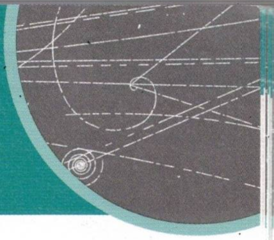
26.2 Binding energy

Specification reference: 6.4.4



Learning outcomes

Demonstrate knowledge, understanding, and application of:

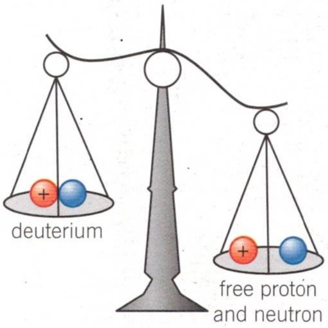
- → mass defect; binding energy; binding energy per nucleon
- → binding energy per nucleon against nucleon number curve; energy changes in reactions
- → binding energy of nuclei using $\Delta E = \Delta mc^2$ and masses of nuclei.

Deuterium nucleus

Deuterium is an isotope of hydrogen. A nucleus of deuterium consists of one proton and one neutron. Now imagine separating these two nucleons. All nucleons are bound together by the strong nuclear force, so they can only be separated by doing work to overcome that force. External energy has to be supplied to make this happen. According to Einstein's mass–energy equation, energy and mass are equivalent, therefore the total mass of the separated nucleons must be greater than the mass the deuterium nucleus.

Is this really true? We can use a mass spectrometer to determine the mass of particles accurately. In terms of unified atomic mass units u (1.661×10^{-27} kg), a deuterium nucleus has mass 2.013553 u, a proton has mass 1.007276 u, and a neutron has mass 1.008665 u. The total mass of the separated proton and neutron is indeed more than the mass of the deuterium nucleus. The difference is 0.002388 u, which is equivalent to an energy of about 3.5×10^{-13} J or 2.2 MeV. In simple terms, this means that a minimum energy of 2.2 MeV is needed to completely separate the nucleons of a deuterium nucleus.

Suppose we could reverse the process and construct a deuterium nucleus from a proton and a neutron. This time, an energy of 2.2 MeV would be released – most likely in the form of a photon.



▲ **Figure 1** *A deuterium nucleus has less mass than its separated nucleons*

Mass defect and binding energy

In the example of the deuterium nucleus above, the difference in mass of 0.002388 u is known as the **mass defect** of the deuterium nucleus.

The mass defect of a nucleus is defined as the difference between the mass of the completely separated nucleons and the mass of the nucleus.

The energy difference of 2.2 MeV for the deuterium nucleus is known as its **binding energy**.

The binding energy of a nucleus is defined as the minimum energy required to completely separate a nucleus into its constituent protons and neutrons.

To calculate the binding energy of a nucleus, you can use Einstein's mass–energy equation.

$$\text{binding energy of nucleus} = \text{mass defect of nucleus} \times c^2$$

The binding energy is not the same for all nuclei. A uranium-235 has 92 protons and 143 neutrons, and you would expect the external energy required to split this nucleus into its constituent protons and neutrons to be much greater than 2.2 MeV – there are many more strong nuclear bonds to be broken.

Study tip

Be careful with your use of the terms 'atoms' and 'nuclei'. Binding energy holds the nucleus together and not the atom.

Worked example: Binding energy of uranium nucleus

The mass of a uranium-235 ($^{235}_{92}U$) nucleus is 235.004393 u. Calculate its binding energy in MeV.

Step 1: Calculate the total mass of the constituent nucleons.

The uranium-235 nucleus has 92 protons and (235 − 92) = 143 neutrons

Therefore, mass of nucleons = (92 × 1.007276) + (143 × 1.008665)
= 236.908487 u

Step 2: Calculate the mass defect for the uranium-235 nucleus.

mass defect = 235.004393 − 236.908487 = (−)1.904094 u

Step 3: Change the mass defect from u to kg ($1\,u = 1.661 \times 10^{-27}$ kg).

mass defect = $1.904094 \times 1.661 \times 10^{-27} = 3.162... \times 10^{-27}$ kg

Step 4: Calculate the binding energy and convert it from J to eV using the conversion factor $1\,eV = 1.60 \times 10^{-19}$ J

binding energy = mass defect × c^2

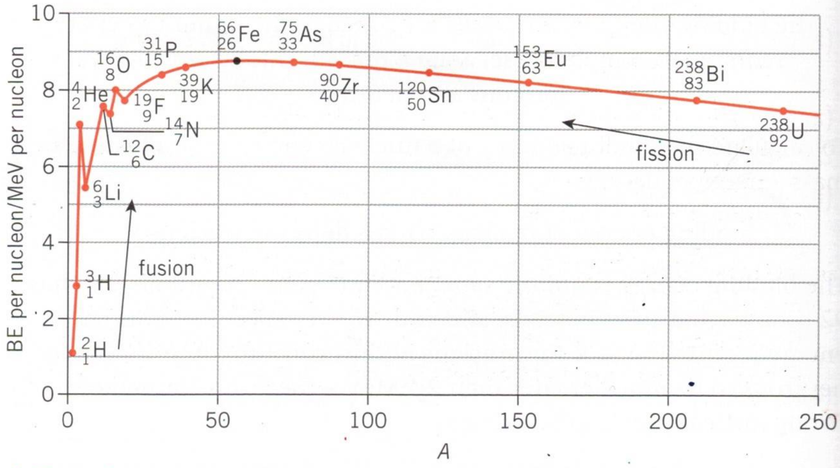
binding energy = $3.162... \times 10^{-27} \times (3.00 \times 10^8)^2 = 2.84... \times 10^{-10}$ J

$$\text{binding energy} = \frac{2.84... \times 10^{-10}}{1.60 \times 10^{-19}} = 1.779... \times 10^9\,\text{eV} = 1780\,\text{MeV (3 s.f.)}$$

Binding energy per nucleon

You would expect a uranium-235 nucleus to have a greater binding energy than a deuterium nucleus because uranium-235 has more nucleons than a deuterium nucleus. To compare how easy it is to break up nuclei, it would be sensible to determine the average **binding energy per nucleon** of nuclei. The greater the binding energy (BE) per nucleon, the more tightly bound are the nucleons within the nucleus, or in other words a nucleus is more stable if it has a greater BE per nucleon.

The binding energy per nucleon for uranium-235 is 1780 MeV/235 ≈ 7.6 MeV and for deuterium is 2.2 MeV/2 ≈ 1.1 MeV per nucleon.



▲ **Figure 2** *Graph of binding energy per nucleon against nucleon number A for nuclei*

Figure 2 is a graph of BE per nucleon against nucleon number *A*. The shape of the graph helps us to understand processes such as natural radioactive decay, **fission**, and **fusion**. The last two processes are covered in greater depth in the next two topics. From the graph you can see that:

- For nuclei with $A < 56$, the BE per nucleon increases as A increases.
- For nuclei with $A > 56$, the BE per nucleon decreases as A increases.
- The nucleus of iron-56 ($^{56}_{26}\text{Fe}$) has the greatest BE per nucleon – it is the most stable isotope in nature.
- The helium-4 nucleus (alpha particle), with its two protons and two neutrons, has an abnormally greater BE per nucleon than its immediate neighbours. The same goes for carbon-12 and oxygen-16 nuclei.
- Energy is released in natural radioactive decay. Figure 2 can be used to show that in cases of spontaneous decay the total binding energy of the parent nucleus is less than the binding energy of the daughter nucleus and the alpha particle. The difference is the energy released in the decay as kinetic energy.
- In a fusion process, two low *A* number nuclei join together to produce a higher *A* number nucleus. The newly formed nucleus has much greater binding energy than the initial nuclei and therefore energy is released. Fusion is the process by which the Sun and other stars produce their energy. Thanks to fusion, we have life on Earth.
- In a fission process, a high *A* number nucleus splits into two lower *A* number nuclei. Energy is released because the two nuclei produced have higher binding energy than the parent nucleus. All fission reactors use this process to produce energy.

Summary questions

1 State the SI units of mass defect and binding energy. (*1 mark*)

2 State the link between binding energy and mass defect. (*1 mark*)

3 Show that a mass defect of 0.002368 u is equivalent to a binding energy of about 3.5×10^{-13} J. (*3 marks*)

4 The binding energy of the nucleus of iron-56 is 7.8×10^{-11} J. Calculate its BE per nucleon in joules per nucleon and in MeV per nucleon. (*3 marks*)

5 Use Figure 2 to estimate the binding energy in MeV of:

a a helium-4 nucleus; (*2 marks*)

b an oxygen-16 nucleus; (*2 marks*)

c a uranium-238 nucleus. (*2 marks*)

6 The mass of the beryllium-8 nucleus ($^{8}_{4}\text{Be}$) is 1.33×10^{-26} kg. The mass of a proton or a neutron is about 1.67×10^{-27} kg. Use this information to calculate the binding energy per nucleon of the beryllium-8 nucleus in both J per nucleon and MeV per nucleon. (*4 marks*)