is $(0, 1\frac{1}{2})$	B1	Mid-point of DF <u>or</u> centre of circle is $(0, 1\frac{1}{2})$
	Radius = 2.5	B1	Radius = 2.5
	$x^2 + (y - \left(\frac{3}{2}\right)^2) = \left(\frac{5}{2}\right)^2$ $x^2 + y^2 - 3y + \frac{9}{4} = \frac{25}{4}$	B1 $\sqrt{}$	$x^2 + (y - \left(\frac{3}{2}\right)^2) = \left(\frac{5}{2}\right)^2$
	$x^2 + y^2 - 3y - 4 = 0$	B1 5	$x^2 + y^2 - 3y - 4 = 0$ obtained correctly with at least one line of intermediate working. <u>SR</u> For working that only shows $x^2 + y^2 - 3y - 4 = 0$ is equation for a circle with centre $(0, 1\frac{1}{2})$ B1 radius 2.5 B1

6.

(i)	$y = \frac{4}{3}x + \frac{5}{3}$ gradient $= \frac{4}{3}$	B1 1	$\frac{4}{3}$ or 1.33 or better
(ii)	gradient of $\perp^r = -\frac{3}{4}$	B1 ft	$-\frac{3}{4}$ seen or implied
	$y - 2 = -\frac{3}{4}(x - 1)$	M1	Attempts equation of straight line through (1, 2) with any gradient
	$4y + 3x = 11$	A1	$y - 2 = -\frac{3}{4}(x - 1)$
		A1 4	$3x + 4y - 11 = 0$ (not aef)
(iii)	$P\left(-\frac{5}{4}, 0\right)$	B1	$\left(-\frac{5}{4}, 0\right)$ seen or implied
	$Q\left(0, \frac{11}{4}\right)$	B1 ft	$\left(0, \frac{11}{4}\right)$ seen or implied (from a straight line equation in (ii))
	$\left(-\frac{5}{8}, \frac{11}{8}\right)$	B1 ft 3	$\left(-\frac{5}{8}, \frac{11}{8}\right)$ aef

(iv)	$\sqrt{\left(\frac{5}{4}\right)^2+\left(\frac{11}{4}\right)^2}$	M1		Correct method to find line length using Pythagoras' theorem
	$\frac{\sqrt{146}}{4}$	A1		$\sqrt{\left(\frac{5}{4}\right)^2+\left(\frac{11}{4}\right)^2}$
		A1	3	$\frac{\sqrt{146}}{4}$

7.

(i)	Length AC = $\sqrt{(8-5)^2+(2-1)^2}$	M1		Uses $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
	$=\sqrt{3^2+1^2}$			
	$=\sqrt{10}$	A1		$\sqrt{10}$ ($\pm\sqrt{10}$ scores A0)
	Length AB = $\sqrt{(p-5)^2+(7-1)^2}$	A1		$\sqrt{(p-5)^2+(7-1)^2}$
	$=\sqrt{(p-5)^2+36}$			
	$\sqrt{(p-5)^2+36}=2\sqrt{10}$	M1		AB = 2AC (with algebraic expression) used
	$p^2-10p+25+36=40$			
	$p^2-10p+21=0$	M1		Obtains 3 term quadratic = 0 suitable for solving <u>or</u> $(p-5)^2=4$
	$(p-7)(p-3)=0$			
	$p=7,3$	A1		$p=7$
		A1	7	$p=3$
				SR <u>If no working seen</u>, and one correct value found, award B2 in place of the final 4 marks in part (i)
(ii)	$7=3x-14$	M1		Correct method to find x
	$x=7$	A1		$x=7$
	(5, 1) (7, 7)	M1		Use $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$
	Mid-point (6, 4)	A1√	**4**	(6, 4) or correct midpoint for their AB
				<u>Alternative method</u> y coordinate of midpoint $=4$ M1 A1 sub 4 into equation of line M1 obtains $x=6$ A1

8.

Question		Answer	Marks	Guidance	
	(i)	grad AB = $\frac{7-1}{4-2}$ oe or 3	M1		
		$y-7$ = their m $(x-4)$ or $y-1$ = their m $(x-2)$	M1	or use of y = their gradient $x+c$ with coords of A or B or M2 for $\frac{y-1}{7-1}=\frac{x-2}{4-2}$ o.e.	allow step methods used or eg M1 for $7=4m+c$ and $1=2m+c$ then M1 for correctly finding one of m and c
		$y=3x-5$ oe	A1	accept equivalents if simplified eg $3x-y=5$ allow B3 for correct eqn www	allow A1 for $c=-5$ oe if $y=3x+c$ oe already seen B2 for eg $y-1=3(x-2)$
			[3]		
	(ii)	showing grad BC = $\frac{2-1}{-1-2}=-\frac{1}{3}$ oe	B1	may be calculation or showing on diagram	
		and $-1/3\times 3=-1$ or grad BC is neg reciprocal of grad AB, [so 90°]	B1	may be earned for statement / use of $m_1m_2=-1$ oe, even if first B1 not earned for B1+B1, must be fully correct, with 3 as gradient in (i)	eg allow 2[nd] B1 for statement grad BC $=-1/3$ with no working if first B1 not earned
		<u>or</u> for finding AC or AC^2 independently of AB and BC	<u>or</u> B1	working needed such as $AC^2=5^2+5^2=50$	condone any confusion between squares and square roots etc for first B1 and for two M1s eg $AC=25+25=\sqrt{50}$
		for correctly showing $AC^2=BC^2+AB^2$ oe	B1	working needed using correct notation such as $BC^2=3^2+1^2=10$; $AB^2=6^2+2^2=40$, $40+10=50$ [hence $AC^2=BC^2+AB^2$]	accept eg 3 and 1 shown on diagram and $BC^2=10$ etc 0 for eg $\sqrt{40}+\sqrt{10}=\sqrt{50}$

9.

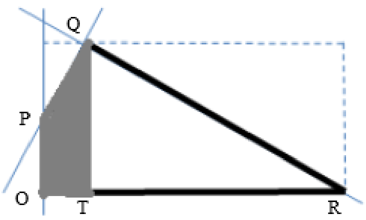
Question Number	Scheme		Marks
(a)	$\frac{5}{4}$ oe	$\frac{5}{4}$ or exact equivalents such as 1.25 but **not** $\frac{5}{4}x$.	B1
			(1)
(b)	$y=\frac{5}{4}x+c$	Uses a line with a parallel gradient $\frac{5}{4}$ oe or their gradient from part (a). Evidence is $y=\text{"}\frac{5}{4}\text{"}x+c$ or similar.	M1
	$12,5\Rightarrow 5='\frac{5}{4}'\times 12+c\Rightarrow c=..$	Method of finding an equation of a line with numerical gradient and passing through $12,5$. Score even for the perpendicular line. Must be seen in part (a).	M1
	$y=\frac{5}{4}x-10$	Correct equation. Allow $-\frac{40}{4}$ for -10	A1
			(3)
(c)	$B=\ \ 0,-10$	$B=\ \ 0,-10$ Follow through on their 'c'. Allow also if -10 is marked in the correct place on the diagram. Allow $x=0$, $y=-10$ (the $x=0$ may be seen "embedded" but not just $y=-10$ with no evidence that $x=0$)	B1ft
	$C=\ \ 8,0$	$C=\ \ 8,0$ Correct coordinates. Allow also if 8 is marked in the correct place on the diagram. Allow $y=0$, $x=8$ (the $y=0$ may be seen "embedded" but not just $x=8$ with no evidence that $y=0$)	B1
	Do not penalise lack of "0" twice so penalise it at the first occurrence but check the diagram if necessary.		
			(2)

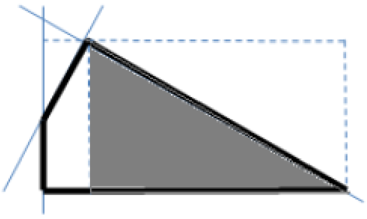
(d)	Area of Parallelogram = $3+'10' \times '8'$	Uses area of parallelogram = $bh = 3+'10' \times "8"$ Follow through on their 10 and their 8	M1
	$= 104$	cao	A1
	Correct answer only scores both marks		**(2)**

10.

Question Number	Scheme	Notes	Marks
(a)	l_1 : passes through (0, 2) and (3, 7) l_2 : goes through (3, 7) and is perpendicular to l_1		
	Gradient of l_1 is $\frac{7-2}{3-0}\left(=\frac{5}{3}\right)$	$m(l_1)=\frac{7-2}{3-0}$. Allow un-simplified. May be implied.	B1
	$m(l_2) = -1 \div their\ \frac{5}{3}$	Correct application of perpendicular gradient rule	M1
	$y-7 = "-\frac{3}{5}"(x-3)$ or $y = "-\frac{3}{5}"x + c,\ 7 = "-\frac{3}{5}"(3)+c \Rightarrow c = \frac{44}{5}$	M1: Uses $y-7=m(x-3)$ with their <u>**changed**</u> gradient or uses $y=mx+c$ with (3, 7) and their <u>**changed**</u> gradient to find a value for c A1ft: Correct ft equation for their perpendicular gradient **(this is dependent on both M marks)**	M1A1ft
	$3x+5y-44=0$	Any positive or negative integer multiple. Must be seen in (a) and must include "= 0".	A1
			[5]
(b)	When $y=0$ $x=\frac{44}{3}$	M1: Puts $y=0$ and finds a value for x from their equation A1: $x=\frac{44}{3}\left(\text{or } 14\frac{2}{3} \text{ or } 14.\dot{6}\right)$ or exact equivalent. ($y=0$ not needed)	M1 A1
	Condone 3x – 5y – 44 = 0 only leading to the correct answer and condone coordinates written as (0, 44/3) but allow recovery in (c)		
			[2]

(c)			
	$OPQT = \frac{1}{2}(2+7)\times 3$ or $TRQ = \frac{1}{2}\times 7\times\left(\frac{44}{3}-3\right)$	M1: Correct method for *OPQT* or *TRQ* A1ft: $OPQT = \frac{1}{2}(2+7)\times 3$ or $TRQ=\frac{1}{2}\times 7\times\left(\frac{44}{3}-3\right)$	M1A1ft
	$\frac{1}{2}(2+7)\times 3 + \frac{1}{2}\times 7\times\left(\frac{44}{3}-3\right)$	dM1: Correct numerical combination of areas that have been calculated correctly A1: **Fully Correct** numerical expression for the area *ORQP*	dM1A1
	$54\frac{1}{3}$	Any exact equivalent e.g. $\frac{163}{3}, \frac{326}{6}, 54.\dot{3}$	A1





$$\frac{1}{2}\times(7+2)\times 3 + \frac{1}{2}\times\frac{"35"}{3}\times 7$$
$$=\frac{27}{2}+\frac{245}{6}=\frac{326}{6}$$

11.

Question			Answer	Marks	Guidance	
	(i)		$AB^2 = (1-(-1))^2 + (5-1)^2$	M1	oe, or square root of this; condone poor notation re roots; condone $(1+1)^2$ instead of $(1-(-1))^2$ allow M1 for vector AB = $\begin{pmatrix}-2\\-4\end{pmatrix}$, condoning poor notation, or triangle with hyp AB and lengths 2 and 4 correctly marked	
			$BC^2 = (3-(-1))^2 + (-1-1)^2$	M1	oe, or square root of this; condone poor notation re roots; condone $(3+1)^2$ instead of $(3-(-1))^2$ oe allow M1 for vector BC = $\begin{pmatrix}4\\-2\end{pmatrix}$, condoning poor notation, or triangle with hyp BC and lengths 4 and 2 correctly marked	
			shown equal eg $AB^2 = 2^2 + 4^2$ [=20] and $BC^2 = 4^2 + 2^2$ [=20] with correct notation for final comparison	A1	or statement that AB and BC are each the hypotenuse of a right-angled triangle with sides 2 and 4 so are equal SC2 for just $AB^2 = 2^2 + 4^2$ and $BC^2 = 4^2 + 2^2$ (or roots of these) with no clearer earlier working; condone poor notation	eg A0 for AB = 20 etc
				[3]		

Question			Answer	Marks	Guidance	
	(ii)		[grad. of AC =] $\frac{5-(-1)}{1-3}$ or $\frac{6}{-2}$ oe	M1	award at first step shown even if errors after	
			[grad. of BD =] $\frac{5-1}{11-(-1)}$ or $\frac{4}{12}$ oe	M1		if one or both of grad AC = -3 and grad BD = 1/3 seen without better working for both gradients, award one M1 only. For M1M1 it must be clear that they are obtained independently
			showing or stating product of gradients = −1 or that one gradient is the negative reciprocal of the other oe	B1	eg accept $m_1 \times m_2 = -1$ or 'one gradient is negative reciprocal of the other' B0 for 'opposite' used instead of 'negative' or 'reciprocal'	may be earned independently of correct gradients, but for all 3 marks to be earned the work must be fully correct
				[3]		

Question			Answer	Marks	Guidance	
	(iii)		midpoint E of AC = (2, 2) www	B1	condone missing brackets for both B1s	0 for ((5+ −1)/2, (1+3)/2) = (2, 2)
			eqn BD is $y = \frac{1}{3}x + \frac{4}{3}$ oe	M1	accept any correct form isw or correct ft their gradients or their midpt F of BD this mark will often be gained on the first line of their working for BD	may be earned using (2, 2) but then must independently show that B or D or (5, 3) is on this line to be eligible for A1
			eqn AC is $y = -3x + 8$ oe	M1	accept any correct form isw or correct ft their gradients or their midpt E of AC this mark will often be gained on the first line of their working for AC [see appendix for alternative methods instead showing E is on BD for this M1]	if equation(s) of lines are seen in part ii, allow the M1s if seen/used in this part
			using both lines and obtaining intersection E is (2, 2) (NB must be independently obtained from midpt of AC)	A1		[see appendix for alternative ways of gaining these last two marks in different methods]
			midpoint F of BD = (5,3)	B1	this mark is often earned earlier see the appendix for some common alternative methods for this question; for all methods, for A1 to be earned, all work for the 5 marks must be correct	for all methods show annotations M1 B1 etc then omission mark or A0 if that mark has not been earned
				[5]		

12.

(i)	grad CD = $\frac{5-3}{3-(-1)}$ $\left[= \frac{2}{4} \text{ o.e.}\right]$ isw grad AB = $\frac{3-(-1)}{6-(-2)}$ or $\frac{4}{8}$ isw same gradient so parallel www	**M1** **M1** **A1**	NB needs to be obtained independently of grad AB must be explicit conclusion mentioning 'same gradient' or 'parallel' if M0, allow **B1** for 'parallel lines have same gradient' o.e.
(ii)	[BC^2=] $3^2 + 2^2$ [BC^2 =] 13 showing $AD^2 = 1^2 + 4^2$ [=17] [$\neq BC^2$] isw	**M1** **A1** **A1**	accept $(6-3)^2 + (3-5)^2$ o.e. or [BC =] $\sqrt{13}$ or [AD =] $\sqrt{17}$ or equivalent marks for finding AD or AD^2 first alt method: showing AC ≠ BD – mark equivalently
(iii)	[BD eqn is] $y = 3$ eqn of AC is $y - 5 = 6/5 \times (x - 3)$ o.e [$y = 1.2x + 1.4$ o.e.] M is (4/3, 3) o.e. isw	**M1** **M2** **A1**	eg allow for 'at M, $y = 3$' or for 3 subst in eqn of AC or **M1** for grad AC = 6/5 o.e. (accept unsimplified) and M1 for using their grad of AC with coords of A(−2, −1) or C (3, 5) in eqn of line or **M1** for 'stepping' method to reach M allow : at M, $x = 16/12$ o.e. [eg =4/3] isw A0 for 1.3 without a fraction answer seen
(iv)	midpt of BD = (5/2, 3) or equivalent simplified form cao midpt AC = (1/2, 2) or equivalent simplified form cao or 'M is 2/3 of way from A to C' conclusion 'neither diagonal bisects the other'	**M1** **M1** **A1**	or showing BM ≠ MD oe [BM = 14/3, MD = 7/3] or showing AM ≠ MC or $AM^2 \neq MC^2$ in these methods A1 is dependent on coords of M having been obtained in part (iii) or in this part; the coordinates of M need not be correct; it is also dependent on midpts of both AC and BD attempted, at least one correct alt method: show that mid point of BD does not lie on AC (M1) and vice-versa (M1), A1 for both and conclusion