17.4 Damping and driving

Specification reference: 5.3.3

Shocking

The suspension forks found on modern mountain bikes can be adjusted in several ways, affecting features like the travel, rebound, and, importantly, the amount of **damping**. Without damping to absorb the energy of the shock, after hitting a bump the front end of the bike would oscillate up and down like a mass–spring system.

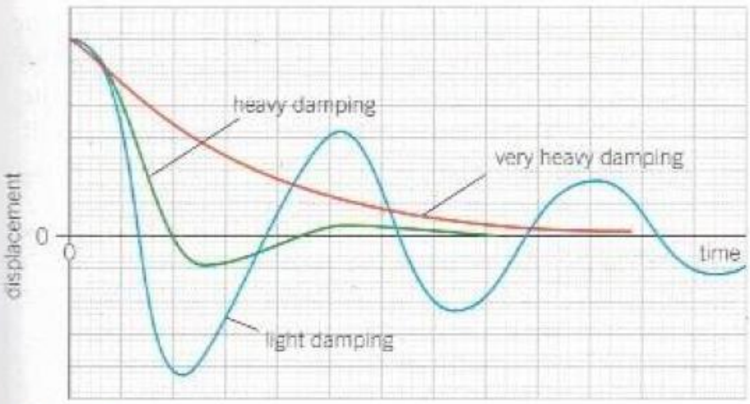
Damping

An oscillation is damped when an external force that acts on the oscillator has the effect of reducing the amplitude of its oscillations. For example, a pendulum moving through air experiences air resistance, which damps the oscillations until eventually the pendulum comes to rest.

There are many forms of damping. When the damping forces are small, the amplitude of the oscillator gradually decreases with time, but the period of the oscillations is almost unchanged. This type of damping is referred to as **light damping**. This would be the case for a pendulum oscillating in air.

For larger damping forces, the amplitude decreases significantly, and the period of the oscillations also increases slightly. This type of **heavy damping** would occur for a pendulum oscillating in water. Now imagine an oscillator, such as a pendulum, moving through treacle or oil. In this example of very heavy damping, there would be no oscillatory motion. Instead the oscillator would slowly move towards its equilibrium position. Figure 2 shows the displacement–time graphs for light damping, heavy damping, and very heavy damping.

In all cases of damped motion, the kinetic energy of the oscillator is transferred to other forms (usually heat).



▲ Figure 2 *The effects of different types of damping*

Learning outcomes

Demonstrate knowledge, understanding, and application of:

- → the effects of damping on an oscillatory system
- → free and forced oscillations
- → natural frequency and resonance
- → observing forced and damped oscillations for a range of systems.

▲ Figure 1 *Damping is an essential feature of all suspension systems, absorbing the energy from bumps*

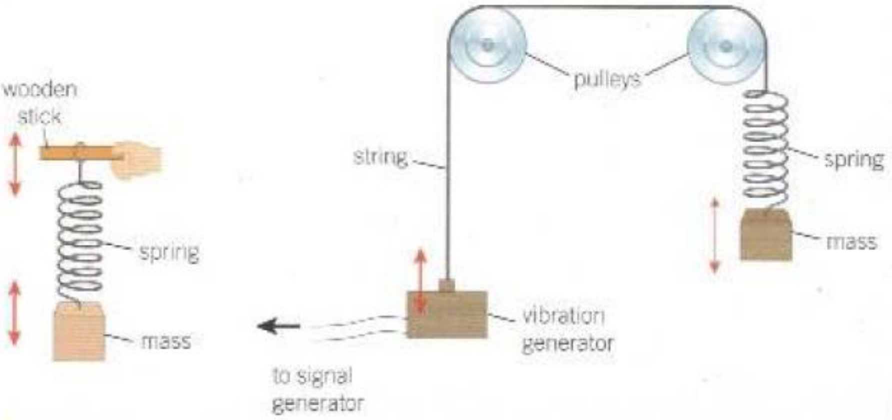
Synoptic link

There are many other examples of exponential decay in nature, including capacitance and radioactive decay – see Topic 21.4, Discharging capacitors, and Topic 25.3, Half-life and activity.

Free and forced oscillations

When a mechanical system is displaced from its equilibrium position and then allowed to oscillate without any external forces, its motion is referred to as **free oscillation**. The frequency of the free oscillations is known as the **natural frequency** of the oscillator.

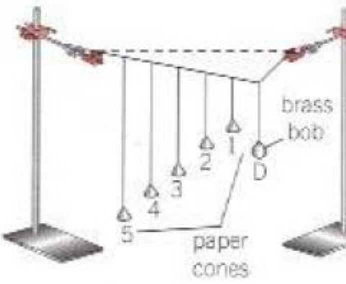
A **forced oscillation** is one in which a periodic driver force is applied to an oscillator. In this case the object will vibrate at the frequency of the driving force (the **driving frequency**). For example, a mass hanging on a vertical spring can be forced to oscillate up and down at a given frequency if the top of the spring is held and the hand moves up and down. The hand is the driver and its motion provides a driver frequency that forces the mass–spring system to oscillate (Figure 3).



▲ **Figure 3** *Two examples of forced oscillation – to investigate damping, the object could be placed in water or oil*

If the driving frequency is equal to the natural frequency of an oscillating object, then the object will **resonate**. This will cause the amplitude of the oscillations to increase dramatically, and if not damped, the system may break. There is more on this in Topic 17.5.

Barton's pendulums provide another example of a forced oscillation (Figure 4). A number of paper cone pendulums of varying lengths are suspended from a string along with a heavy brass bob **D**. This heavy pendulum acts as the driver for the paper cone pendulums. The pendulum **D** oscillates at its natural frequency and forces all the other pendulums to oscillate at the same frequency. As pendulum 2 has the same length as pendulum **D** it has the same natural frequency. It will resonate and its amplitude will be greater than the other pendulums.



▲ **Figure 4** *Forced oscillation – Barton's pendulums*

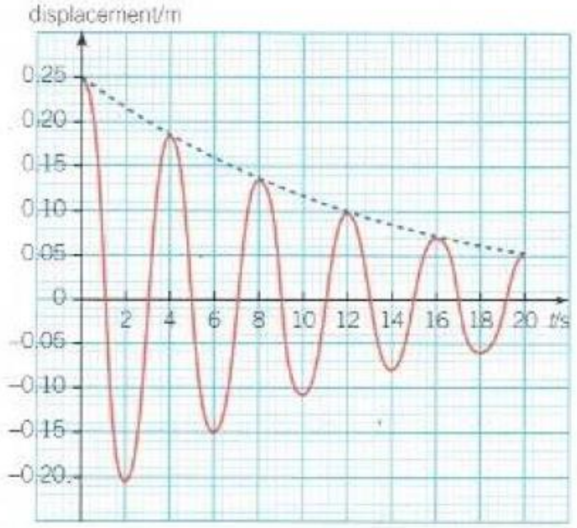
Damping and exponential decrease

In some examples of damping, the amplitude of a damped oscillating system decreases exponentially with respect to time. This is referred to as an **exponential decay**. A good example is a pendulum oscillating in air or a spring–mass system damped by air (Figure 5).

In any exponential decay the physical quantity (in this case amplitude) decreases by the same factor in equal time intervals. For example, for an amplitude A that decays exponentially, if it is measured every 4 seconds, then

$$\frac{A_4}{A_0} = \frac{A_8}{A_4} = \frac{A_{12}}{A_8} = \text{constant}$$

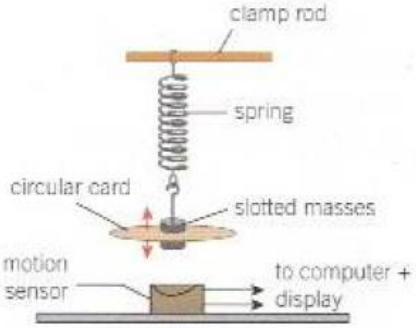
This constant-ratio property is the defining characteristic of an exponential decay.



▲ **Figure 6** *Decay of the amplitude of a pendulum oscillating in air*

1 Determine the initial amplitude, the period, and the angular frequency of the pendulum in Figure 4.

2 Use the graph to determine if the amplitude decays exponentially.



▲ **Figure 5** *An example of a damped spring–mass system*

Summary questions

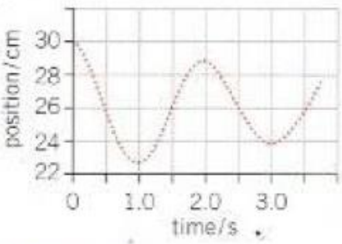
1 Give one example of a free oscillation and one of forced oscillation. *(2 marks)*

2 Describe how damping affects the amplitude of an oscillating object. *(1 mark)*

3 Sketch a graph of displacement against time for a simple pendulum with
   - **a** no damping
   - **b** a small amount of damping. *(4 marks)*

4 A data-logger is used to monitor the oscillations of a lightly damped oscillator. The results are shown in Figure 7.
   - **a** State why the amplitude decreases with time. *(1 mark)*
   - **b** State one quantity that remains constant for the oscillations. *(1 mark)*
   - **c** Determine the natural frequency of the oscillator. *(2 marks)*

5 The amplitude of a damped oscillator decreases exponentially with time. At time $t = 0$, the amplitude is 5.0 cm. The amplitude decreases to 90% after each period. The period of the oscillations is 1.0 s. Sketch the displacement–time graph for this damped oscillator up to a time of 5.0 s. *(4 marks)*



▲ **Figure 7**

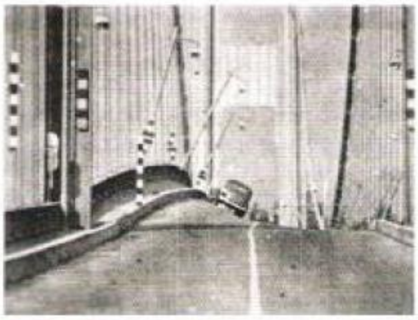
17.5 Resonance

Specification reference: 5.3.3

Learning outcomes

Demonstrate knowledge, understanding, and application of:

- → resonance and natural frequency
- → amplitude-driving frequency graphs for forced oscillators
- → practical examples of forced oscillations and resonance.



▲ **Figure 1** *The main span of the Tacoma Narrows Bridge in Washington state, USA, oscillated with an amplitude of several metres until eventually the bridge collapsed*

Oscillating to destruction

The original Tacoma Narrows Bridge was first opened in 1940. It was designed to withstand hurricane-force winds, yet a wind of just 40 mph famously brought it crashing down just four months later. The wind caused a type of resonance, causing the bridge to oscillate with increasing amplitude until eventually the structure failed and the bridge collapsed.

Resonance

Resonance is the effect that allows an opera singer to break a wine glass with just their voice. It occurs when the driving frequency of a forced oscillation is equal to the natural frequency of the oscillating object. In the case of the wine glass, resonance occurs when the frequency of the sound produced by the singer is equal to the natural frequency of the wine glass.

For a forced oscillator with negligible damping, at resonance

driving frequency = natural frequency of the forced oscillator

When an object resonates, the amplitude of the oscillation increases considerably. If the system is not damped, the amplitude will increase to the point at which the object fails – the glass breaks, or the bridge collapses. The greatest possible transfer of energy from the driver to the forced oscillator occurs at the resonant frequency. This is why the amplitude of the forced oscillator is maximum. In the case of the Tacoma Narrows Bridge, the kinetic energy from the wind was efficiently transferred to the bridge, leading to its ultimate collapse.

Examples of resonance

As well as causing a problem for engineers designing buildings and bridges, resonance can have useful effects:

- Many clocks keep time using the resonance of a pendulum or of a quartz crystal:
- Many musical instruments have bodies that resonate to produce louder notes.
- Some types of tuning circuits (for example in car radios) use resonance effects to select the correct frequency radio wave signal.
- Magnetic resonance imaging (MRI) enables diagnostic scans of the inside of our bodies to be obtained without surgery or the use of harmful X-rays.

Magnetic resonance imaging

Magnetic resonance imaging (MRI) relies on the resonance of hydrogen nuclei found in the water molecules within tissues inside the body.

Inside the scanner there is a strong magnetic field created by superconducting electromagnets. The hydrogen nuclei behave like tiny magnets and precess (a kind of rotation effect) in this magnetic field. The precession occurs at different natural frequencies depending on the type of molecule and thus occurs at different natural frequencies for different tissues in which the hydrogen nuclei are found. Radio waves from transmitting coils inside the scanner cause the nuclei to resonate and absorb energy.

When the radio waves from the transmitter are switched off, the hydrogen nuclei 'relax' and re-emit the energy gained as radio wave photons of specific wavelengths. These are detected by numerous receiving coils surrounding the scanner. The signals from these coils are processed by high-speed computers and the software helps to produce a three-dimensional image of the patient.

MRI scanning offers a number of advantages over some other forms of medical imaging. For example, unlike X-rays they do not expose the patent to ionising radiation and can produce clear images of soft tissue like the brain and heart (Figure 2).

1. Identify the driver and forced oscillators in an MRI scanner.
2. The natural frequency of the hydrogen nuclei depends on the magnetic field inside the MRI. Inside a certain MRI scanner, the natural frequency of the hydrogen nuclei is 128 MHz. Calculate the wavelength of the radio waves that would cause the nuclei to resonate.
3. Calculate the energy of the radio wave photons emitted from the relaxing hydrogen nuclei.

▲ **Figure 2** *An MRI scan of the brain*

Synoptic link

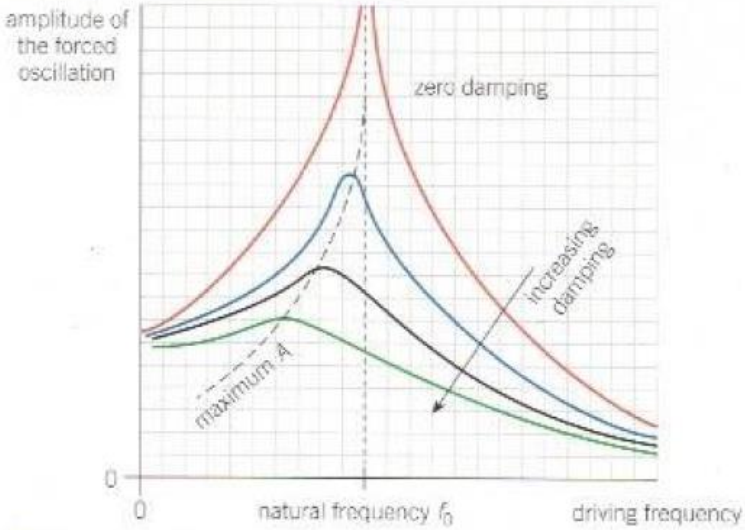
You can learn about other forms of medical imaging in Chapter 27, Medical imaging.

Resonance and damping

The Millennium Bridge in London was opened in June 2000 (Figure 3). It was quickly nicknamed the "wobbly bridge" after it was discovered to resonate when large numbers of people walked across it. As the bridge started to sway, pedestrians tended to match their step to the sway, providing a driving force that was very close to the natural frequency of the bridge. To prevent a possible collapse like that over the Tacoma Narrows, the bridge was closed for two years to allow engineers to install dampers.

▲ **Figure 3** *The Millennium Bridge across the Thames, now with added dampers*

Damping a forced oscillation has the effect of reducing the maximum amplitude at resonance. The degree of damping also has an effect on the frequency of the driver when maximum amplitude occurs. Figure 4 shows the effect of damping on the graph of amplitude against forcing frequency for an oscillator with a natural frequency f_0.



▲ **Figure 4** *Increasing damping can prevent the effects of resonance from becoming severe*

From Figure 4 you can see that:

- For light damping, the maximum amplitude occurs at the natural frequency f_0 of the forced oscillator.
- As the amount of damping increases:
  - the amplitude of vibration at any frequency decreases
  - the maximum amplitude occurs at a lower frequency than f_0
  - the peak on the graph becomes flatter and broader.

Summary questions

1 Sketch a graph of amplitude against driver frequency for an experiment on mass–spring forced oscillation carried out like the one in Figure 3 of Topic 17.4. Use this graph to explain the effect of resonance on an object. *(5 marks)*

2 Explain why a glass shatters when exposed to a sound at the natural frequency of the glass. *(3 marks)*

3 The side panel of a tumble dryer vibrates loudly when the dryer spins at a specific angular frequency. Explain why it only happens at a certain frequency and suggest a technique to reduce vibration. *(4 marks)*

4 An object is suspended from a spring. When displaced, the object executes 180 complete oscillations in a time of 1 minute. The upper end of the spring is then suspended from a mechanical oscillator. The spring–mass system is forced to oscillate. The frequency of the mechanical oscillator is gradually increased from zero. Sketch a graph to show the variation of amplitude of the object with the frequency of the mechanical oscillator. *(3 marks)*

5 An old van with an undamped suspension system drives over three speed bumps 10 m apart at a speed of 2.5 m s^{-1}. The front end of the van begins to resonate. State the natural frequency of the suspension and explain why driving over the bumps at a different speed would reduce the amplitude of the oscillations. *(4 marks)*