Mixed Exercise - 5

1.

The nth term of the linear sequence   2   7   12   17   ...   is $5n - 3$

A new sequence is formed by squaring each term of the linear sequence and adding 1.

Prove algebraically that **all** the terms in the new sequence are multiples of 5.

2.

Simplify $\dfrac{x^2 + 4x - 12}{x^2 - 25} \div \dfrac{x + 6}{x^2 - 5x}$

3.

$x^{\frac{3}{2}} = 8$ where $x > 0$ and $y^{-2} = \dfrac{25}{4}$ where $y > 0$
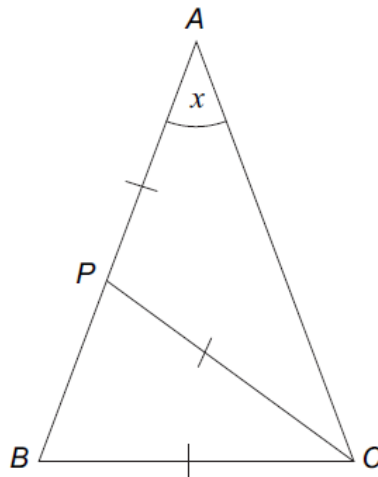
Work out the value of $\dfrac{x}{y}$.

4.

ABC is a triangle.
P is a point on AB such that $AP = PC = BC$
Angle $BAC = x$

A

x

Not drawn accurately

P

B   C

(a)   Prove that angle $ABC = 2x$

(b)   You are also given that $AB = AC$

Work out the value of x.

5.

$1 \leqslant m \leqslant 5$ and $-9 \leqslant n \leqslant 2$

(a)   Work out an inequality for $m + n$.

Answer.............................. $\leqslant m + n \leqslant$ ....

(b)   Work out an inequality for $(m + n)^2$.

Answer.............................. $\leqslant (m + n)^2 \leqslant$ ....
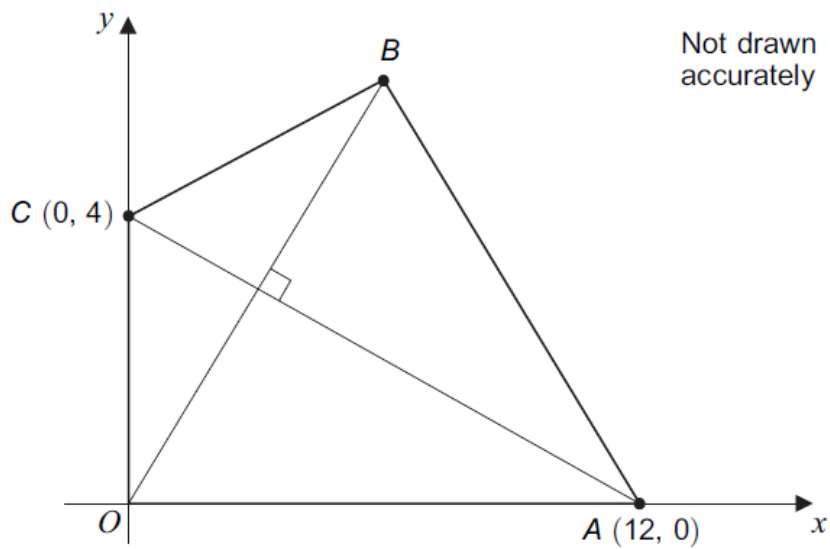

6.

A sphere has radius x centimetres.
A hemisphere has radius y centimetres.
The shapes have equal volumes.

Work out the value of $\dfrac{y}{x}$.

Give your answer in the form $a^{\frac{1}{3}}$ where a is an integer.

7.

OABC is a kite.

(a) Work out the equation of AC.

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Answer... *(2 marks)*

(b) Work out the coordinates of *B*.

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Answer (........................ , ...) *(6 marks)*

8.

A sketch of $y = ax^2 + bx + c$ is shown.
The minimum point is $(2, -3)$.
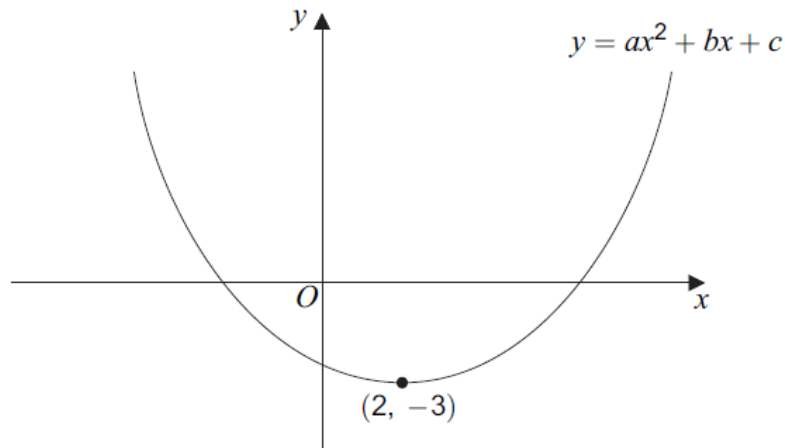
For the sketch shown, circle the correct answer in each of the following.

(a) The value of *a* is

      zero   positive   negative   *(1 mark)*

(b) The value of *c* is

      zero   positive   negative   *(1 mark)*

(c)   The solutions of $ax^2 + bx + c = 0$ are

        both zero   both positive   both negative   one positive and
one negative

        (1 mark)

(d)   The **number** of solutions of $ax^2 + bx + c = -6$ is

        0   1   2   3   *(1 mark)*

(e)   The equation of the tangent to $y = ax^2 + bx + c$ at $(2, -3)$ is

        $x = 2$ $y = 2$ $x = -3$ $y = -3$ *(1 mark)*

9.

Solve the simultaneous equations

$$x + y = 4$$
$$y^2 = 4x + 5$$

Do **not** use trial and improvement.
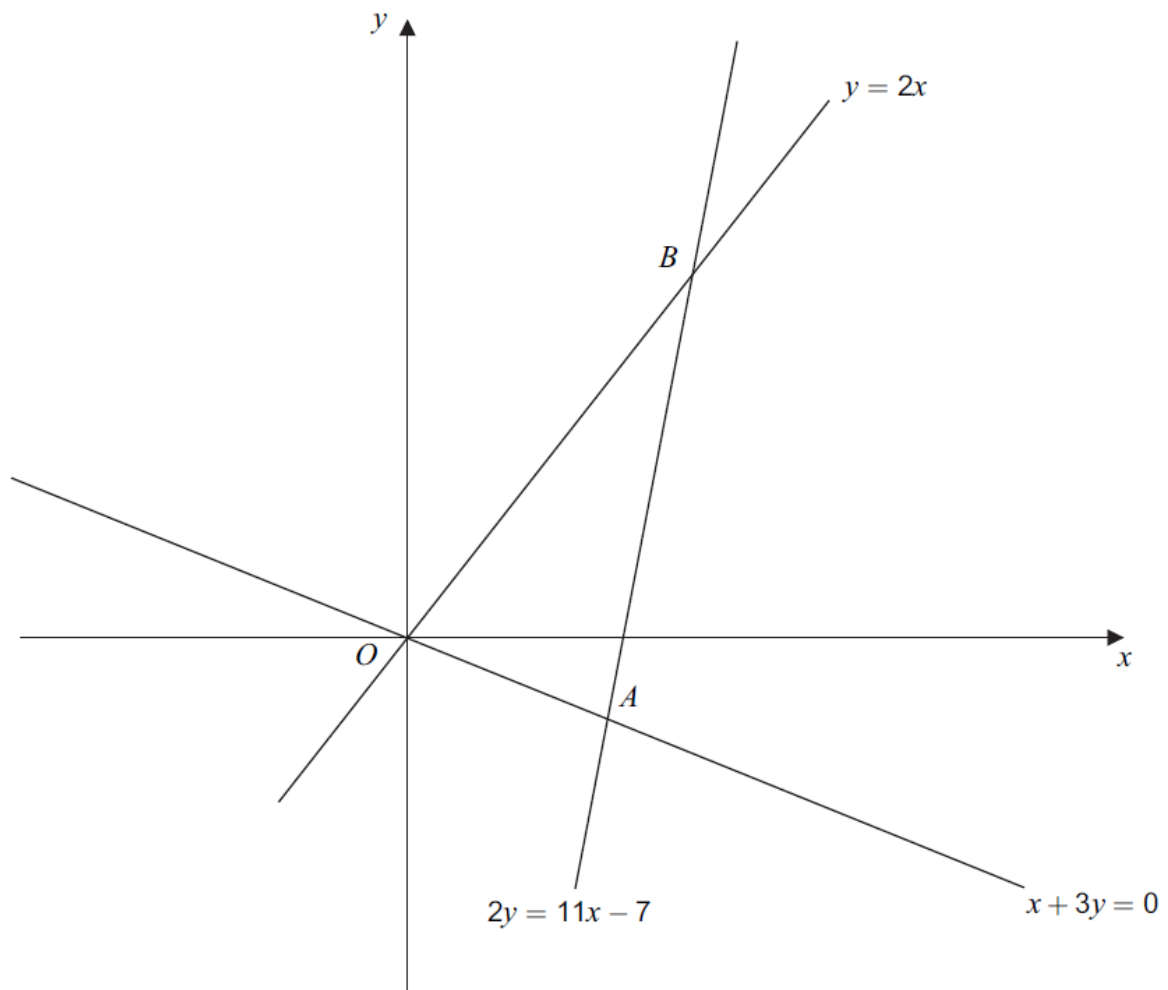
Answer... *(6 marks)*

10.

The equations of three straight lines are

$$y = 2x \qquad x + 3y = 0 \qquad 2y = 11x - 7$$

The lines intersect at the points O, A and B as shown on this sketch.

Show that   length OB = length AB