

Answers: Completing the Square

①

$$\begin{aligned} 1. (a) \quad & x^2 + 10x + 3 \\ & = (x + 5)^2 - 25 + 3 \\ & = \underline{\underline{(x + 5)^2 - 22}} \end{aligned}$$

$$\begin{aligned} (b) \quad & x^2 + 14x - 5 \\ & = (x + 7)^2 - 49 - 5 \\ & = \underline{\underline{(x + 7)^2 - 54}} \end{aligned}$$

$$\begin{aligned} (c) \quad & x^2 - 6x + 3 \\ & = (x - 3)^2 - 9 + 3 \\ & = \underline{\underline{(x - 3)^2 - 6}} \end{aligned}$$

$$\begin{aligned} (d) \quad & y^2 - 16y - 10 \\ & = (y - 8)^2 - 64 - 10 \\ & = \underline{\underline{(y - 8)^2 - 74}} \end{aligned}$$

$$\begin{aligned} (e) \quad & \cancel{y^2 + 8y + 1} \\ & y^2 + 8y - 1 \\ & = (y + 4)^2 - 16 - 1 \\ & = \underline{\underline{(y + 4)^2 - 17}} \end{aligned}$$

$$\begin{aligned} (f) \quad & x^2 - 20x + 7 \\ & = (x - 10)^2 - 100 + 7 \\ & = \underline{\underline{(x - 10)^2 - 93}} \end{aligned}$$

$$\begin{aligned} (g) \quad & x^2 + 4x - 7 \\ & = (x + 2)^2 - 4 - 7 \\ & = \underline{\underline{(x + 2)^2 - 11}} \end{aligned}$$

$$\begin{aligned} (h) \quad & x^2 + 10x \\ & = \underline{\underline{(x + 5)^2 - 25}} \end{aligned}$$

$$\begin{aligned} (i) \quad & y^2 + 18y \\ & = \underline{\underline{(y + 9)^2 - 81}} \end{aligned}$$

$$\begin{aligned} (j) \quad & x^2 - 12x \\ & = \underline{\underline{(x - 6)^2 - 36}} \end{aligned}$$

$$\begin{aligned} (k) \quad & x^2 + 6x \\ & = \underline{\underline{(x + 3)^2 - 9}} \end{aligned}$$

$$\begin{aligned} (l) \quad & x^2 + 14x \\ & = \underline{\underline{(x + 7)^2 - 49}} \end{aligned}$$

②

$$\begin{aligned} (a) \quad & 2x^2 - 20x - 6 \\ & = 2[x^2 - 10x - 3] \\ & = 2[(x - 5)^2 - 25 - 3] \\ & = 2[(x - 5)^2 - 28] \\ & = \underline{\underline{2(x - 5)^2 - 56}} \end{aligned}$$

$$\begin{aligned}
 (b) \quad & 2x^2 - 12x + 8 \\
 &= 2 [x^2 - 6x + 4] \\
 &= 2 [(x-3)^2 - 9 + 4] \\
 &= 2 [(x-3)^2 - 5] \\
 &= \underline{\underline{2(x-3)^2 - 10}}
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad & 2x^2 + 16x + 4 \\
 &= 2 [x^2 + 8x + 2] \\
 &= 2 [(x+4)^2 - 16 + 2] \\
 &= 2 [(x+4)^2 - 14] \\
 &= \underline{\underline{2(x+4)^2 - 28}}
 \end{aligned}$$

$$\begin{aligned}
 (d) \quad & 3x^2 + 12x + 9 \\
 &= 3 [x^2 + 4x + 3] \\
 &= 3 [(x+2)^2 - 4 + 3] \\
 &= 3 [(x+2)^2 - 1] \\
 &= \underline{\underline{3(x+2)^2 - 3}}
 \end{aligned}$$

$$\begin{aligned}
 (e) \quad & 5x^2 - 10x - 20 \\
 &= 5 [x^2 - 2x - 4] \\
 &= 5 [(x-1)^2 - 1 - 4] \\
 &= 5 [(x-1)^2 - 5] \\
 &= \underline{\underline{5(x-1)^2 - 25}}
 \end{aligned}$$

$$\begin{aligned}
 (f) \quad & 2x^2 - 8x \\
 &= 2 [x^2 - 4x] \\
 &= 2 [(x-2)^2 - 4] \\
 &= \underline{\underline{2(x-2)^2 - 8}}
 \end{aligned}$$

$$\begin{aligned}
 (g) \quad & 4x^2 + 24x \\
 &= 4 [x^2 + 6x] \\
 &= 4 [(x+3)^2 - 9] \\
 &= \underline{\underline{4(x+3)^2 - 36}}
 \end{aligned}$$

$$\begin{aligned}
 (h) \quad & 3x^2 - 18x \\
 &= 3 [x^2 - 6x] \\
 &= 3 [(x-3)^2 - 9] \\
 &= \underline{\underline{3(x-3)^2 - 27}}
 \end{aligned}$$

3

$$\begin{aligned}
 (a) \quad & x^2 + 9x + 2 \\
 &= \left(x + \frac{9}{2}\right)^2 - \frac{81}{4} + 2 \\
 &= \left(x + \frac{9}{2}\right)^2 - \frac{81}{4} + \frac{8}{4} \\
 &= \underline{\underline{\left(x + \frac{9}{2}\right)^2 - \frac{73}{4}}}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad & x^2 + 7x + 3 \\
 &= \left(x + \frac{7}{2}\right)^2 - \frac{49}{4} + 3
 \end{aligned}$$

$$= \left(x + \frac{7}{2}\right)^2 - \frac{49}{4} + \frac{12}{4}$$

$$= \left(x + \frac{7}{2}\right)^2 - \frac{37}{4}$$

(f) $x^2 - 5x$

$$= \left(x - \frac{5}{2}\right)^2 - \frac{25}{4}$$

(c) $x^2 - 11x + 8$

$$= \left(x - \frac{11}{2}\right)^2 - \frac{121}{4} + \frac{32}{4}$$

$$= \left(x - \frac{11}{2}\right)^2 - \frac{89}{4}$$

(g) $x^2 - 9x$

$$= \left(x - \frac{9}{2}\right)^2 - \frac{81}{4}$$

(d) $x^2 - 3x - 10$

$$= \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - 10$$

$$= \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{40}{4}$$

$$= \left(x - \frac{3}{2}\right)^2 - \frac{49}{4}$$

(h) $2x^2 + 10x - 6$

$$= 2 \left[x^2 + 5x - 3 \right]$$

$$= 2 \left[\left(x + \frac{5}{2}\right)^2 - \frac{25}{4} - 3 \right]$$

$$= 2 \left(x + \frac{5}{2}\right)^2 - \frac{25}{2} - 6$$

$$= 2 \left(x + \frac{5}{2}\right)^2 - \frac{25}{2} - \frac{12}{2}$$

$$= 2 \left(x + \frac{5}{2}\right)^2 - \frac{37}{2}$$

(e) $x^2 + x - 5$

$$= \left(x + \frac{1}{2}\right)^2 - \frac{1}{4} - 5$$

$$= \left(x + \frac{1}{2}\right)^2 - \frac{1}{4} - \frac{20}{4}$$

$$= \left(x + \frac{1}{2}\right)^2 - \frac{21}{4}$$

(i) $2x^2 - 6x + 4$

$$= 2 \left[x^2 - 3x \right] + 4$$

$$= 2 \left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} \right] + 4$$

$$= 2 \left(x - \frac{3}{2}\right)^2 - \frac{9}{2} + 4$$

$$= 2 \left(x - \frac{3}{2}\right)^2 - \frac{9}{2} + \frac{8}{2}$$

$$= 2 \left(x - \frac{3}{2}\right)^2 - \frac{1}{2}$$

(j) $2x^2 - 7x - 8$

$$= 2 \left[x^2 - \frac{7}{2}x \right] - 8$$

$$= 2 \left[\left(x - \frac{7}{4} \right)^2 - \frac{49}{16} \right] - 8$$

$$= 2 \left(x - \frac{7}{4} \right)^2 - \frac{49}{8} - 8$$

$$= 2 \left(x - \frac{7}{4} \right)^2 - \frac{49}{8} - \frac{64}{8}$$

$$= 2 \left(x - \frac{7}{4} \right)^2 - \frac{113}{8}$$

$$= 2 \left(x - \frac{7}{4} \right)^2 - \frac{137}{8}$$

(k) $2x^2 + 5x - 2$

$$= 2 \left[x^2 + \frac{5}{2}x \right] - 2$$

$$= 2 \left[\left(x + \frac{5}{4} \right)^2 - \frac{25}{16} \right] - 2$$

$$= 2 \left(x + \frac{5}{4} \right)^2 - \frac{25}{8} - 2$$

$$= 2 \left(x + \frac{5}{4} \right)^2 - \frac{25}{8} - \frac{16}{8}$$

$$= 2 \left(x + \frac{5}{4} \right)^2 - \frac{41}{8}$$

(l) $2x^2 - 9x - 7$

$$= 2 \left[x^2 - \frac{9}{2}x \right] - 7$$

$$= 2 \left[\left(x - \frac{9}{4} \right)^2 - \frac{81}{16} \right] - 7$$

$$= 2 \left(x - \frac{9}{4} \right)^2 - \frac{81}{8} - 7$$

$$= 2 \left(x - \frac{9}{4} \right)^2 - \frac{81}{8} - \frac{56}{8}$$