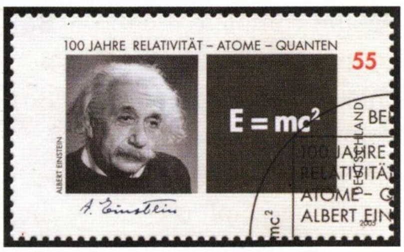
26 NUCLEAR PHYSICS
26.1 Einstein's mass–energy equation

Specification reference: 6.4.4

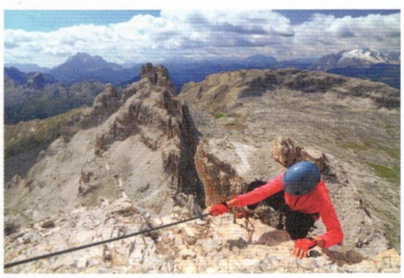
Learning outcomes

Demonstrate knowledge, understanding, and application of:

- → Einstein's mass–energy equation, $\Delta E = \Delta mc^2$
- → energy released or absorbed in simple nuclear reactions
- → creation and annihilation of particle–antiparticle pairs.



▲ **Figure 1** *Albert Einstein, shown here on a German stamp, suggested that mass and energy are equivalent*



▲ **Figure 2** *The mass of this climber is more than her rest mass because of her gain in gravitational potential energy*

$E = mc^2$

The idea that mass and energy are equivalent was proposed by Albert Einstein in 1905 with his famous equation $E = mc^2$, where E is energy, m is mass, and c is the speed of light in a vacuum. This equation has two interpretations.

The first is that *mass is a form of energy*. The interaction of an electron–positron pair illustrates this idea well – the particles completely destroy each other (**annihilation**) and the entire mass of the particles is transformed into two gamma photons.

The second interpretation is that *energy has mass*. The change in mass Δm of an object, or a system, is related to the change in its energy ΔE by the equation $\Delta E = \Delta mc^2$. A moving ball has kinetic energy, implying that its mass is greater than its **rest mass**. The same happens to electrons in particle accelerators. However, because they can have speeds close to the speed of light, their mass could be a hundred times greater than their rest mass.

Similarly, a decrease in the energy of a system means the mass of the system must also decrease. For example, the mass of a mug of hot tea decreases as it cools and loses thermal energy. However, the change in mass is negligibly small (the conversion factor c^2 is enormous).

Everyday situations

Consider a person with rest mass 70 kg sitting in a stationary car. Now imagine the car travelling at a steady speed of $15\,\mathrm{m\,s^{-1}}$ ($\approx 55\,\mathrm{km\,h^{-1}}$). The person has gained kinetic energy, an increase in energy ΔE. The person will therefore have increased mass. The change in mass Δm can be calculated using the mass–energy equation $\Delta E = \Delta mc^2$.

$$\Delta E = \Delta mc^2$$

$$\Delta m = \frac{\Delta E}{c^2} = \frac{\frac{1}{2}mv^2}{c^2} = \frac{\frac{1}{2} \times 70 \times 15^2}{(3.00 \times 10^8)^2} = 8.8 \times 10^{-14} \approx 10^{-13}\,\mathrm{kg}$$

This is a minuscule change in mass and is not noticeable.

Natural radioactive decay

Unstable nuclei decay by emitting either particles or photons. In alpha decay, the parent nucleus emits an alpha particle, creating a daughter nucleus, which recoils in the opposite direction. The alpha particle and the daughter nucleus have kinetic energy. You cannot simply use the principle of conservation of energy to explain this event. It makes more sense to discuss it, and other nuclear reactions, in terms of conservation of mass–energy.

The total amount of mass and energy in a system is conserved. Since energy is released in radioactive decay, there must be an accompanying decrease in mass. In simple terms, this means that the total mass of the alpha particle and the daughter nucleus in the example above must be less than the mass of the parent nucleus. This decrease in mass Δm is equivalent to the energy released ΔE.

Similarly, beta decay is accompanied by a decrease in mass.

Worked example: Decay of carbon-14

The decay of a carbon-14 nucleus is represented by the decay equation

$$^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e} + \overline{\nu}_{\text{e}}$$

A carbon-14 nucleus is initially at rest. Use Table 1 to calculate the total kinetic energy released by the decay of a single carbon-14 nucleus.

Step 1: Determine the change in mass Δm in this reaction.

initial mass = $2.3253914 \times 10^{-26}$ kg

final mass = $(2.3252723 + 0.0000911) \times 10^{-26}$ kg

$\Delta m = [(2.3252723 + 0.0000911) - 2.3253914] \times 10^{-26}$
$= -2.800 \times 10^{-31}$ kg

The minus sign shows that the mass decreases, therefore energy must be released.

Step 2: Use Einstein's mass–energy equation to calculate the energy released.

kinetic energy released = ΔE

$\Delta E = \Delta mc^2 = 2.800 \times 10^{-31} \times (3.00 \times 10^8)^2 = 2.52 \times 10^{-14}$ J

Annihilation and creation

Positrons are the antiparticles of electrons. When they meet, they annihilate each other, and their entire mass is transformed into energy in the form of two identical gamma photons. This is not science fiction. It does happen, and medical physicists have exploited this phenomenon in positron emission tomography (PET). A PET scanner is used to examine the function of organs, including the brain.

Consider an electron–positron pair annihilating each other.

- change in mass $\Delta m = 2m_e$ (m_e = mass of electron or positron $= 9.11 \times 10^{-31}$ kg)
- energy released $\Delta E = \Delta mc^2 = 2m_ec^2$
- minimum energy of two gamma photons $= 2m_ec^2$
- minimum energy of each gamma photon $= m_ec^2$

Therefore, the minimum energy of each photon is 8.2×10^{-14} J or about 0.51 MeV. If the interacting particles also have kinetic energy, then the energy of each photon would be even greater.

Synoptic link

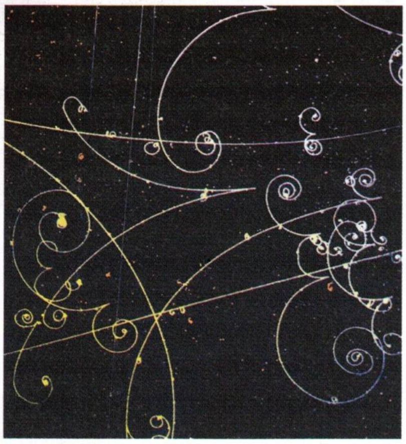
You can review the radioactive decay of unstable nuclei in Topic 25.2, Nuclear decay equations.

Table 1 *Rest masses of some particles*

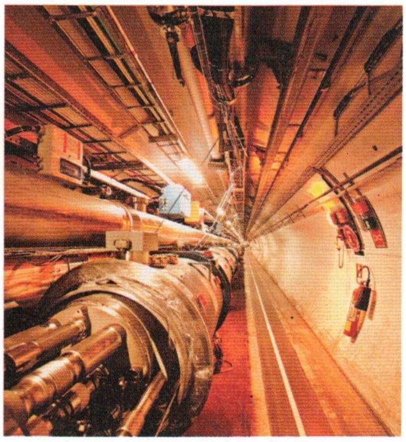
Particle	Mass / 10^{-26} kg
$^{14}_{6}$C nucleus	2.3253914
$^{14}_{7}$N nucleus	2.3252723
$^{0}_{-1}$e (electron)	0.0000911
$\overline{\nu}_e$ (electron antineutrino)	negligible

Synoptic link

You will learn more about PET scanners in Topic 27.5, PET scans.



▲ **Figure 3** *A photon creates an electron and a positron in a bubble chamber – the electron and the positron curve in opposite directions in a magnetic field*



▲ **Figure 4** *The LHC can create two opposing proton beams that smash into each other, with each individual proton having up to 4 TeV (0.64 μJ) of kinetic energy!*

In **pair production**, a single photon vanishes and its energy creates a particle and a corresponding antiparticle. Figure 3 shows such an event. The pair produced here is an electron–positron pair. Since an electron is equivalent to a minimum energy of 0.51 MeV, the minimum energy of the photon creating the electron–positron pair must be $2 \times 0.51 = 1.02$ MeV.

Nuclear reactions

In a particle accelerator, like the LHC at CERN in Geneva, very energetic protons are smashed together. Their kinetic energy is transformed into matter. Under the right conditions, energy in whatever form can be transformed into matter, just as a gamma photon, which is electromagnetic energy, can change into an electron–positron pair.

Soon after the Big Bang and the creation of the Universe, the temperatures were so high that particle–antiparticle pairs of all sorts were being created and destroyed in interactions. Particle accelerators provide a means of recreating the conditions in the very early Universe.

Consider the nuclear reaction below. Two protons, travelling at speeds close to that of light, collide and produce a proton, a neutron, and a hadron called a π^+ meson.

$$^1_1\text{p} + {}^1_1\text{p} \rightarrow {}^1_1\text{p} + {}^1_0\text{n} + \pi^+$$

The total rest mass of the particles after the collision is greater than that before. The increase Δm multiplied by c^2 must be equal to the minimum kinetic energy of the colliding protons.

Summary questions

1 Use Einstein's idea about mass and energy to state and explain whether there is an increase or a decrease in the mass of the following systems:
- **a** a person running; *(1 mark)*
- **b** wood burning; *(1 mark)*
- **c** electrons decelerating. *(1 mark)*

2 Calculate the equivalent energy for the masses below:
- **a** mass of a proton 1.673×10^{-27} kg; *(2 marks)*
- **b** 1 kg mass. *(2 marks)*

3 There is a decrease in mass of 9.6×10^{-30} kg when a single nucleus of polonium-210 emits an alpha particle. Calculate the energy released in a single decay of polonium-210. *(2 marks)*

4 Calculate the increase in the mass of an electron with kinetic energy 1.0 keV. *(3 marks)*

5 Compare the increase in the mass of an electron accelerated through a potential difference of 1.0 MV with its rest mass. *(4 marks)*

6 The nuclear transformation equation below shows the decay of a single thorium-228 nucleus.

$$^{228}_{90}\text{Th} \rightarrow {}^{224}_{88}\text{Ra} + {}^4_2\text{He}$$

Use the information given below to calculate the energy released in the single decay of thorium-228. *(4 marks)*

Mass of thorium-228 nucleus = 3.7853×10^{-25} kg; mass of radium-224 nucleus = 3.7187×10^{-25} kg; mass of helium-4 (alpha particle) = 6.625×10^{-27} kg.