25 RADIOACTIVITY

25.1 Radioactivity

Specification reference: 6.4.3

Learning outcomes

Demonstrate knowledge, understanding, and application of:

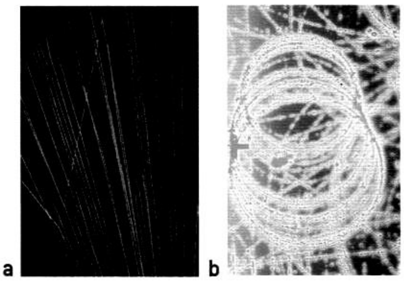
- → radioactive decay
- → α-particles, β-particles, and γ-rays
- → nature, penetration, and range of these radiations, and techniques used to investigate their absorption.

Types of radiation

Radioactivity was accidentally discovered in 1896 by the French physicist Henri Becquerel. He thought that uranium salts might produce X-rays when exposed to sunlight, but after postponing an experiment in which he intended to use photographic plates to record these rays he found that even in the dark the uranium salts had emitted invisible radiation that fogged plates wrapped in lightproof paper.

Investigations carried out by Becquerel, Ernest Rutherford, Marie and Pierre Curie, and Frederick Soddy at the turn of the 20th century showed that radioactive substances emitted different types of radiation – alpha (α), beta (β), and gamma (γ). All three are described as **ionising radiations** because they can ionise atoms by removing some of their electrons, leaving positive ions.

A **cloud chamber** can be used to detect the presence of these types of radiation. It contains air saturated with vapour at a very low temperature. When air molecules are ionised, liquid condenses onto the ions to leave tracks of droplets marking the path of the radiation (Figure 1).



▲ **Figure 1** *a) In this false-colour version of a cloud-chamber picture from the 1920s, alpha particles leave tracks (green) as they shoot upwards through the chamber, where one (yellow) collides with a proton (red) – this image was one of several taken by English physicist Patrick Blackett as he studied alpha-particle scattering; b) spiral tracks left in a cloud chamber by beta particles*

Study tip

Remember that the source is radioactive and not the radiation.

Synoptic link

The changes within nuclei that result in the emission of radiation will be explored in Chapter 26, Nuclear physics.

The nature of alpha, beta, and gamma radiations

Alpha radiation consists of positively charged particles. Each alpha particle comprises two protons and two neutrons (a helium nucleus), and has charge $+2e$, where e is the elementary charge.

Beta radiation consists of fast-moving electrons (β^-) or fast-moving positrons (β^+). A beta-minus particle has charge $-e$ and a beta-plus particle has charge $+e$.

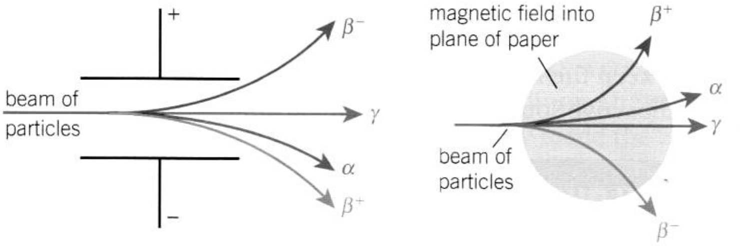
Gamma radiation (or rays) consists of high-energy photons with wavelengths less than about 10^{-13} m. They travel at the speed of light and carry no charge.

All are emitted from the nuclei of atoms as a result of changes within unstable nuclei.

The effect of electric and magnetic fields

Figure 2 shows how a uniform electric field provided by two oppositely charged parallel plates can distinguish between the different types of radiation. The negative beta-minus particles (electrons) are deflected towards the positive plate, whilst the positive alpha and beta-plus (positron) particles are deflected towards the negative plate. Alpha particles are deflected less than beta particles because of their greater mass. The paths of the beta-minus and beta-plus particles are mirror images. Gamma rays are not deflected, because they are uncharged.

For a uniform magnetic field (Figure 2), the direction of the force on each particle can be determined using Fleming's left-hand rule. Again, the uncharged gamma rays are not deflected.



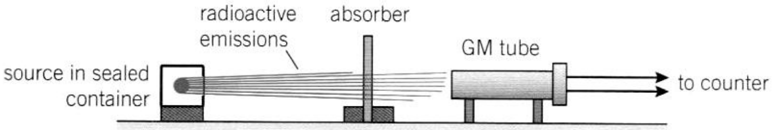
▲ **Figure 2** *The effect of a uniform electric field and a uniform magnetic field on the paths of different types of radiation*

Synoptic link

You already know from Topic 22.4, Charged particles in electric fields, and Topic 23.3, Charged particles in magnetic fields, how these fields affect charged particles.

Absorption experiments

Alpha particles, beta particles, and gamma rays all cause ionisation, which affects how they can penetrate different materials.



▲ **Figure 3** *Experimenting with radiation and different absorbers*

Figure 3 shows how a Geiger–Müller (GM) tube and a counter may be used to investigate the absorption of α, β^-, and γ radiation by different materials. The GM tube is kept at a fixed distance from the radioactive source. Each ionising particle, or photon, detected by the GM tube produces a single count or click.

Everything around us (including your own body) produces a small amount of radiation. This **background radiation** must be measured before you conduct any absorption experiments. The background count rate is the count rate without the radioactive source present, and depends on where in the country you are – it is typically around 0.4 counts per second, about 20 counts per minute.

The count rate for a particular absorber is then determined. You can use different thicknesses of the same absorber to investigate how this affects the count rate. The recorded count rate also includes the background count rate. In order to get the true or **corrected count rate**, you subtract the background count rate from the measured count rate.

1 Why is it not possible to carry out a similar absorption experiment in a school or college laboratory with positrons?

Absorption of alpha, beta, and gamma radiations

The large mass and charge of alpha particles mean they interact with surrounding particles to produce strong ionisation, and therefore they have a very short range in air. It takes only a few centimetres of air to absorb most alpha particles. A thin sheet of paper completely absorbs them.

The small mass and charge of beta particles make them less ionising than alpha particles. This means that they have a much longer range in air, about a metre. It takes about 1–3 mm of aluminium to stop most beta particles.

Gamma rays have no charge, and this makes them even less ionising than beta particles. You can show that for gamma rays the count rate decays exponentially with the thickness of a lead absorber. You need a few centimetres of lead to absorb a significant proportion of gamma rays.

The dangers of radioactivity

All types of radiation cause ionisation, which means that they can damage living cells. This is why radioactive sources are stored in lead-lined storage containers. When transferring radioactive sources, for your own protection you must use a pair of tongs with long handles in order to keep the source as far from your body as possible. Never handle radioactive sources with bare hands.

Summary questions

1 Complete the missing words in Table 1 below. *(3 marks)*

▼ **Table 1**

Alpha radiation	Beta radiation	Gamma radiation
______ nucleus 2 protons + 2 neutrons charge ______	β^- is an ______ charge $-e$ β^+ is a ______ charge $+e$	photon of gamma radiation of ______ less than about 10^{-13} m ______ charge

2 List the different radiations in order of decreasing ionisation effect. *(1 mark)*

Study tip

Background count is the number of counts recorded without the radioactive source present.

3 A student gets 250 counts from a GM tube–counter arrangement for a radioactive source in 2.0 minutes. The background count is 48 counts in this time. Calculate the corrected count rate in counts per minute and counts per second. *(2 marks)*

4 A single alpha particle can produce about 10^4 ions per mm in air. The typical range of an alpha particle in air is about 2.5 cm. It takes about 10 eV of energy to produce a single ion. Estimate the initial kinetic energy in MeV of an alpha particle. *(3 marks)*

5 Use your answer to question 4 to estimate the speed of the alpha particle. (The mass of an alpha particle is 6.6×10^{-27} kg.) *(3 marks)*

6 Table 2 below shows the results from an absorption experiment using gamma rays from a cobalt-60 source and an absorber made of lead.

▼ **Table 2**

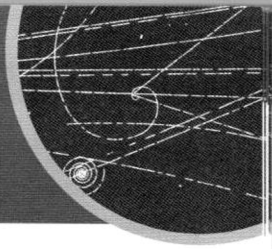
Thickness of lead x / mm	1.8	3.1	6.7	9.8	15.7	19.6
Corrected count rate C / counts s^{-1}	3.8	3.5	2.7	2.3	1.6	1.4

a Plot a graph of ln$C$ against x. *(3 marks)*

b Use this graph to estimate the half-thickness of lead – this is the thickness of lead that will absorb half of the gamma ray photons. Explain your answer. *(5 marks)*

25.2 Nuclear decay equations

Specification reference: 6.4.3



Transmutation

In the early 1900s, the hands and numbers of clocks were painted with a mixture of zinc sulfide and radioactive radium. The energetic particles emitted from radium's radioactive decay made the zinc sulfide glow in the dark. Many painters developed cancers, and the use of radium in paints was stopped when the risks of radioactivity were better understood.

The nuclei of radium atoms emit alpha particles, and in doing so, they change (transmute) into the new nuclei of radon atoms. The unstable radon nuclei in turn decay into nuclei of another element. This process does stop eventually when stable nuclei are formed. The nucleus before the decay is known as the **parent nucleus**, and the new nucleus after the decay is called the **daughter nucleus**.

Basic characteristics and conservation rules

Table 1 provides a reminder of the basic characteristics of types of radiation.

▼ **Table 1** *The different types of radiation*

Radiation	Symbol	Charge	Mass / u	Typical speed / m s^{-1}
alpha	${}^{4}_{2}$He or α	+2e	4.00151	~ 10^6
beta-minus	${}^{0}_{-1}$e or β^- or e^-	−e	0.00055	~ 10^8
beta-plus	${}^{0}_{+1}$e or β^+ or e^+	+e	0.00055	~ 10^8
gamma	γ (also ${}^{0}_{0}\gamma$)	0	0	speed of light, 3.00×10^8

You already know that conservation laws are important in physics. These conservation ideas can also be applied when nuclei decay. In all nuclear reactions, the nucleon number A and proton (atomic) number Z must be conserved. However, as Albert Einstein showed, mass and energy are interchangeable — the energy released in nuclear reactions is produced from mass.

Alpha decay

The nuclear transformation equation below shows a parent nucleus X decaying into a daughter nucleus Y when it emits an alpha particle.

$${}^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He}$$

parent nucleus daughter nucleus

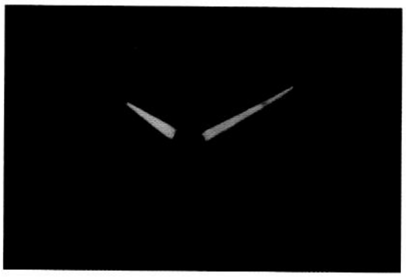
Loss of an alpha particle removes two protons and two neutrons from a parent nucleus, so the nucleon number drops by four. The daughter has a different proton number so is a different element. The equation is balanced, with the total nucleon and proton numbers before and after being the same. Energy is also released in the decay.

Learning outcomes

Demonstrate knowledge, understanding, and application of:

→ nuclear decay equations for alpha, beta-minus, and beta-plus decays

→ balancing nuclear transformation equations.



▲ **Figure 1** *The glow from the hands and numbers of this old clock are caused by a radioactive substance*

Synoptic link

In Newtonian physics, you came across the principles of conservation of energy and momentum (Topic 5.2, Conservation of energy, and Topic 7.5, Collisions in two dimensions). Kirchhoff's second law is related to the idea that charge too is conserved (Topic 10.1, Kirchhoff's laws and circuits). In fact, the conservation of mass and energy is a little more complicated – as you will learn in Topic 26.1, Einstein's mass – energy equation, mass and energy are interchangeable.

Synoptic link

In Topic 24.5, Beta decay, you learnt that there are two types of beta decay: β^- and β^+.

Worked example: Radium

A radium-226 nucleus (${}^{226}_{88}\text{Ra}$) decays by alpha emission to become a nucleus of radon (Rn). Predict the isotope of radon produced in this decay.

Step 1: Determine the final nucleon and proton numbers for the radon nucleus.

$$A = 226 - 4 = 222 \text{ and } Z = 88 - 2 = 86$$

Step 2: Represent the daughter nucleus using the correct chemical symbol and A and Z numbers.

daughter nucleus: ${}^{222}_{86}\text{Rn}$

Beta decay

Beta decay is caused by the weak nuclear force. Radioactive nuclei that emit beta-minus radiation are characterised as having too many neutrons for stability. The weak nuclear force is responsible for one of the neutrons decaying into a proton. In the process an electron (${}^{0}_{-1}\text{e}$) is emitted, together with an electron anti-neutrino ($\overline{\nu}_e$). The nucleon and proton numbers must balance, as shown in the general nuclear transformation equation for beta-minus decay below, together with a couple of examples.

$$\underset{\text{parent nucleus}}{{}^{A}_{Z}\text{X}} \rightarrow \underset{\text{daughter nucleus}}{{}^{A}_{Z+1}\text{Y}} + {}^{0}_{-1}\text{e} + \overline{\nu}_e$$

- strontium-90: ${}^{90}_{38}\text{Sr} \rightarrow {}^{90}_{39}\text{Y} + {}^{0}_{-1}\text{e} + \overline{\nu}_e$
- helium-6: ${}^{6}_{2}\text{He} \rightarrow {}^{6}_{3}\text{Li} + {}^{0}_{-1}\text{e} + \overline{\nu}_e$

Radioactive nuclei that emit beta-plus radiation often have too many protons for stability. Once again, the weak nuclear force initiates changes within the parent nucleus by transforming one of the protons into a neutron. In the process a positron (${}^{0}_{+1}\text{e}$) is emitted together with an electron neutrino (ν_e). Again, the nucleon and proton numbers balance in the general nuclear transformation equation:

$$\underset{\text{parent nucleus}}{{}^{A}_{Z}\text{X}} \rightarrow \underset{\text{daughter nucleus}}{{}^{A}_{Z-1}\text{Y}} + {}^{0}_{+1}\text{e} + \nu_e$$

- potassium-37: ${}^{37}_{19}\text{K} \rightarrow {}^{37}_{18}\text{Ar} + {}^{0}_{+1}\text{e} + \nu_e$
- fluorine-17: ${}^{17}_{9}\text{F} \rightarrow {}^{17}_{8}\text{O} + {}^{0}_{+1}\text{e} + \nu_e$

Study tip

Potassium-37 is found in most foods, including bananas.

Gamma decay

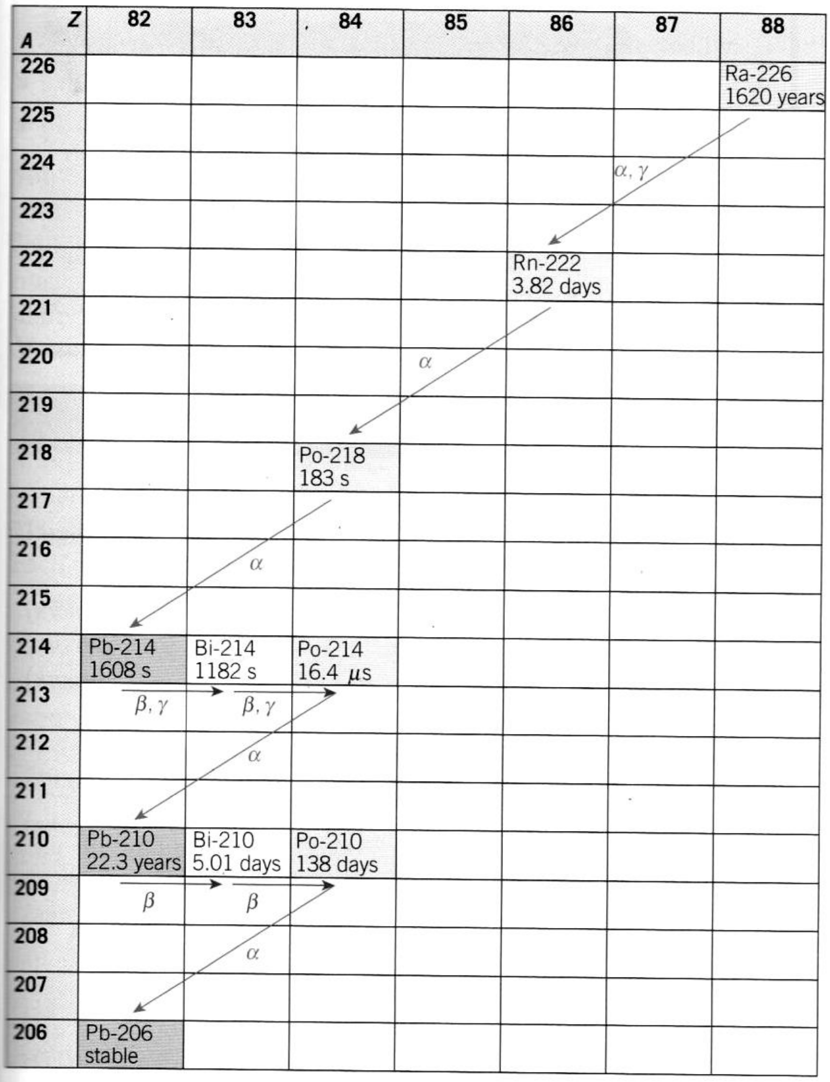
Gamma photons are emitted if a nucleus has surplus energy following an alpha or beta emission. The composition of the nucleus remains the same. The nuclear decay equation when a gamma photon is emitted is shown below.

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z}X + \gamma$$

Decay chains – a complete story

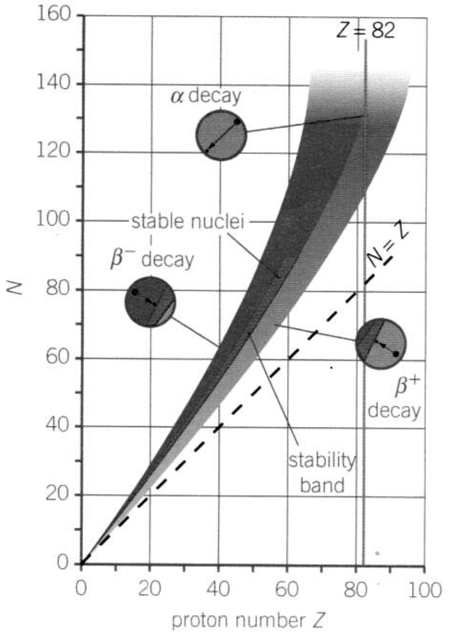
The radioactive decay of nuclei is complex, because the daughter nuclei can themselves be radioactive. An ancient rock containing uranium will therefore also contain its daughters, their daughters, and so on. All of them will emit their own characteristic radiation.

Figure 2 shows the decay chain for a parent radium-226 nucleus. After a very long time, following many transformations, the chain ends in a stable isotope of lead-206. The half-life of each isotope is also shown in Figure 2. Half-life will be covered in greater depth in Topic 25.3, Half-life and activity.



▲ **Figure 2** *The decay chain for radium-226 – a sample initially made of pure radium-226 can end up as 10 different isotopes*

Patterns for stability



▲ **Figure 3** *N–Z plot of nuclei*

Figure 3 shows a graph of number of neutrons N against proton number Z. All stable nuclei lie on a very narrow band known as the stability band (brown). The ratio of neutrons to protons in stable nuclei gradually increases as the number of protons in the nuclei increases. Only nuclei with proton numbers less than about 20 are stable with an equal number of protons and neutrons. Most nuclei have more neutrons than protons.

The stability band is surrounded by possible unstable nuclei. You can determine the likely decay of an unstable nucleus from its position relative to the stability band.

- Nuclei with more than 82 protons are likely to decay by emitting alpha particles.
- Nuclei to the right of the band have too many protons (proton-rich) and will likely decay by beta-plus.
- Nuclei to the left of the band have too many neutrons (neutron-rich) and will likely undergo beta-minus decay.

1 The only stable isotope of aluminium is $^{27}_{13}\text{Al}$. State and explain whether the isotope $^{29}_{13}\text{Al}$ is proton-rich or neutron-rich.

2 The only stable isotope of phosphorus is $^{31}_{15}\text{P}$. There are six other phosphorus isotopes with nucleon numbers ranging from 28 to 34. List all six of these isotopes and identify whether they are likely to be β^+ or β^- emitters.

Summary questions

1 State two numbers conserved in the nuclear transformation shown below.

$^{14}_{6}\text{C} \rightarrow\, ^{14}_{?}\text{N} + \,^{0}_{-1}\text{e} + \overline{\nu}_e$ *(1 mark)*

2 For the reason shown in question 1, state:

a the force responsible for the decay; *(1 mark)*

b the type of decay; *(1 mark)*

c the nucleon number of the daughter nucleus. *(1 mark)*

3 Complete the following nuclear transformation equations for alpha decay.

a $^{238}_{92}\text{U} \rightarrow\, ^{4}_{2}\text{He} + \,^{?}_{?}\text{Th}$ *(1 mark)*

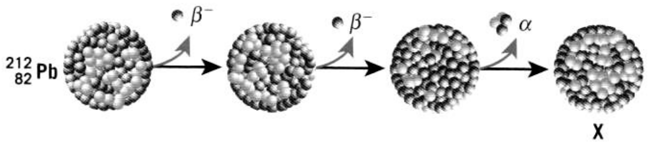
b $^{222}_{?}\text{Rn} \rightarrow\, ^{4}_{2}\text{He} + \,^{?}_{84}\text{Po}$ *(2 marks)*

4 Complete the following nuclear transformation equation for beta-plus decay.

$^{?}_{?}\text{N} \rightarrow\, ^{13}_{6}\text{C} + ? + \nu_e$ *(2 marks)*

5 A nucleus of uranium-234 ($^{234}_{92}\text{U}$) transforms into an isotope of lead (Pb) after emitting five alpha particles. Predict the lead isotope formed. *(3 marks)*

6 In Figure 4, the final nucleus X is an isotope of lead (Pb). Use Figure 4 to identify the isotope X. *(4 marks)*



◀ **Figure 4**

25.3 Half-life and activity

Specification reference: 6.4.3

Random and spontaneous

When doing experiments with a radioactive source, you would have noticed that the counts, or clicks, from a GM tube do not show a regular pattern. The clicks are random. This suggests that the radioactive nuclei themselves must also decay in a random manner. In fact, radioactive decay is described as a random and a spontaneous event.

It is *random* because:

- we cannot predict when a particular nucleus in a sample will decay or which one will decay next
- each nucleus within a sample has the same chance of decaying per unit time.

It is *spontaneous* because the decay of nuclei is not affected by:

- the presence of other nuclei in the sample
- external factors such as pressure.

You can simulate the random behaviour of unstable nuclei by flipping coins or rolling a large number of dice. You can even use popcorn cooking in a microwave oven. The kernels represent the undecayed nuclei and a single pop represents a single decay. At the start, there are many unpopped kernels and the popping rate is high. As the amount of unpopped corn decreases, so does the popping rate.

Half-life

A large number of six-sided dice can be used to simulate the decay of naturally decaying radioactive nuclei. Imagine starting off with 216 dice. Each die represents a single undecayed nucleus in a sample. Each throw represents a small interval of time. Assume the number '1' appearing on the top face represents a decay. The probability of decay for each die is $\frac{1}{6}$. This means that with each throw about 1 in 6 dice will 'decay' and about 5 in 6 will remain undecayed. The actual number decaying and remaining will be determined by chance. The decay is random because you cannot predict which dice will decay – you can only state their probability of decay.

After the first throw, you would expect about $\frac{216}{6} = 36$ dice to 'decay' and 180 to remain. After the second throw, you would expect about another 30 to decay, with about 150 left. After n throws, you would expect $\left(\frac{5}{6}\right)^n \times 216$ dice to be left. This constant-ratio property means that the number of dice left decays exponentially with the number of throws. You can show that it takes about 3.8 throws to halve the number of dice each time.

A radioactive sample behaves similarly. The two main differences are the number of nuclei, which could be many trillions, and the probability of decay, which depends on the isotope. The probability of decay governs how quickly the nuclei decay and therefore the **half-life** of the isotope.

Learning outcomes

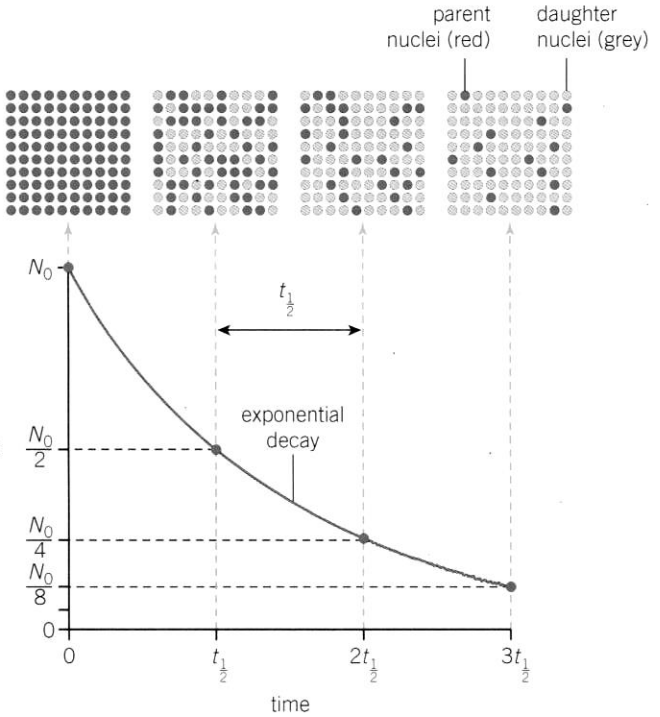
Demonstrate knowledge, understanding, and application of:

- → the spontaneous and random nature of decay
- → activity of a source
- → decay constant λ of an isotope; $A = \lambda N$
- → simulation of radioactive decay.

▲ **Figure 1** *What does making popcorn have in common with radioactive nuclei?*

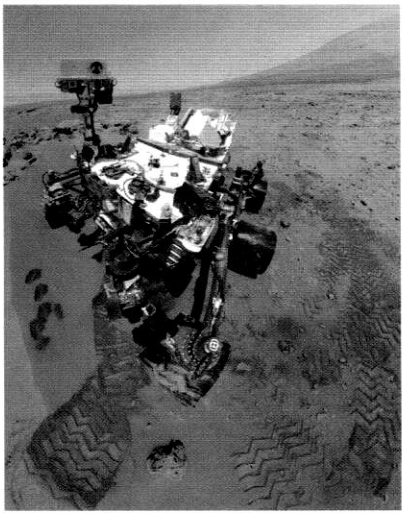
The half-life of an isotope is the average time it takes for half the number of active nuclei in the sample to decay.

This means that after a time equal to one half-life $t_{\frac{1}{2}}$, the number N of undecayed nuclei in a sample will have halved. The number N must therefore decay exponentially with time (Figure 2).



▲ **Figure 2** *Exponential decay in the number of undecayed nuclei from $N = N_0$ at time $t = 0$ – notice how the graph starts to show more statistical variations as the number of nuclei becomes smaller*

Unstable isotopes have half-lives ranging from fractions of a second to billions of years. For example, beryllium-8 has a half-life of about 8×10^{-17} s, whereas thorium-232 has a half-life of 14 billion years. Lead-204 is extraordinary – it has a half-life 10 million times longer than the age of our Universe.



▲ **Figure 3** *Some spacecraft, such as NASA's Curiosity rover [shown], are powered using radioactive isotopes. The radioactive source is housed in a thermally insulated container, which absorbs all the radiation energy emitted. Thermocouples transfer this heat to electrical energy*

Activity *A*

The count rate measured using a GM tube and a counter is only a fraction of the rate at which particles, or photons are emitted by a radioactive source. The actual **activity** of the source will be much higher than the count rate.

The activity A of a source is the rate at which nuclei decay or disintegrate.

You can also think of the activity as the number of alpha, beta, or gamma photons emitted from the source per unit time. Activity is measured in decays per second. An activity of one decay per second is one **becquerel** (1 Bq). An activity of 2000 Bq from an alpha source means 2000 nuclei decay per second or 2000 alpha particles are emitted per second. The activity depends on the number of undecayed nuclei present in the source and on the half-life of the isotope.

Worked example: Power from a radioactive source

The activity of an alpha-emitting source is 5.0×10^{12} Bq. The kinetic energy of each alpha particle is 4.0 MeV. Calculate the power emitted by this source.

Step 1: Calculate the energy of each alpha particle.

$$1\,\text{eV} = 1.60 \times 10^{-19}\,\text{J}$$

$$\therefore \text{ energy of an } \alpha\text{-particle} = 4.0 \times 10^{6} \times 1.60 \times 10^{-19} = 6.4 \times 10^{-13}\,\text{J}$$

Step 2: The activity means that there are 5.0×10^{12} α-particles emitted per second.

Therefore, energy emitted per second $= 5.0 \times 10^{12} \times 6.4 \times 10^{-13} = 3.2\,\text{J}\,\text{s}^{-1}$ (2 s.f.)

Step 3: Power is the rate of energy emitted.

Therefore, power = 3.2 W

Decay constant λ

Consider a source with a very large number of nuclei, with N undecayed nuclei at time $t = 0$ that decay into stable daughter nuclei. As you have learnt, the decay is both random and spontaneous. In a small interval of time Δt, it would be reasonable to assume that the number of nuclei disintegrating would be directly proportional to both N and Δt, that is

$$\Delta N \propto N\Delta t$$

Therefore

$$\frac{\Delta N}{\Delta t} \propto -N$$

The minus sign is included to show that the number of nuclei is decreasing with time. $\frac{\Delta N}{\Delta t}$ is the rate of decay of the nuclei, that is, the activity A of the source. For a source containing a known isotope, the relationship may be written with a constant λ, the **decay constant** of the isotope. Therefore

$$A = \lambda N$$

The minus sign has now been omitted because we just need to know the value of the activity. The decay constant has the SI unit s^{-1} (or h^{-1} or even y^{-1}, but not Bq).

The decay constant can be defined as the probability of decay of an individual nucleus per unit time.

Summary questions

1 Define activity and state its SI unit. *(2 marks)*

2 A beta-emitting source has an activity of 4.0 kBq. Calculate the number of beta particles emitted in a period of 1.0 minute. State any assumption made. *(3 marks)*

3 At time $t = 0$, there are 5000 undecayed nuclei in a source. The half-life of the isotope is 20 s.

- a Predict the number of undecayed nuclei after 100 s. *(2 marks)*
- b State and explain what will happen to the activity after 100 s. *(2 marks)*

4 A GM tube detects 2.5% of the activity of a source and measures 200 counts per second. Estimate the activity of the source in Bq. *(2 marks)*

5 An alpha-emitting source has an activity of 8.6×10^{6} Bq. The decay constant of the isotope is $2.0 \times 10^{-6}\,\text{s}^{-1}$. Calculate the number of nuclei in the source. *(2 marks)*

6 Calculate the power emitted from an alpha-emitting source with an activity of 1.0 MBq and each alpha particle having kinetic energy 4.6 MeV. *(3 marks)*

7 A strontium-90 source has mass 3.0 μg. The decay constant of strontium-90 isotope is $1.1 \times 10^{-9}\,\text{s}^{-1}$. Calculate the activity of the source in MBq. The molar mass of strontium is $0.090\,\text{kg}\,\text{mol}^{-1}$. *(4 marks)*

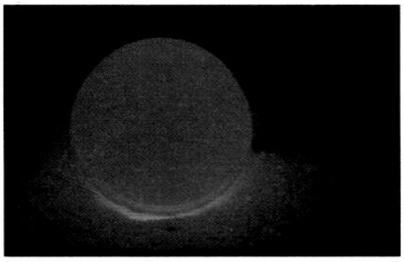
25.4 Radioactive decay calculations

Specification reference: 6.4.3

Learning outcomes

Demonstrate knowledge, understanding, and application of:

- → the half-life of an isotope; $\lambda t_{\frac{1}{2}} = \ln(2)$
- → techniques used to determine the half-life of an isotope
- → the equations $A = A_0 e^{-\lambda t}$ and $N = N_0 e^{-\lambda t}$.



▲ **Figure 1** *A pellet of plutonium, illuminated by the glow of its own radioactivity*

Study tip

You can also calculate the number of nuclei N left in a sample if you know the number of half-lives elapsed, n, using the equation: $N = (0.5)^n N_0$, where $n = \frac{t}{t_{\frac{1}{2}}}$.

Synoptic link

You have met other types of exponential decay already, such as the damping of simple harmonic motion in Topic 17.4, Damping and driving, and the discharge of capacitors in Topic 21.4, Discharging capacitors.

Determining half-life

Plutonium is a highly toxic and carcinogenic substance and is very dangerous even in tiny amounts. The isotope plutonium-239 has a half-life of 24 000 years. Do you have to record results for decades before you can determine its half-life? In fact, to determine the half-life of any isotope, all you need to know is how many nuclei are present in the source and its activity. The activity can easily be determined using radiation detectors, and the number of nuclei can either be measured directly with a mass spectrometer or calculated from its mass.

Exponential decay

The mathematical solution to the decay equation $\frac{\Delta N}{\Delta t} = -\lambda N$ is $N = N_0 e^{-\lambda t}$, where N_0 is the number of undecayed nuclei at time $t = 0$, N is the number of undecayed nuclei in the sample at time t, and e is the base of natural logarithms, 2.718. You do not need to be able to derive this equation, but you are expected to apply it to solve problems.

The number of undecayed nuclei decreases exponentially with time. The activity A of the source is directly proportional to N. Therefore, the activity also decreases exponentially with time and is given by the equation $A = A_0 e^{-\lambda t}$, where A_0 is the activity at time t.

Decay constant and half-life

The decay constant λ of an isotope is related to its half-life $t_{\frac{1}{2}}$. You can use your knowledge of natural logarithms and the decay equation $N = N_0 e^{-\lambda t}$ to determine this link.

After a time $t = t_{1/2}$, $N = \frac{N_0}{2}$.

Therefore

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}} \quad \text{or} \quad \frac{1}{2} = e^{-\lambda t_{1/2}}$$

This equation can also be written as

$$e^{\lambda t_{1/2}} = 2$$

By taking natural logarithms (ln) of both sides, we end up with

$$\ln(e^{\lambda t_{1/2}}) = \ln(2) \quad \text{or} \quad \lambda t_{1/2} = \ln(2)$$

The value of ln (2) is about 0.693. The decay constant and half-life are inversely proportional to each other. The decay constant of uranium-237 isotope, with a half-life of 6.8 days, is going to be much smaller than that of nitrogen-16, which has a half-life of 7.4 s.

Worked example: Thorium-227

A freshly prepared sample of thorium-227 has 4.0×10^{12} nuclei. The isotope of thorium-227 has a half-life of 18 days. Calculate its activity after 22 days.

Step 1: You can work in days, but it is best to convert the half-life into seconds when calculating the decay constant λ.

$$\lambda t_{1/2} = \ln(2)$$

$$\lambda = \frac{\ln(2)}{t_{1/2}} = \frac{\ln(2)}{18 \times 24 \times 3600} = 4.457 \times 10^{-7}\,\mathrm{s}^{-1}$$

To avoid rounding errors, you must leave more significant figures in this intermediate answer.

Step 2: Calculate the initial activity of the source using $A = \lambda N$.

Initial activity $A_0 = 4.457 \times 10^{-7} \times 4.0 \times 10^{12} = 1.783 \times 10^{6}\,\mathrm{Bq}$

Step 3: Use $A = A_0 e^{-\lambda t}$ to calculate the activity after 22 days. Once again, you need to convert the time t into seconds.

$$A = A_0 e^{-\lambda t} = 1.783 \times 10^{6} \times e^{-(4.457 \times 10^{-7} \times 22 \times 24 \times 3600)} = 7.6 \times 10^{5}\,\mathrm{Bq}\ (2\ \text{s.f.})$$

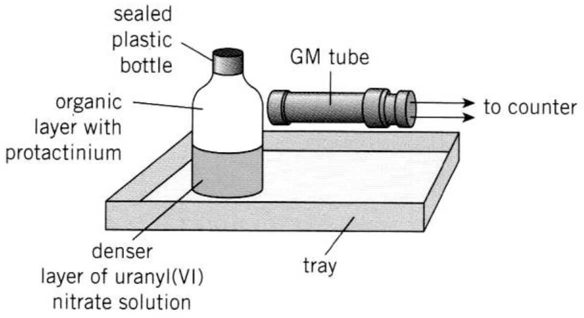
You can do this calculation all at once on a calculator, but it is best to double-check that you have entered the data correctly.

Note: You could have also used the following method for the last step. The number of half-lives $n = \frac{22}{18} = 1.222...$

$$A = A_0 e^{-\lambda t} = 1.783 \times 10^{6} \times (0.5)^{1.222...} = 7.6 \times 10^{5}\,\mathrm{Bq}\ (2\ \text{s.f.})$$

Measuring half-life

Protactinium-234 is a suitable isotope to use in an experiment to measure half-life, because its half-life is short. The protactinium-234 isotope is produced from the decay of thorium-234, which is itself produced from the decay of uranium-238. A sealed plastic bottle containing an organic solvent and a solution of uranyl(VI) nitrate in water is used to separate the protactinium from thorium. This works because the compound of the protactinium daughter isotope is soluble in the organic solvent, whereas the parent thorium compound is not.



▲ **Figure 2** *A practical arrangement for determining the half-life of protactinium-234*

The background count rate is firstly determined in the absence of the source. The plastic bottle is shaken for about 15 s to dissolve the protactinium in the organic solvent, which floats to the top. The end-window of the GM tube is placed opposite the organic layer (Figure 2). In order to avoid contamination, the GM tube must not touch the bottle. The counts from the decaying protactinium can be recorded by taking a 10 s count every half-minute. The corrected count rate is directly proportional to the activity of the source. Therefore, the half-life of protactinium-234 can be determined by plotting a graph of corrected count rate against time.

Analysing data

Unlike the half-life of protactinium-234, that of radon-222 cannot safely be measured in a school or college laboratory. Radon is a gas, and because of possible leakage problems, it is safest to leave the collection of data to specialists.

Table 1 shows data collected by a university researcher. The corrected count rate at time t is C. The absolute uncertainty in each value of C is also provided.

▼ **Table 1**

t / s	C / counts s^{-1}
0	50.0 ± 2.2
30	32.1 ± 1.8
60	22.9 ± 1.5
90	15.2 ± 1.3
120	9.5 ± 1.0
150	6.8 ± 0.8

1 Use the table to plot a graph of ln (C) against t. Include the error bars for the ln (C) values. Draw a best-fit straight line through the error bars.

2 Explain why the plot produces a straight line.

3 Determine the gradient of the best-fit line and therefore the half-life of radon-222.

4 Describe how you can determine the absolute uncertainty in your value for the half-life.

Summary questions

1 Calculate the decay constant in s^{-1} of the following isotopes:

a lithium-8: half-life = 0.84 s; *(2 marks)*

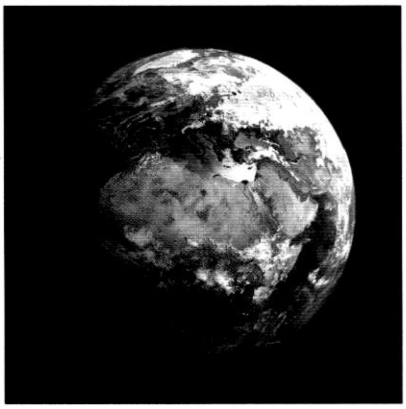
b sodium-24: half-life = 15 h. *(2 marks)*

2 The decay constant of uranium-238 is $4.9 \times 10^{-18}\,s^{-1}$. Calculate its half-life in seconds and in years. *(3 marks)*

3 The isotope of polonium-210 has a half-life of 140 days. A radioactive source has 8.0×10^{10} nuclei of this isotope. Calculate the initial activity of the source. *(4 marks)*

4 Use the information given in this topic to determine the ratio

$$\frac{\text{decay constant of uranium-237}}{\text{decay constant of nitrogen-16}}$$ *(3 marks)*



▲ **Figure 3** *How old is the Earth?*

5 Americium-241 is used in domestic smoke detectors. The half-life of this isotope is 430 y. The activity of a particular americium-241 source is 4.8 kBq. Calculate:

a the number of americium-241 nuclei present in the source; *(4 marks)*

b the activity of the source after 25 y. *(2 marks)*

6 Estimates show that since the creation of the Earth, the amount of uranium-235 on the Earth has dropped to 1.2% of its initial amount. The half-life of the isotope of uranium-235 is 710 million years. Estimate the age of the Earth in years. *(4 marks)*