Coordinate Geometry - Straight Line Graphs

1.

The line l_1 passes through the points $P(-1, 2)$ and $Q(11, 8)$.

(*a*) Find an equation for l_1 in the form $y = mx + c$, where m and c are constants.

(4)

The line l_2 passes through the point $R(10, 0)$ and is perpendicular to l_1. The lines l_1 and l_2 intersect at the point S.

(*b*) Calculate the coordinates of S.

(5)

(*c*) Show that the length of RS is $3\sqrt{5}$.

(2)

(*d*) Hence, or otherwise, find the exact area of triangle PQR.

(4)

2.

The line l_1 passes through the point $(9, -4)$ and has gradient $\frac{1}{3}$.

(*a*) Find an equation for l_1 in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

The line l_2 passes through the origin O and has gradient -2. The lines l_1 and l_2 intersect at the point P.

(*b*) Calculate the coordinates of P.

(4)

Given that l_1 crosses the y-axis at the point C,

(*c*) calculate the exact area of $\triangle OCP$.

(3)

3.

The line l_1 has equation $y = 3x + 2$ and the line l_2 has equation $3x + 2y - 8 = 0$.

(a) Find the gradient of the line l_2.

(2)

The point of intersection of l_1 and l_2 is P.

(b) Find the coordinates of P.

(3)

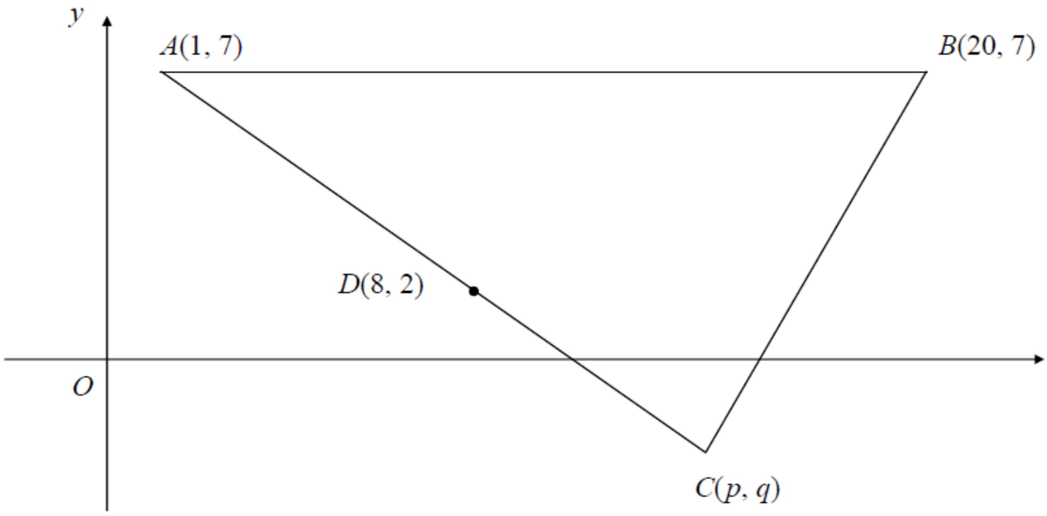
The lines l_1 and l_2 cross the line $y = 1$ at the points A and B respectively.

(c) Find the area of triangle ABP.

(4)

4.

Figure 2



The points $A(1, 7)$, $B(20, 7)$ and $C(p, q)$ form the vertices of a triangle ABC, as shown in Figure 2. The point $D(8, 2)$ is the mid-point of AC.

(*a*) Find the value of p and the value of q.

(2)

The line l, which passes through D and is perpendicular to AC, intersects AB at E.

(*b*) Find an equation for l, in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

(*c*) Find the exact x-coordinate of E.

(2)

5.

The points D, E and F have coordinates $(-2, 0)$, $(0, -1)$ and $(2, 3)$ respectively.

(i) Calculate the gradient of DE. [1]

(ii) Find the equation of the line through F, parallel to DE, giving your answer in the form $ax + by + c = 0$. [3]

(iii) By calculating the gradient of EF, show that DEF is a right-angled triangle. [2]

(iv) Calculate the length of DF. [2]

(v) Use the results of parts **(iii)** and **(iv)** to show that the circle which passes through D, E and F has equation $x^2 + y^2 - 3y - 4 = 0$. [5]

6.

(i) Find the gradient of the line l_1 which has equation $4x - 3y + 5 = 0$. [1]

(ii) Find an equation of the line l_2, which passes through the point (1, 2) and which is perpendicular to the line l_1, giving your answer in the form $ax + by + c = 0$. [4]

The line l_1 crosses the x-axis at P and the line l_2 crosses the y-axis at Q.

(iii) Find the coordinates of the mid-point of PQ. [3]

(iv) Calculate the length of PQ, giving your answer in the form $\frac{\sqrt{a}}{b}$, where a and b are integers. [3]

7.

The points A, B and C have coordinates (5, 1), (p, 7) and (8, 2) respectively.

(i) Given that the distance between points A and B is twice the distance between points A and C, calculate the possible values of p. [7]

(ii) Given also that the line passing through A and B has equation $y = 3x - 14$, find the coordinates of the mid-point of AB. [4]

8.

Point A has coordinates (4, 7) and point B has coordinates (2, 1).

(i) Find the equation of the line through A and B. **[3]**

(ii) Point C has coordinates (–1, 2). Show that angle ABC = 90° and calculate the area of triangle ABC. **[5]**

(iii) Find the coordinates of D, the midpoint of AC.

Explain also how you can tell, without having to work it out, that A, B and C are all the same distance from D. **[3]**

Question 9 is on the next page.

9.

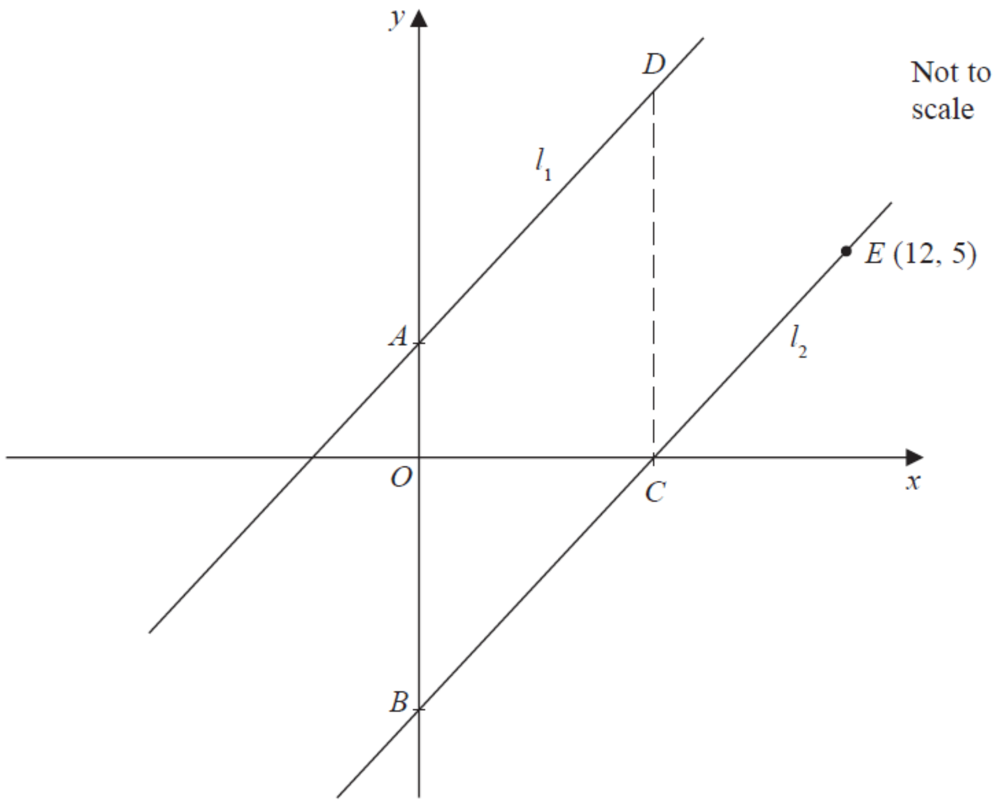


Figure 2

Figure 2 shows the straight line l_1 with equation $4y = 5x + 12$

(a) State the gradient of l_1

(1)

The line l_2 is parallel to l_1 and passes through the point E (12, 5), as shown in Figure 2.

(b) Find the equation of l_2. Write your answer in the form $y = mx + c$, where m and c are constants to be determined.

(3)

The line l_2 cuts the x-axis at the point C and the y-axis at the point B.

(c) Find the coordinates of

(i) the point B,

(ii) the point C.

(2)

The line l_1 cuts the y-axis at the point A.

The point D lies on l_1 such that $ABCD$ is a parallelogram, as shown in Figure 2.

(d) Find the area of $ABCD$.

(2)

10.

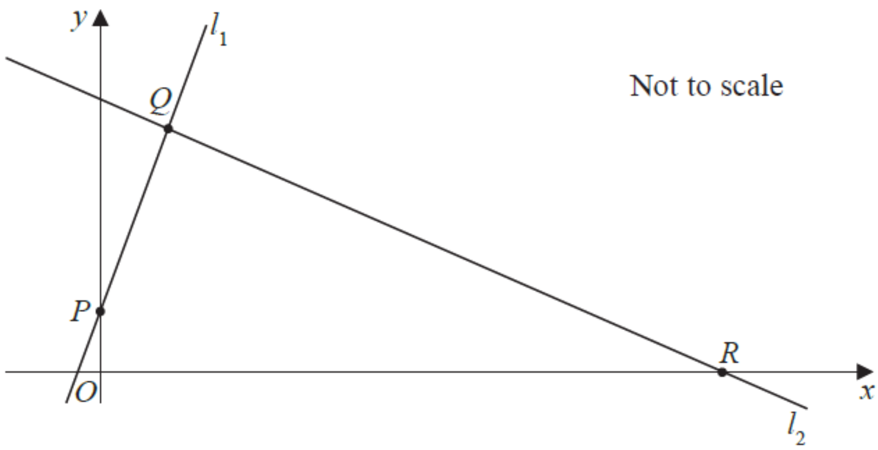


Figure 2

The points P (0, 2) and Q (3, 7) lie on the line l_1, as shown in Figure 2.

The line l_2 is perpendicular to l_1, passes through Q and crosses the x-axis at the point R, as shown in Figure 2.

Find

(a) an equation for l_2, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers,

(5)

(b) the exact coordinates of R,

(2)

(c) the exact area of the quadrilateral $ORQP$, where O is the origin.

(5)

11.

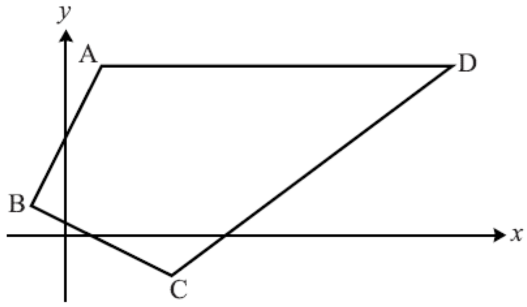


Fig. 10

Fig. 10 is a sketch of quadrilateral ABCD with vertices A (1, 5), B (–1, 1), C (3, –1) and D (11, 5).

(i) Show that AB = BC. **[3]**

(ii) Show that the diagonals AC and BD are perpendicular. **[3]**

(iii) Find the midpoint of AC. Show that BD bisects AC but AC does not bisect BD. **[5]**

12.

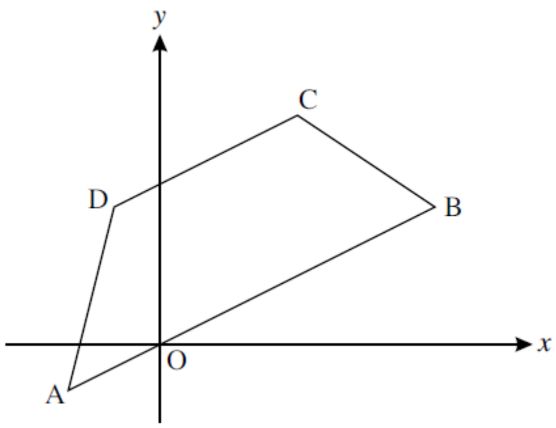


Fig. 10

Fig. 10 shows a trapezium ABCD. The coordinates of its vertices are A (−2, −1), B (6, 3), C (3, 5) and D (−1, 3).

(i) Verify that the lines AB and DC are parallel. **[3]**

(ii) Prove that the trapezium is not isosceles. **[3]**

(iii) The diagonals of the trapezium meet at M. Find the exact coordinates of M. **[4]**

(iv) Show that neither diagonal of the trapezium bisects the other. **[3]**