19.7 Stellar luminosity

Specification reference: 5.5.2

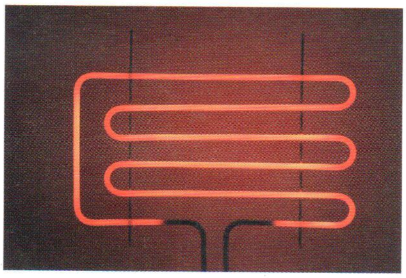
Learning outcomes

Demonstrate knowledge, understanding, and application of:

→ use of Wien's displacement law $\lambda_{max} \propto \frac{1}{T}$ to estimate the peak surface temperature of a star

→ luminosity L of a star; Stefan's law $L = 4\pi r^2 \sigma T^4$, where σ is the Stefan constant

→ use of Wien's displacement law and Stefan's law to estimate the radius of a star.



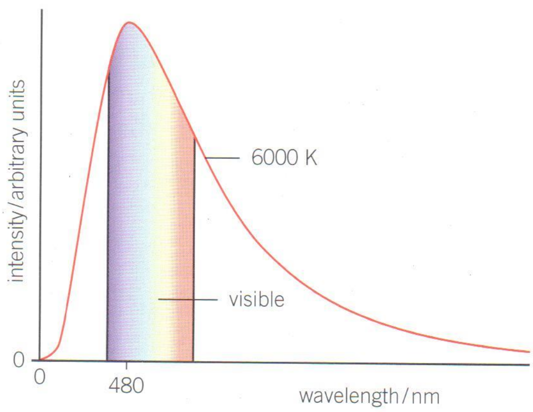
▲ **Figure 1** *A grill glows as it is heated, with a colour directly related to its temperature*

Red hot

You have already seen that the temperature of the surface of a star affects its colour, with the hottest stars glowing blue-white, and cooler stars a deeper shade of red. We can see the same effect when a metal is heated (Figure 1). At first the metal glows dull red, then reddish-orange as its temperature increases. If it does not melt, it will eventually glow white-hot if the temperature gets high enough.

Black-body radiation

At any given temperature above absolute zero, an object emits electromagnetic radiation of different wavelengths and different intensities. We can model a hot object as a **black body**. A black body is an idealised object that absorbs all the electromagnetic radiation that shines onto it and, when in thermal equilibrium, emits a characteristic distribution of wavelengths at a specific temperature. Figure 2 shows a graph of intensity against wavelength for electromagnetic radiation emitted by a black body at 6000 K.



▲ **Figure 2** *The intensity–wavelength graph for a black body at a temperature of 6000 K*

Wien's displacement law

Wien's displacement law is a simple law that relates the absolute temperature T of a black body to the peak wavelength λ_{max} at which the intensity is a maximum. It can be applied to most objects, from stars to filament lamps, and even to mammals. Wien's displacement law states that λ_{max} is inversely proportional to T, that is

$$\lambda_{max} \propto \frac{1}{T}$$

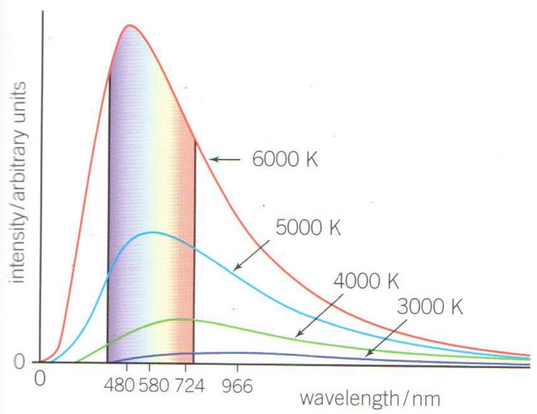
It follows that for any black-body emitter $\lambda_{max} T$ = constant. The value of this constant is 2.90×10^{-3} m K and it is known as Wien's constant. (You do not need to memorise the value of this constant for this course).

Many objects, including stars, can be modelled as approximate black bodies (Table 1). This helps scientists to determine temperatures of objects simply by analysing the electromagnetic radiation they emit.

▼ **Table 1** λ_{max} *values for a number of different objects*

Object	λ_{max} / m	T / K
Healthy human	9.4×10^{-6}	310
Wood fire	1.9×10^{-6}	1500
Betelgeuse (red supergiant)	8.5×10^{-7}	3400
Sun	5.0×10^{-7}	5800
Sirius B (white dwarf)	1.2×10^{-7}	25 000

As the temperature of an object changes, so does the distribution of the emitted wavelengths. The peak wavelength reduces as the temperature increases, and the peak of the intensity–wavelength graph becomes sharper (Figure 3).



▲ **Figure 3** *The distribution of wavelengths changes as the temperature of the black-body emitter changes*

Stefan's law

Stefan's law, also known as the Stefan–Boltzmann law, states that the total power radiated per unit surface area of a black body is directly proportional to the fourth power of the absolute temperature of the black body. The total power radiated by a star is called luminosity (see Topic 19.3, The Hertzsprung–Russell diagram).

According to Stefan's law, the equation for the luminosity L in watts (W) of a star is given by the equation

$$L = 4\pi r^2 \sigma T^4$$

where r is the radius of the star in metres (m), T is the surface absolute temperature of the star in kelvin (K), and σ is the **Stefan constant**, $5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$.

Stefan's law shows that the luminosity of a star is directly proportional:

- to its radius^2 ($L \propto r^2$)
- to its surface area ($L \propto 4\pi r^2$)
- to its surface absolute temperature^4 ($L \propto T^4$)