Upper and Lower Bounds

Exercise A

1. In a DIY store the height of a door is given as 195 cm to the nearest cm. Write down the upper bound for the height of the door.

2. A vet weighs a sick goat at 37 kg to the nearest kg. What is the least possible weight of the goat?

3. A cook's weighing scales weigh to the nearest 0·1 kg. What is the upper bound for the weight of a chicken which she weighs at 3·2 kg?

4. A surveyor using a laser beam device can measure distances to the nearest 0·1 m. What is the least possible length of a warehouse which he measures at 95·6 m?

5. In the county sports Jill was timed at 28·6 s for the 200 m. What is the upper bound for the time she could have taken?

6. Copy and complete the table.

	Measurement	Lower bound	Upper bound
(a)	temperature in a fridge = 2°C to the nearest degree		
(b)	mass of an acorn = 2·3 g to 1 d.p.		
(c)	length of telephone cable = 64 m to nearest m		
(d)	time taken to run 100 m = 13·6 s to nearest 0·1 s		

7. The length of a telephone is measured as 193 mm, to the nearest mm. The length lies between:

A	B	C
192 and 194 mm	192·5 and 193·5 mm	188 and 198 mm

8. The weight of a labrador is 35 kg, to the nearest kg. The weight lies between:

A	B	C
30 and 40 kg	34 and 36 kg	34·5 and 35·5 kg

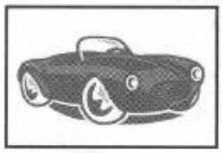
9. Liz and Julie each measure a different worm and they both say that their worm is 11 cm long to the nearest cm.
(a) Does this mean that both worms are the same length?
(b) If not, what is the maximum possible difference in the length of the two worms?

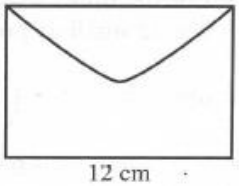
10. To the nearest cm, the length, l of a stapler is 12 cm. As an inequality we can write $11{\cdot}5 \leqslant l < 12{\cdot}5$.

For parts (a) to (j) you are given a measurement. Write the possible values using an inequality as above.

(a) mass = 17 kg	(2 s.f.)	(b) $d = 256$ km	(3 s.f.)
(c) length = 2·4 m	(1 d.p.)	(d) $m = 0{\cdot}34$ grams	(2 s.f.)
(e) $v = 2{\cdot}04$ m/s	(2 d.p.)	(f) $x = 12{\cdot}0$ cm	(1 d.p.)
(g) $T = 81{\cdot}4°C$	(1 d.p.)	(h) $M = 0{\cdot}3$ kg	(1 s.f.)
(i) mass = 0·7 tonnes	(1 s.f.)	(j) $n = 52\,000$	(nearest thousand)

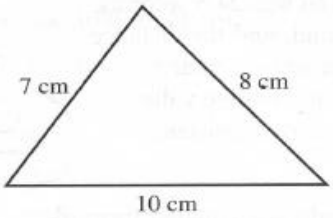
11. A card measuring 11·5 cm long (to the nearest 0·1 cm) is to be posted in an envelope which is 12 cm long (to the nearest cm).
Can you guarantee that the card will fit inside the envelope?
Explain your answer.


11.5 cm

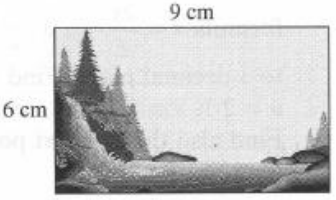

12 cm

Exercise B

1. The sides of the triangle are measured correct to the nearest cm.
(a) Write down the upper bounds for the lengths of the three sides.
(b) Work out the maximum possible perimeter of the triangle.



2. The dimensions of a photo are measured correct to the nearest cm. Work out the minimum possible area of the photo.



3. In this question the value of a is either exactly 4 or 5, and the value of b is either exactly 1 or 2.
Work out:
(a) the maximum value of $a + b$ (b) the minimum value of $a + b$
(c) the maximum value of ab (d) the maximum value of $a - b$
(e) the minimum value of $a - b$ (f) the maximum value of $\frac{a}{b}$
(g) the minimum value of $\frac{a}{b}$ (h) the maximum value of $a^2 - b^2$.

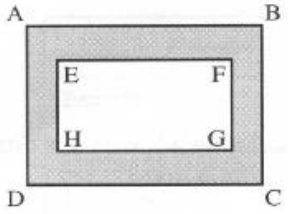
4. If $p = 7$ cm and $q = 5$ cm, both to the nearest cm, find:
(a) the largest possible value of $p + q$
(b) the smallest possible value of $p + q$
(c) the largest possible value of $p - q$
(d) the largest possible value of $\frac{p^2}{q}$.

5. If $a = 3{\cdot}1$ and $b = 7{\cdot}3$, correct to 1 decimal place, find the largest possible value of:
(i) $a + b$ (ii) $b - a$

6. If $x = 5$ and $y = 7$ to one significant figure, find the smallest possible values of

(i) $x + y$ (ii) $y - x$ (iii) $\frac{x}{y}$

7. In the diagram, ABCD and EFGH are rectangles with AB = 10 cm, BC = 7 cm, EF = 7 cm and FG = 4 cm, all figures accurate to the nearest cm.
Find the largest possible value of the shaded area.



8. When a voltage V is applied to a resistance R the power consumed P is given by $P = \frac{V^2}{R}$.

If you measure V as 12·2 and R as 2·6, correct to 1 d.p., calculate the smallest possible value of P.

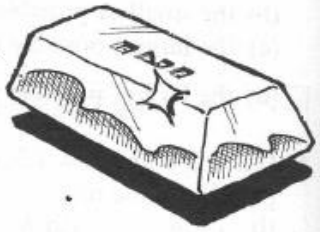
9. A cyclist was timed along a straight piece of road. The time taken was 24·5 seconds, to the nearest 0·1 second, and the distance was 420 metres, to the nearest metre.
Calculate the maximum possible value for the speed of the cyclist consistent with this data.

10. The velocity v of a body is calculated from the formula $v = \frac{2s}{t} - u$ where u, s and t are measured correct to 1 decimal place. Find the largest possible value for v when $u = 2{\cdot}1$, $s = 5{\cdot}7$ and $t = 2{\cdot}2$.
Find also the smallest possible value for v consistent with these figures.

11. Use the formula $z = \frac{a - x^2}{2t}$ to find the largest value that z could have when $a = 71{\cdot}4$, $x = 5{\cdot}3$ and $t = 5{\cdot}4$, all correct to one decimal place.

12. A formula for velocity is $v = \sqrt{(u^2 + 2as)}$. Find the smallest possible value for v if $u = 11{\cdot}5$, $a = -9{\cdot}8$ and $s = 4{\cdot}0$, all correct to one decimal place.

13. The price of gold is \$22·65 per gram, and the density of gold is 19·2 g/cm^3 (i.e. 1 cm^3 of gold weighs 19·2 g). These figures can be used as exact. A solid gold bar in the shape of a cuboid has dimensions 8·6 × 4·1 × 2·4, all measurements in cm correct to the nearest 0·1 cm.



(a) What is the range of possible prices for this gold bar?
Write '... < price < ...' giving your answers to the nearest \$10.

(b) The gold bar was in fact weighed on some scales giving a weight of 1574 g to the nearest gram. Using this value, work out the range of possible prices for the gold bar.
Again write '... < price < ...'.

(c) Explain why the weighing method appears to give a smaller range of possible prices.