23.4 Electromagnetic induction

Specification reference: 6.3.3

Learning outcomes

Demonstrate knowledge, understanding, and application of:

- → magnetic flux ϕ, the unit weber, $\phi = BA\cos\theta$
- → magnetic flux linkage.

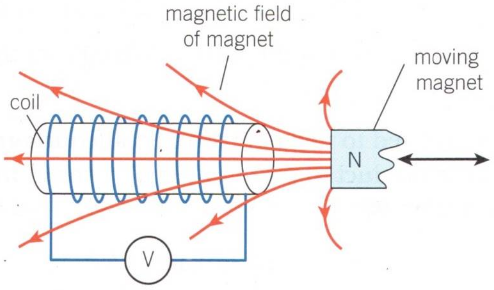
Turbines

You know that a current-carrying conductor produces magnetism, but can you produce electrical currents using magnetism? This question was tackled in the 1800s by the eminent scientist Michael Faraday, whose pioneering experiments revealed much about electromagnetic induction. Electromagnetic induction occurs in the generators in power stations, and in wind turbines. Figure 1 shows the inside of a large wind turbine. It generates electricity – induces an e.m.f. – by relative motion between a conductor and a magnetic field.

▲ **Figure 1** *The generator inside a wind turbine produces electrical energy by electromagnetic induction*

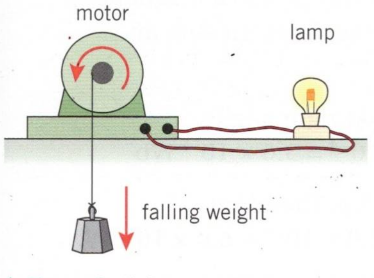
Investigating electromagnetic induction

To induce an e.m.f. all you need is a coil and a magnet (Figure 2). A sensitive voltmeter attached to the coil shows no reading when the coil and the magnet are stationary. When the magnet is pushed towards the coil, an e.m.f. is induced across the ends of the coil, and when the magnet is pulled away a reverse e.m.f. is induced. Repeatedly pushing and pulling the magnet will induce an alternating current in the coil. The faster the magnet is moved, the larger is the induced e.m.f.

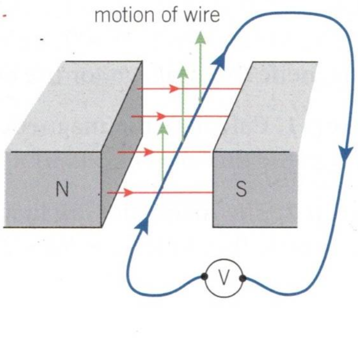


▲ **Figure 2** *Inducing an e.m.f. across the ends of a coil using a moving magnet*

There are other methods of inducing an e.m.f. in conductors. You can use a simple d.c. electric motor in reverse, for example using a falling mass to rotate the coil between the poles of the stationary magnet. The induced e.m.f. can be large enough to operate a lamp (Figure 3). An e.m.f. is induced in a loop of copper wire when it is moved perpendicular to the magnetic field lines of a magnet (Figure 4). The magnitude of the e.m.f. is bigger when the wire is pulled away faster from the magnetic field.



▲ **Figure 3** *Using a motor as a generator*



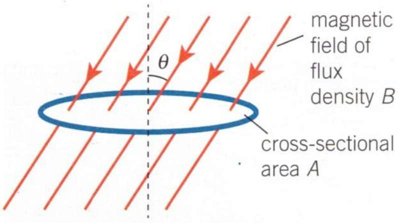
◀ **Figure 4** *Using a wire to produce an e.m.f.*

Explaining electromagnetic induction

Energy is always conserved – this principle cannot be violated. So where does the electrical energy produced in the coil shown in Figure 2 come from? Some of the work done to move the magnet is transferred into electrical energy. The motion of the coil (and the electrons in it) relative to the magnetic field makes the electrons move because they experience a magnetic force given by Bev, where B is the magnetic flux density, e is the elementary charge, and v is the relative speed between the coil and magnet. The moving electrons constitute an electrical current within the coil, so the process has produced electrical energy.

Magnetic flux and magnetic flux linkage

Every experiment demonstrating electromagnetic induction can be explained in terms of **magnetic flux**, ϕ.



▲ **Figure 5** *Magnetic flux ϕ is the product of the component of the magnetic flux density perpendicular to the area and the cross-sectional area*

Figure 5 shows a uniform magnetic field of flux density B passing through a region with a cross-sectional area A at an angle θ to the normal. The magnetic flux ϕ is defined as the product of the component of the magnetic flux density perpendicular to the area and the cross-sectional area, that is

$$\phi = (B\cos\theta) \times A \qquad \text{or} \qquad BA\cos\theta$$

When the field is normal to the area, $\cos 0° = 1$ and $\phi = BA$.

The SI unit for magnetic flux is the weber (Wb). From the equation above you can show that $1\,\text{Wb} = 1\,\text{T}\,\text{m}^2$.

Study tip

It is easy to confuse magnetic flux density B and magnetic flux ϕ, but they are very different. The units, T and Wb respectively, are the clue for identifying these two quantities.

Another quantity related to magnetic flux is called **magnetic flux linkage**. This is the product of the number of turns in the coil N and the magnetic flux, that is,

$$\text{magnetic flux linkage} = N\phi$$

The SI unit of magnetic flux linkage is also the **weber**, but sometimes weber-turns is also used to distinguish it from magnetic flux.

An e.m.f. is induced when...

An e.m.f. is induced in a circuit whenever there is a *change* in the magnetic flux linking the circuit. Since $\phi = BA\cos\theta$, you can induce an e.m.f. by changing B, A, or θ.

Worked example: Getting the terminology right

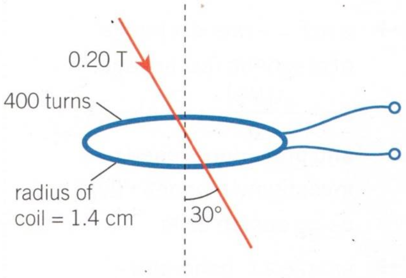
A coil has 200 turns and a core of cross-sectional area $1.0 \times 10^{-4}\,\text{m}^2$. The coil is placed at right angles to a magnetic field of flux density 0.30 T. Calculate the magnetic flux and magnetic flux linkage for the coil.

Step 1: Calculate the magnetic flux. At right angles, magnetic flux $\phi = BA = 0.30 \times 1.0 \times 10^{-4} = 3.0 \times 10^{-5}\,\text{Wb}$

Step 2: The magnetic flux linkage is $N\phi$. Therefore Magnetic flux linkage $= N\phi = 200 \times 3.0 \times 10^{-5} = 6.0 \times 10^{-3}\,\text{Wb}$

Summary questions

1. State the SI units for magnetic flux density, magnetic flux, and magnetic flux linkage. *(1 mark)*
2. Use the idea of magnetic flux to explain why an e.m.f. is induced in the coil shown in Figure 2. *(2 marks)*
3. A single loop of wire coil has a cross-sectional area $1.4 \times 10^{-4}\,\text{m}^2$. Calculate the maximum magnetic flux for this loop in a field of flux density 0.02 T. *(2 marks)*
4. Calculate the magnetic flux linkage for the coil shown in Figure 6. *(2 marks)*
5. The direction of the magnetic field is reversed for the coil shown in Figure 6. Calculate the change in the magnetic flux linkage. *(2 marks)*
6. In London, the Earth's magnetic field makes an angle of 66° with the horizontal and has flux density $4.9 \times 10^{-5}\,\text{T}$. Estimate the magnetic flux for a small coin lying on flat ground. *(3 marks)*



▲ **Figure 6**

23.5 Faraday's law and Lenz's law

Specification reference: 6.3.3

Learning outcomes

Demonstrate knowledge, understanding, and application of:

- → Faraday's law of electromagnetic induction
- → Lenz's law
- → e.m.f. = − rate of change of magnetic flux linkage, $\varepsilon = -\frac{\Delta(N\phi)}{\Delta t}$, techniques and procedures used to investigate magnetic flux using search coils
- → simple a.c. generator.

▲ Figure 1 *The first generator in the world, made by Michael Faraday in 1831*

The first generator

Figure 1 shows the first ever electric generator – a coil of copper wound around a hollow core. Moving a magnetised iron rod through the coil induced an e.m.f. and hence a current in the coil. Faraday's imagination and inventiveness helped him to formulate a law for electromagnetic induction – a law that we now call **Faraday's law**.

Faraday's law of electromagnetic induction

In Topic 23.4, Electromagnetic induction, the idea of magnetic flux linkage was introduced. Faraday's law relates it to the magnitude of the induced e.m.f. in conductors.

Faraday's law: The magnitude of the induced e.m.f. is directly proportional to the rate of change of magnetic flux linkage.

We can write this mathematically as

$$\varepsilon \propto \frac{\Delta(N\phi)}{\Delta t}$$

where ε is the induced e.m.f and $\Delta(N\phi)$ is the change in magnetic flux linkage in a time interval Δt.

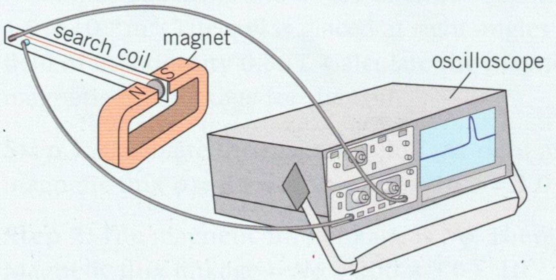
This relationship can be written as an equation where the constant of proportionality is equal to −1. The reasons for the negative sign will be given later when we examine **Lenz's law**.

$$\varepsilon = -\frac{\Delta(N\phi)}{\Delta t}$$

The equation above is as simple and elegant as Newton's second law in mechanics, and like all fundamental laws, it can explain a variety of phenomena.

Worked example: Search coil

A search coil (used to measure variations in magnetic flux) is made of thin copper wire with 2000 turns and a mean cross-sectional area of $1.4\,\text{cm}^2$. It is placed between the poles of a strong magnet at right angles to the magnetic field of flux density 0.30 T and then quickly removed from the field in a time of 80 ms. The ends of the search coil are connected to an oscilloscope (Figure 2). Calculate the magnitude of the average e.m.f. induced across the ends of the search coil.



▲ Figure 2 *A search coil in use*

Step 1: To find ε you first need to calculate $\Delta(N\phi)$. The final flux linkage for the coil is zero. The initial flux linkage can be calculated using $N\phi = NBA\cos\theta$ (where $\theta = 90°$). It is important to convert the cross-sectional area of the coil into m^2 when calculating the change in the flux linkage.

$$(N\phi) = \text{final flux linkage} - \text{initial flux linkage}$$

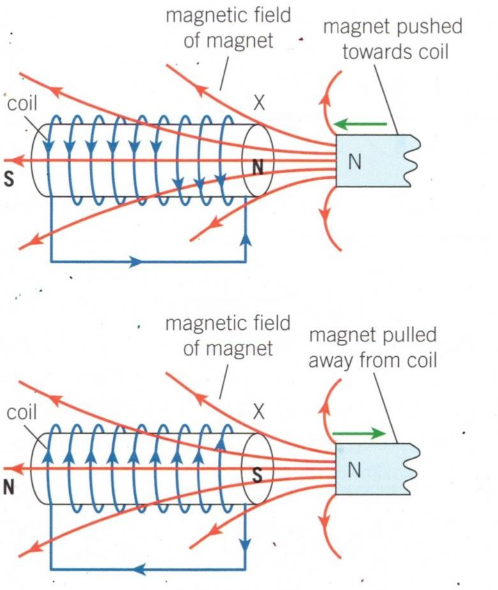
$$\begin{aligned}\Delta(N\phi) &= 0 - 2000 \times (0.30 \times 1.4 \times 10^{-4})\\ &= -8.40 \times 10^{-2}\,\text{Wb}\ (1\,\text{cm}^2 = 10^{-4}\,\text{m}^2)\end{aligned}$$

Step 2: Calculate the induced e.m.f. ε using Faraday's law.

$$\varepsilon = -\frac{\Delta(N\phi)}{\Delta t} = \frac{8.40 \times 10^{-2}}{0.08} = 1.1\,\text{V (2 s.f.)}$$

Lenz's law

Figure 3 shows the coil and magnet arrangement that you have already met in Topic 23.4. The only difference here is that there is no voltmeter – instead the wires are connected together so that any induced currents in the coils are large enough to create their own strong magnetic fields. The direction of the induced e.m.f., and hence the current, changes direction when the magnet is pulled away from coil instead of being pushed into the coil. Why does this happen?



▲ **Figure 3** *The coil and the magnet repel (above) or attract (below) each other.*

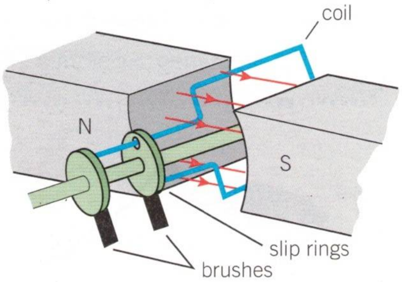
Figure 3 shows what happens when the magnet and the end X of the coil are brought closer together. In the upper image, the induced current is such that the end X of the coil has a north polarity. You have to do work to push the magnet towards the coil. The work done on the magnet is equal to the electrical energy produced in the coil. The end X cannot be a south pole. If it could be, then the principle of conservation of energy would be violated, with attraction between the coil and the magnet creating electrical energy from nowhere.

When the magnet is pulled away from the coil, the motion of the magnet must once again be opposed so that you must do work. The end X therefore has a south polarity and the induced e.m.f. and current are reversed (lower part of Figure 3). **Lenz's law** is an expression of conservation of energy.

Lenz's law: The direction of the induced e.m.f. or current is always such as to oppose the change producing it.

The negative sign in the equation for Faraday's law is mathematical way of expressing Lenz's law. In most calculations, you can ignore this minus sign. However, it is a reminder that energy cannot be created from nothing.

The alternating current generator

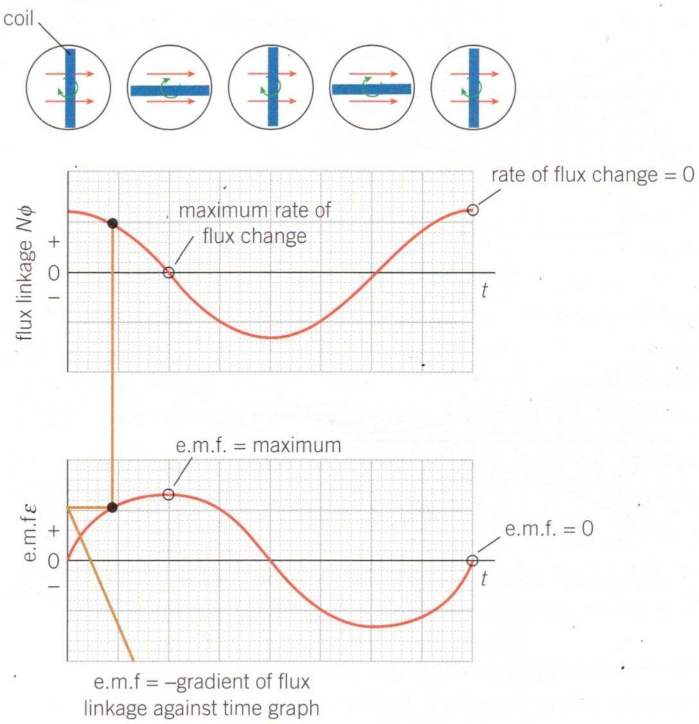


▲ **Figure 4** *An a.c. generator*

Our lives would be completely different without the mains electricity from generators spinning away to producing an alternating e.m.f. of frequency 50 Hz. We can explain the principles of an alternating current (a.c.) generator using Faraday's law.

The simple a.c. generator in Figure 4 consists of a rectangular coil of cross-sectional area A and N turns of coil rotating in a uniform magnetic field of flux density B. The flux linkage for the coil is

$$\text{flux linkage} = N\phi = N(BA\cos\theta) = BAN\cos\theta$$



▲ **Figure 5** *The variation of flux linkage with time (above) and of the induced e.m.f. with time (below)*

As the coil rotates at a steady frequency, the flux linkage changes with time t as shown in the first graph in Figure 5. This variation is referred to as sinusoidal and is caused by the changing $\cos\theta$ factor.

According to Faraday's law, the induced e.m.f. $\varepsilon = -\frac{\Delta(BAN\cos\theta)}{\Delta t}$.

- The magnitude of the gradient from the magnetic flux linkage against time graph is equal to the induced e.m.f. ε.
- For a given generator, B, A, and N are all constant, therefore $\varepsilon \propto -\frac{\Delta(\cos\theta)}{\Delta t}$.

The lower graph in Figure 5 shows the variation of e.m.f. ε with time t. The maximum induced e.m.f. is directly proportional to:

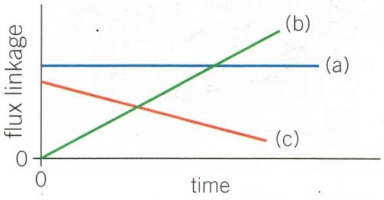
- the magnetic flux density B
- the cross-sectional area A of the coil
- the number of turns N
- the frequency f of the rotating coil.

Study tip

For a generator, the induced e.m.f. ε is at its maximum when the flux linkage is zero and ε is zero when the flux linkage is at its maximum.

Summary questions

1. State what the minus sign represents in the equation for Faraday's law. *(1 mark)*
2. Figure 6 shows the variation of flux linkage with time for three coils. State and explain the e.m.f. induced in the coil in each case. *(3 marks)*
3. A coil connected to a voltmeter is placed next to one end of a long current-carrying solenoid. The voltmeter reads zero. When the current in the solenoid is switched off, the voltmeter shows a reading for a very short interval of time and then goes back to zero. Explain these observations. *(3 marks)*
4. The north pole of a bar magnet is placed on top of a square coil of cross-sectional area $3.0 \times 10^{-4}\ \text{m}^2$. The coil has 800 turns. The magnet is quickly removed from the coil in a time of 0.12 s. The average induced e.m.f. in the coil is 32 mV. Calculate the magnetic flux density at the pole of the magnet. *(4 marks)*
5. Explain why a large current-carrying coil can produce dangerously high 'back' e.m.f. when the current is suddenly switched off. *(3 marks)*
6. A horizontal copper wire of length L forms part of a circuit. It is moved with a constant speed v in a region of vertical magnetic field of flux density B. Use Faraday's law to show that the induced e.m.f. ε across the ends of the wire is given by the expression $\varepsilon = BvL$. *(3 marks)*



▲ Figure 6

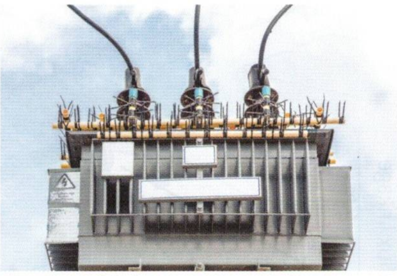
23.6 Transformers

Specification reference: 6.3.3

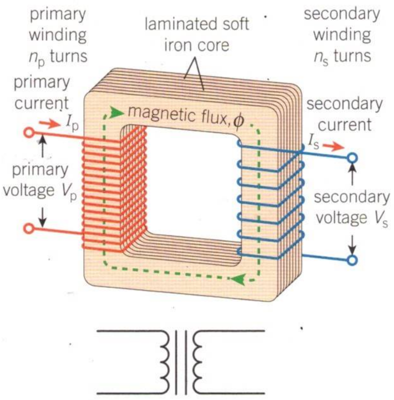
Learning outcomes

Demonstrate knowledge, understanding, and application of:

- → the simple laminated, iron-cored transformer
- → $\frac{n_s}{n_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$ for an ideal transformer
- → techniques and procedures used to investigate transformers.



▲ **Figure 1** *A transformer – note the fins for air cooling*



▲ **Figure 2** *The structure of an iron-core transformer and its circuit symbol*

Study tip

Transformers will not work with steady direct current because there is no changing magnetic flux.

Transformers change voltages

One important use of electromagnetic induction is in transformers, which change alternating voltages to higher or lower values. Power stations use transformers to convert the supply from 25 kV up to 400 kV. Mobile phone chargers have transformers that change the mains voltage of 230 V down to lower values such as 5 V. In this topic you will learn about iron-core transformers.

Step-up and step-down transformers

A simple transformer (Figure 2) consists of a laminated iron core, a primary (input) coil, and a secondary (output) coil. An alternating current is supplied to the primary coil. This produces a varying magnetic flux in the soft iron core. The secondary coil, which is wound round the same core, is linked by this changing flux. The iron core ensures that all the magnetic flux created by the primary coil links the secondary coil and none is lost. According to Faraday's law of electromagnetic induction, a varying e.m.f. is produced across the ends of the secondary coil.

The input voltage V_p and the output voltage V_s are related to the number n_p of turns on the primary coil and number n_s of turns on the secondary coil by the **turn-ratio equation**

$$\frac{n_s}{n_p} = \frac{V_s}{V_p} \text{ for an ideal transformer}$$

- A **step-up transformer** has more turns on the secondary than on the primary coil, and $V_s > V_p$.
- A **step-down transformer** has fewer turns on the secondary than on the primary coil, and $V_s < V_p$.

Worked example: Step-down transformer

A step-down transformer changes 230 V mains voltage to 5.0 V. The transformer has 920 turns on its primary coil. Calculate the number of turns on its secondary coil.

Step 1: Rearrange the turn-ratio equation.

$$\frac{n_s}{n_p} = \frac{V_s}{V_p}$$

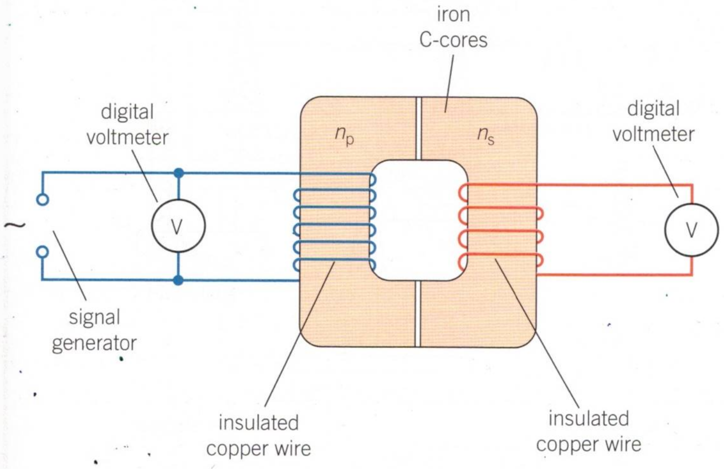
$$n_s = \frac{V_s n_p}{V_p}$$

Step 2: Calculate the number of turns on the secondary coil.

$$n_s = \frac{V_s n_p}{V_p} = \frac{5.0 \times 920}{230} = 20 \text{ turns}$$

Experimenting with transformers

Figure 3 shows an arrangement that you can use in the laboratory to investigate transformers. A multimeter set to 'alternating voltage' can be used to measure the input V_p and output V_s voltages, or you can use an oscilloscope instead. Thin insulated copper wires are used to make primary and secondary coils. You can change the number of turns on one or both coils to see what happens to V_s for a fixed value of V_p and vice versa.



▲ **Figure 3** *Apparatus for investigating transformers*

Efficient transformers

For a 100% efficient transformer, the output power from the secondary coil is equal to the input power into its primary coil. Since power is the product of voltage and current, we have

$$V_s I_s = V_p I_p$$

or

$$\frac{I_p}{I_s} = \frac{V_s}{V_p}$$

Thus, in a step-up transformer, the voltage is stepped up but the current is stepped down. Increasing the voltage by a factor of 100 will decrease the output current by a factor of 100. Similarly, in a step-down transformer, the voltage is stepped down and the current is stepped up.

Transformers can be made efficient by using low-resistance windings to reduce power losses due to the heating effect of the current. Making a laminated core with layers of iron separated by an insulator helps to minimise currents induced in the core itself (eddy currents), so this too minimises loses due to heating. The core is made of soft iron, which is very easy to magnetise and demagnetise, and this also helps to improve the overall efficiency of the transformer.

Summary questions

1 Explain the purpose of the iron core in a transformer. *(1 mark)*

2 Design a step-up transformer that will increase the output voltage by a factor of 20. *(1 mark)*

3 State two reasons why a transformer may not have 100% efficiency. *(2 marks)*

4 An old mobile phone charger has an inbuilt transformer that produces an output voltage of 5.2 V. The input voltage is 230 V and the primary coil has 500 turns. Calculate the number of turns on the secondary coil. *(2 marks)*

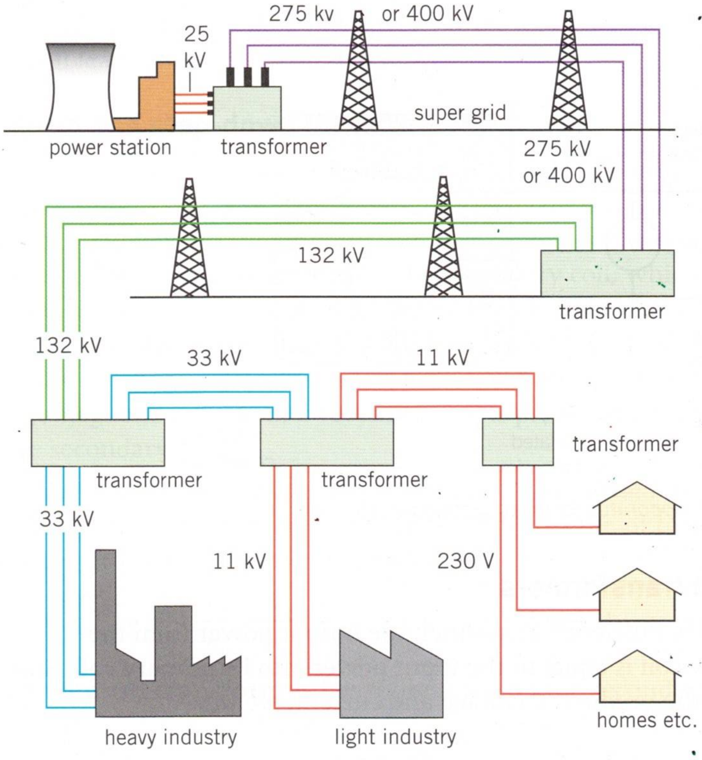
5 An electronic device uses a transformer with turns of ratio 20 : 1 to step down the mains voltage from 230 V. Calculate the output voltage from the transformer. *(2 marks)*

6 A transformer is used to step down 230 V mains voltage to 12 V. A 60 W lamp connected to the secondary coil is lit normally. The primary coil has 1000 turns. Calculate:

- **a** the number of turns on the secondary coil; *(2 marks)*
- **b** the current in the primary coil. *(2 marks)*

The National Grid

In the UK, electrical power is transported across the country by the National Grid. This network consists of transformers and cables on pylons and underground. All a.c. generators in large power stations produce an alternating voltage of about 25 kV at a precise frequency of 50 Hz. Figure 4 shows how a system of cables and transformers distributes electrical power across the country.



▲ **Figure 4** *The National Grid system*

Electrical power is transmitted at high voltage so as to minimise heat losses in the transmission cables. To deliver a power P_0 at a voltage V, the current I required is given by the equation $I = \frac{P_0}{V}$. For transmission cables of resistance R, the power loss P_L due to heating in the cables is given by the equation $P_L = I^2R = \frac{P_0{}^2R}{V^2}$. The higher the transmission voltage V, the smaller are the power losses through heating ($P_L \propto \frac{1}{V^2}$).

A small power station produces 1 MW. Calculate:

1 the current in the transmission cables operating at 400 kV
2 the power losses when the resistance of the cables is 500 Ω
3 the percentage of power lost when power is transmitted at 400 kV
4 the percentage of power lost when the same power is transmitted along the same cables at 40 kV. Comment on your answers to questions 3 and 4.