Mathematical Proof 2

(P) 1 Prove that when n is an integer and $1 \leq n \leq 6$, then $m = n + 2$ is not divisible by 10.

Hint You can try each integer for $1 \leq n \leq 6$.

(P) 2 Prove that every odd integer between 2 and 26 is either prime or the product of two primes.

(P) 3 Prove that the sum of two consecutive square numbers from 1^2 to 8^2 is an odd number.

(E/P) 4 Prove that all cube numbers are either a multiple of 9 or 1 more or 1 less than a multiple of 9. **(4 marks)**

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(P) 5 Find a counter-example to disprove each of the following statements:

a If n is a positive integer then $n^4 - n$ is divisible by 4.

b Integers always have an even number of factors.

c $2n^2 - 6n + 1$ is positive for all values of n.

d $2n^2 - 2n - 4$ is a multiple of 3 for all integer values of n.

(E/P) 6 A student is trying to prove that $x^3 + y^3 < (x + y)^3$.

The student writes:

> $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
> which is less than $x^3 + y^3$ since
> $3x^2y + 3xy^2 > 0$

a Identify the error made in the proof. **(1 mark)**

b Provide a counter-example to show that the statement is not true. **(2 marks)**

Problem-solving For part **b** you need to write down suitable values of x and y and show that they do not satisfy the inequality.

(E/P) 7 Prove that for all real values of x

$$(x + 6)^2 \geq 2x + 11$$

(3 marks)

(E/P) 8 Given that a is a positive real number, prove that:

$$a + \frac{1}{a} \geq 2$$

Watch out Remember to state how you use the condition that a is positive.

(2 marks)

(E/P) 9 **a** Prove that for any positive numbers p and q:

$$p + q \geq \sqrt{4pq}$$

(3 marks)

b Show, by means of a counter-example, that this inequality does not hold when p and q are both negative. **(2 marks)**

Problem-solving Use jottings and work backwards to work out what expression to consider.

E/P **10** It is claimed that the following inequality is true for all negative numbers x and y:

$$x + y \geqslant \sqrt{x^2 + y^2}$$

The following proof is offered by a student:

$x + y \geqslant \sqrt{x^2 + y^2}$
$(x + y)^2 \geqslant x^2 + y^2$
$x^2 + y^2 + 2xy \geqslant x^2 + y^2$
$2xy > 0$ which is true because x and y are both negative, so xy is positive.

a Explain the error made by the student. **(2 marks)**

b By use of a counter-example, verify that the inequality is not satisfied if both x and y are negative. **(1 mark)**

c Prove that this inequality is true if x and y are both positive. **(2 marks)**