23 MAGNETIC FIELDS

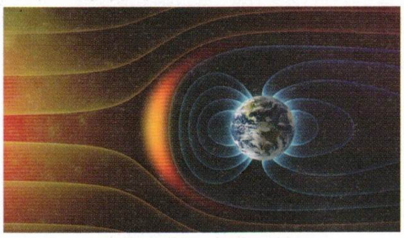
23.1 Magnetic fields

Specification reference: 6.3.1

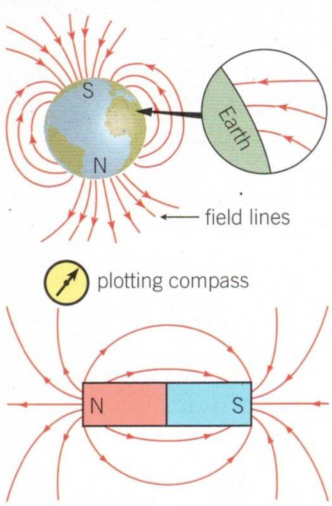
Learning outcomes

Demonstrate knowledge, understanding, and application of:

- → moving charges or permanent magnets as causes of magnetic fields
- → magnetic field lines to map magnetic fields
- → magnetic field patterns for a long straight current-carrying conductor, a flat coil, and a long solenoid.



▲ **Figure 1** *The Earth's magnetic field protects us from the solar wind*



▲ **Figure 2** *The magnetic fields of the Earth and a bar magnet look similar – note the poles marked on the Earth*

Earth's magnetic field

Our Earth has a magnetic field, just like that of a bar magnet, with magnetic north and south poles. It is caused by the electrical currents circulating in the molten iron of the Earth's core.

The Sun emits streams of charged particles travelling at up to $1000\,km\,s^{-1}$. Most of this solar wind is deflected by the Earth's magnetic field before it reaches the surface. Without this field, the Earth would be swept by ionising radiation and we could not live.

Magnetic fields

A magnetic field is a field surrounding a permanent magnet or a current-carrying conductor in which magnetic objects experience a force. You can detect the presence of a magnetic field with a small plotting compass. The needle will deflect in the presence of a magnetic field.

We use **magnetic field lines** (or lines of force) to map **magnetic field patterns** around magnets and current-carrying conductors. Magnetic field patterns are useful visual representations that help us to interpret the direction and the strength of the magnetic fields.

- The arrow on a magnetic field line is the direction in which a free north pole would move — the arrow points from north to south.
- Equally spaced and parallel magnetic field lines represent a uniform field, that is, the strength of the magnetic field does not vary.
- The magnetic field is stronger when the magnetic field lines are closer. For a bar magnet, the field is strongest at its north (N) and south (S) poles.
- Like poles (N–N or S–S) repel and unlike poles (S–N) attract.

Figure 2 shows the magnetic field patterns for a bar magnet and the Earth. You may already have seen how iron filings can reveal the magnetic field around a bar magnet. The field induces magnetism in the filings, which line up in the field. Figure 3 shows the magnetic field patterns between two unlike poles and two like poles.

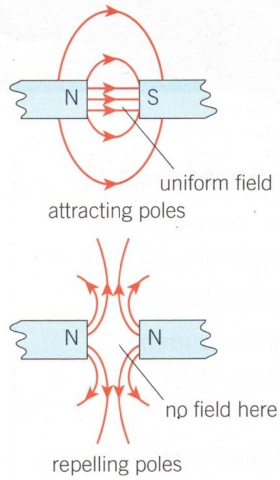
Electromagnetism

When a wire carries a current, a magnetic field is created around the wire. The field is created by the electrons moving within the wire. Any charged particle that moves creates a magnetic field in the space around it. But how do we explain the magnetic field of a bar magnet? In fact, it is created by the electrons whizzing around the iron nuclei. You can visualise the iron atoms as tiny magnets, all aligned in the same direction.

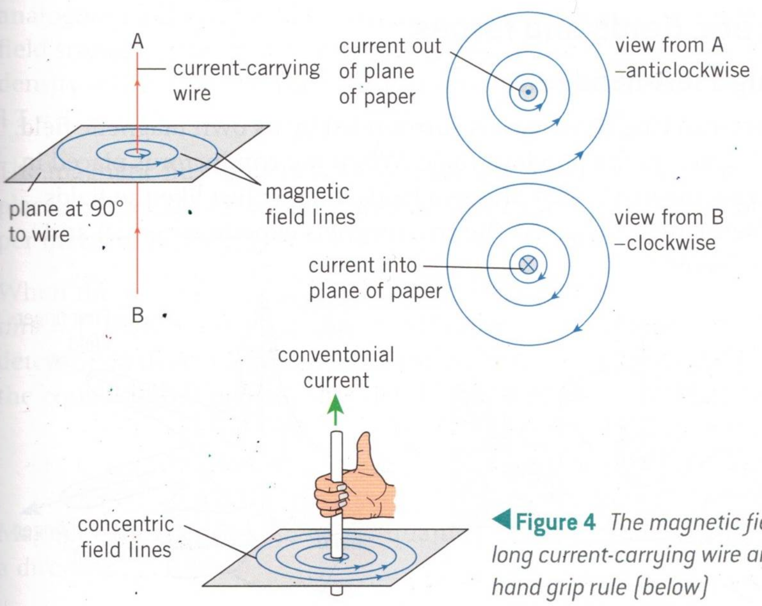
Current-carrying conductors

For a current-carrying wire, the magnetic field lines are concentric circles centred on the wire and perpendicular to it. The direction of the magnetic field can be determined using the **right-hand grip rule**, shown in Figure 4. The thumb points in the direction of the conventional current, and the direction of the field is given by the direction in which the fingers curl around the wire.

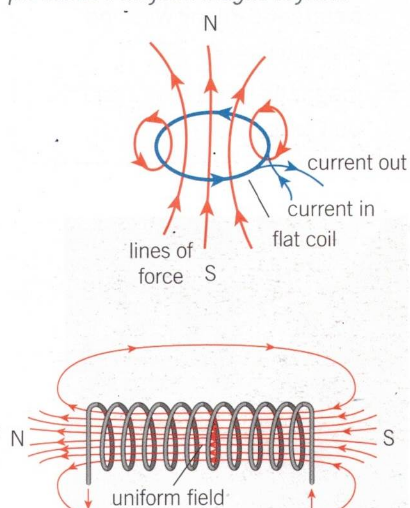
The magnetic field patterns produced by a single coil and a solenoid are shown in Figure 5. Both the coil and the solenoid produce north and south poles at their opposite faces. The magnetic field pattern outside solenoid is similar to that for a bar magnet, and at the centre of the core of the solenoid it is uniform – you can tell this from parallel and equidistant magnetic field lines.



▲ **Figure 3** *Field patterns for attracting and repelling poles – which pair of poles produces a uniform magnetic field?*



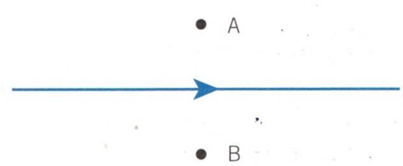
◀ **Figure 4** *The magnetic field around a long current-carrying wire and the right-hand grip rule (below)*



▲ **Figure 5** *Magnetic fields created by a current-carrying flat coil and a solenoid. You can use your right hand again to get the direction of the magnetic field for the solenoid. The fingers point in the direction of the conventional current and the thumb gives the direction of the magnetic field within the core of the solenoid.*

Summary questions

1. State and explain whether each of the following moving particles will produce a magnetic field.
   a an electron; **b** a proton; **c** a neutron. *(3 marks)*
2. State two methods of producing a uniform magnetic field. *(2 marks)*
3. A horizontal current-carrying wire is shown in Figure 6. State the direction of the magnetic field at points A and B. *(2 marks)*
4. Suggest how the magnetic field pattern for a solenoid within its core (Figure 4) would change when the current is both reversed and increased. *(2 marks)*
5. Figure 7 shows the top view of two long and straight current-carrying wires placed very close to each other. The current in each wire is into the plane of the paper. Sketch the magnetic field pattern around these current-carrying wires. *(2 marks)*



▲ **Figure 6**

▲ **Figure 7**

23.2 Understanding magnetic fields

Specification reference: 6.3.1

Learning outcomes

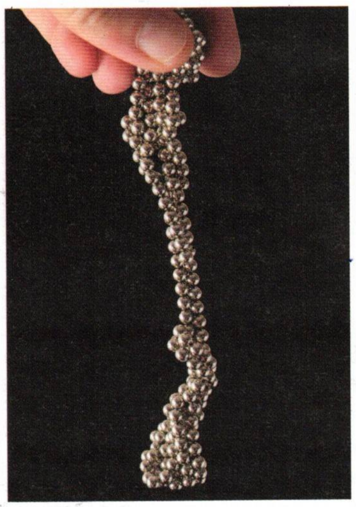
Demonstrate knowledge, understanding, and application of:

→ Fleming's left-hand rule

→ the force on a current-carrying conductor, $F = BIL\sin\theta$

→ the techniques and procedures used to determine the uniform magnetic flux density between the poles of a magnet using a current-carrying wire and digital balance

→ magnetic flux density and the unit tesla.



▲ **Figure 1** *Strong spherical magnets made from rare earth elements*

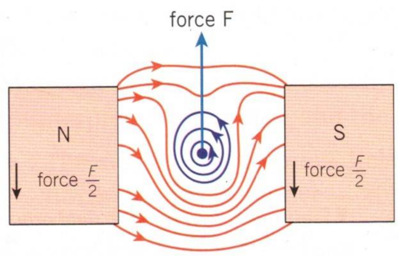
How strong?

Magnets made of alloys of neodymium, a rare earth element, are amongst the strongest available in the world. A coin-sized neodymium magnet can lift about 9 kg. Such magnets have enabled designers and engineers to reduce the size of devices such as the wafer-thin speakers used in mobile phones and electric motors for hybrid cars. The strength of magnets, and magnetic fields, is measured in **tesla** (T). The strength of a rare-earth magnet is about 1.3 T, compared with about 30 μT for the Earth's magnetic field at the equator.

Magnetic fields and forces

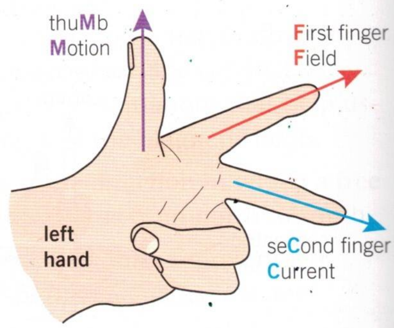
Fleming's left-hand rule

A current-carrying conductor is surrounded by its own magnetic field, as you learned in the previous topic. When the conductor is placed in an external magnetic field, the two fields interact just like the fields of two permanent magnets. The two magnets experience equal and opposite forces.



example of three-dimensional version:

▲ **Figure 2** *The distorted magnetic field of the wire is responsible for catapulting it away from the poles*



▲ **Figure 3** *Fleming's left-hand rule*

Figure 2 shows the resultant field pattern when a current-carrying wire is placed between the poles of a magnet. The direction of the force experienced by a current-carrying conductor placed perpendicular to the external magnetic field can be determined using **Fleming's left-hand rule**. Use your left hand as shown in Figure 3. The direction of your:

- first finger gives the direction of the external magnetic field
- second finger gives the direction of the conventional current
- thumb gives the direction of motion (force) of the wire

Magnetic flux density and the tesla

The magnitude of the force experienced by a wire in an external magnetic field depends on a number of factors. For example, the force is a maximum when the wire is perpendicular to the field and zero when it is parallel to the magnetic field. Experiments show that the magnitude of the force F experienced by the wire is directly proportional to the

- current I
- length L of the wire in the magnetic field
- $\sin\theta$, where θ is the angle between the magnetic field and the current direction
- the strength of the magnetic field.

Therefore

$$F = BIL\sin\theta$$

where B is the **magnetic flux density** – the strength of the field. It is analogous to electric field strength E for electric fields and gravitational field strength g for gravitational fields. The SI unit for magnetic flux density is the tesla (T). You can see from the equation above that $1\,\text{T} = 1\,\text{N}\,\text{m}^{-1}\,\text{A}^{-1}$.

The magnetic flux density is 1 T when a wire carrying a current of 1 A placed perpendicular to the magnetic field experiences a force of 1 N per metre of its length.

When the wire is perpendicular to the magnetic field, $\theta = 90°$ and $\sin\theta = 1$, therefore $F = BIL$. The direction of the force can be determined using Fleming's left-hand rule. You can therefore write the equation for magnetic flux density as

$$B = \frac{F}{IL}$$

Magnetic flux density is a vector quantity. It has both magnitude and a direction.

Worked example: Lifting up

A thin wire of weight $1.8 \times 10^{-3}\,\text{N}\,\text{cm}^{-1}$ is horizontal and perpendicular to a magnetic field of uniform flux density 0.15 T. The current in the wire is slowly increased from zero. The wire experiences a force vertically upwards. Calculate the size of the current in the wire such that the wire just starts to lift itself vertically.

Step 1: Write down the information given in the question.

$F = 1.8 \times 10^{-3}\,\text{N}$, $L = 0.01\,\text{m}$, $B = 0.15\,\text{T}$, $\theta = 0$

Step 2: For the wire to start moving, the force acting on it must be equal to its weight. Rearrange the equation $F = BIL$ with current I as the subject and then substitute all the values.

$$I = \frac{F}{BL} = \frac{1.8 \times 10^{-3}}{0.15 \times 0.01} = 1.2\text{A}$$

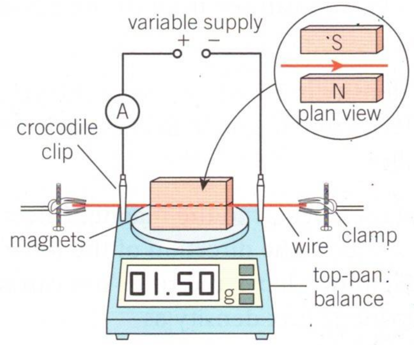
A current of 1.2 A in the wire will just start to lift the wire vertically.

Study tip

Remember that the equation $F = BIL$ only applies when B and I are perpendicular.

Determining magnetic flux density in the laboratory

Figure 4 shows apparatus for determining the magnetic flux density between two magnets. The magnets are placed on a top-pan balance. The magnetic field between them is almost uniform. A stiff copper wire is held perpendicular to the magnetic field between the two poles. The length L of the wire in the magnetic field is measured with a ruler. Using crocodile clips, a section of the wire is connected in series with an ammeter and a variable power supply. The balance is zeroed when there is no current in the wire. With a current I, the wire experiences a vertical upward force (predicted by Fleming's left-hand rule). According to Newton's third law of motion, the magnets experience an equal downward force, F, which can be calculated from the change in the mass reading, m, using $F = mg$, where g is the acceleration of free fall ($9.81\,\mathrm{m\,s^{-2}}$). The magnetic flux density B between the magnets can then be determined from the equation $B = \frac{F}{IL}$.



▲ **Figure 4** *An arrangement for determining B in the laboratory*

▼ **Table 1** *Results of an experiment using the apparatus in Figure 4*

Current I/A	Change in mass m/g	Force F/N
0.00	0	
1.00	0.31	
2.00	0.64	
3.00	0.89	
4.00	1.24	
5.00	1.50	
6.00	1.83	
7.00	2.14	

Analysing results

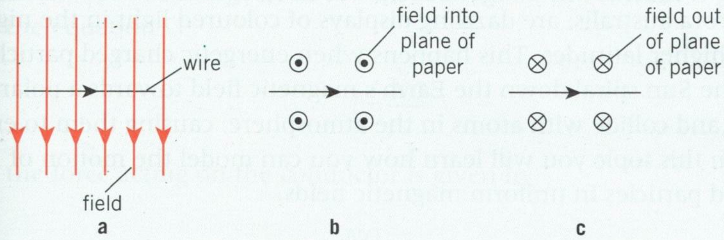
A student uses the arrangement in Figure 4 to determine the magnetic flux density B between two flat magnets held in a yoke. Table 1 shows the results.

1. Copy the table and complete the last column.
2. Plot a graph of force F against current I. Draw a straight best-fit line through the points.
3. Show that the gradient of the line graph is BL, where L is the length of the wire in the magnetic field.
4. The value of L is recorded as 5.0 ± 0.3 cm by the student. Determine the gradient and hence the value for B for the arrangement of the magnets. State the absolute uncertainty in your answer.

Summary questions

1 Explain why a current-carrying wire experiences a force when placed close to a magnet. *(1 mark)*

2 A current-carrying conductor is placed in a uniform magnetic field. In each case in Figure 5, use Fleming's left-hand rule to determine the direction of the force experienced by the conductor. *(3 marks)*



▲ **Figure 5**

3 Calculate the force per centimetre length on a straight wire placed perpendicular to a magnetic field of flux density 120 mT and carrying a current of 5.0 A. *(2 marks)*

4 A current-carrying wire placed perpendicular to a uniform magnetic field experiences a force of 5.0 mN. Determine the force on the wire when, separately:

a the current in the wire is quadrupled *(1 mark)*

b the magnetic flux density is doubled *(1 mark)*

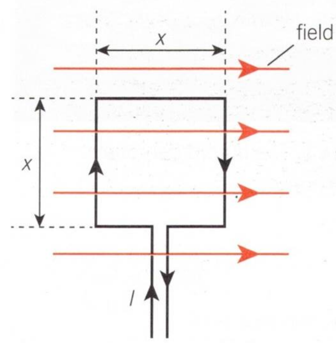
c the length of wire in the wire is reduced to 30% of its original length. *(1 mark)*

5 A 2.8 cm length of copper wire carrying a current of 0.80 A is placed in a uniform magnetic field. The angle between the wire and the magnetic field is 38°. It experiences a force of 4.0 mN. Calculate the magnetic flux density of the field. *(3 marks)*

6 Figure 6 shows a square loop of wire of a simple electric motor placed in a uniform magnetic field of flux density B. The current in the loop is I.

a State and explain the initial direction of rotation of this coil. *(2 marks)*

b Show that torque of the couple acting on the loop is directly proportional to the cross-sectional area of the loop. *(3 marks)*



▲ **Figure 6**

23.3 Charged particles in magnetic fields

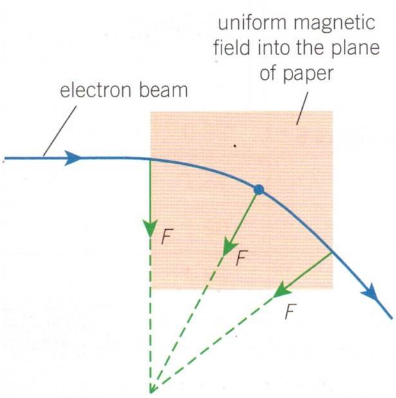
Specification reference: 6.3.2

Learning outcomes

Demonstrate knowledge, understanding, and application of:

- → the force on a charged particle travelling at right angles to a uniform magnetic field, $F = BQv$
- → movement of charged particles in a uniform magnetic field
- → movement of charged particles moving in a region occupied by both electric and magnetic fields
- → velocity selector.

▲ **Figure 1** *The aurora borealis or northern lights*



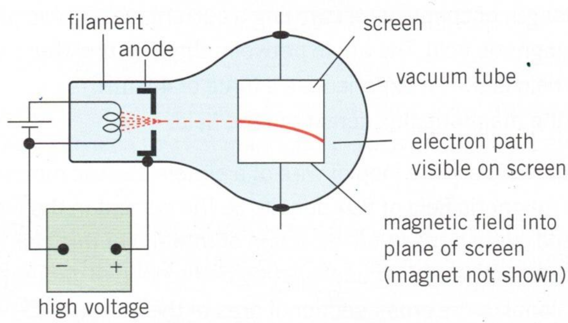
▲ **Figure 3** *The electrons travel in a circular path in the region of the uniform perpendicular magnetic field, and the force on the electrons is always at right angles to their motion*

The aurora

The aurora borealis, or northern lights, and its southern equivalent the aurora australis, are dazzling displays of coloured light in the night sky at higher latitudes. This happens when energetic charged particles from the Sun spiral down the Earth's magnetic field towards a polar region and collide with atoms in the atmosphere, causing them to emit light. In this topic you will learn how you can model the motion of charged particles in uniform magnetic fields.

Circular tracks

A charged particle moving in a magnetic field will experience a force. This effect can be demonstrated for a beam of electrons using an electron deflection tube (Figure 2). The force on the beam of electrons can be predicted using Fleming's left-hand rule. The beam of electrons is moving from left to right into a region of uniform magnetic field, shown in more detail in Figure 3. As the electrons enter the field, they experience a downward force. The electrons change direction, but the force F on each electron always remains perpendicular to its velocity. The speed of the electrons remains unchanged because the force has no component in the direction of motion. Once out of the field, the electrons keep moving in a straight line.



▲ **Figure 2** *An electron deflection tube*

A current-carrying wire in a uniform magnetic field experiences a force because each electron moving within the wire experiences a tiny force.

To find the force F acting on a charged particle of charge Q moving at a speed v at right angles to a uniform magnetic field of flux density B, consider a section of conductor, or a beam of charged particles (Figure 4). In a time t, all the charged particles contained within the shaded region go through section XY. The length L of the shaded region is vt, where v is the speed of the charged particle. The force F on the conductor is given by

$$F = BIL$$

Therefore

$$F = BI(vt)$$

However, current I is the rate of flow of charge. If there are N charged particles, each of charge Q, in the shaded region, the current is given by the equation

$$I = \frac{NQ}{t}$$

So the force acting on the conductor is given by

$$F = B \times \frac{NQ}{t} \times vt = NBQv$$

The force F on *each* charged particle must therefore be

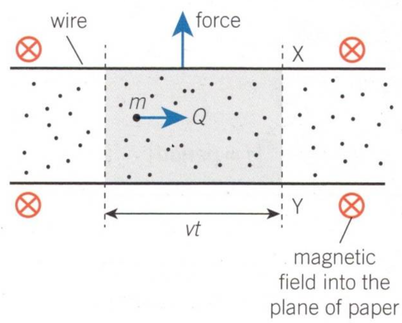
$$F = \frac{NBQv}{N} = BQv$$

For an electron, or a proton, where $Q = e = 1.60 \times 10^{-19}$ C, this equation may be written as

$$F = Bev$$

Study tip

Check the direction in Figure 3 using Fleming's left-hand rule. Remember the conventional current is from right to left.



▲ **Figure 4** *Every charged particle experiences a tiny force within the conductor*

Worked example: Colossal acceleration

An electron travels perpendicular to a magnetic field of flux density 0.15 T. Calculate the acceleration of the electron given its speed is $5.0 \times 10^6\,\text{m s}^{-1}$.

Step 1: Write down the information that you have.

$B = 0.15\,\text{T}$, $v = 5.0 \times 10^6\,\text{m s}^{-1}$, $Q = e = 1.60 \times 10^{-19}\,\text{C}$

Step 2: Select the equations that you need.

The force acting on the electron is given by $F = BQv$ and the acceleration a can be calculated from $F = ma$. (The mass m of the electron is given in the Data Booklet.)

$F = BQv = ma$

$$a = \frac{BQv}{m} = \frac{0.15 \times 1.60 \times 10^{-19} \times 5.0 \times 10^6}{9.11 \times 10^{-31}} = 1.3 \times 10^{17}\,\text{m s}^{-2} \text{ (2 s.f.)}$$

Going round

Consider a charged particle of mass m and charge Q moving at right angles to a uniform magnetic field of flux density B. The particle will describe a circular path because the force acting on it is always perpendicular to its velocity. The centripetal force $\frac{mv^2}{r}$ on the particle is provided by the magnetic force BQv. Therefore

$$BQv = \frac{mv^2}{r} \quad \text{or} \quad r = \frac{mv}{BQ}$$

Synoptic link

In Topics 16.2, Angular acceleration, and 16.3, Centripetal force, you learnt about centripetal forces and accelerations. These ideas are very useful here too.

Study tip

The following equations are very useful when solving problems in which charged particles travel in circular paths:

- $F = BQv$
- $F = ma$
- $F = \frac{mv^2}{r}$
- $v = \frac{2\pi r}{T}$ (T = period)

The equation for the radius r shows that:

- faster-moving particles travel in bigger circles ($r \propto v$)
- more massive particles move in bigger circles ($r \propto m$)
- stronger magnetic fields make the particles move in smaller circles ($r \propto \frac{1}{B}$)
- particles with greater charge move in smaller circles ($r \propto \frac{1}{Q}$).

Worked example: Electrons in a magnetic field

A beam of electrons describes a circular path of radius 15 mm in a uniform magnetic field. The speed of the electrons is $8.0 \times 10^6\,\mathrm{m\,s^{-1}}$. Calculate the magnetic flux density B of the magnetic field.

Step 1: Write down the information given in the question.

$r = 1.5 \times 10^{-2}\,\mathrm{m}$, $v = 8.0 \times 10^6\,\mathrm{m\,s^{-1}}$, $Q = e = 1.60 \times 10^{-19}\,\mathrm{C}$

Step 2: Derive an equation for B from first principles and then substitute the values in.

$$BQv = \frac{mv^2}{r}$$

$$B = \frac{mv}{Qr} = \frac{9.11 \times 10^{-31} \times 8.0 \times 10^6}{1.60 \times 10^{-19} \times 1.5 \times 10^{-2}} = 3.0 \times 10^{-3}\,\mathrm{T}\ (2\ \text{s.f.})$$

The magnetic flux density is about 3.0 mT.

Velocity selector

A **velocity selector** is a device that uses both electric and magnetic fields to select charged particles of specific velocity. It is a vital part of instruments such as mass spectrometers and some particle accelerators. It consists of two parallel horizontal plates connected to a power supply (Figure 5). They produce a uniform electric field of field strength E between the plates. A uniform magnetic field of flux density B is also applied perpendicular to the electric field. The charged particles travelling at different speeds to be sorted enter through a narrow slit Y. The electric and magnetic fields deflect them in opposite directions – only for particles with a specific speed v will these deflections cancel so that they travel in a straight line and emerge from the second narrow slit Z. For an undeflected particle

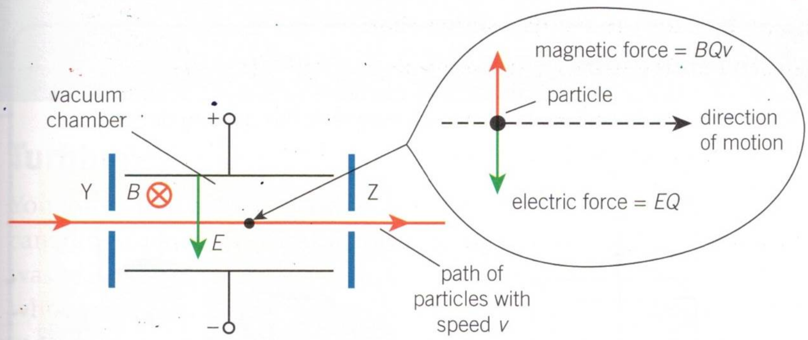
$$\text{electric force} = \text{magnetic force}$$

$$EQ = BQv$$

where Q is the charge on the particle. Thus the speed v depends only on E and B, that is

$$v = \frac{E}{B}$$

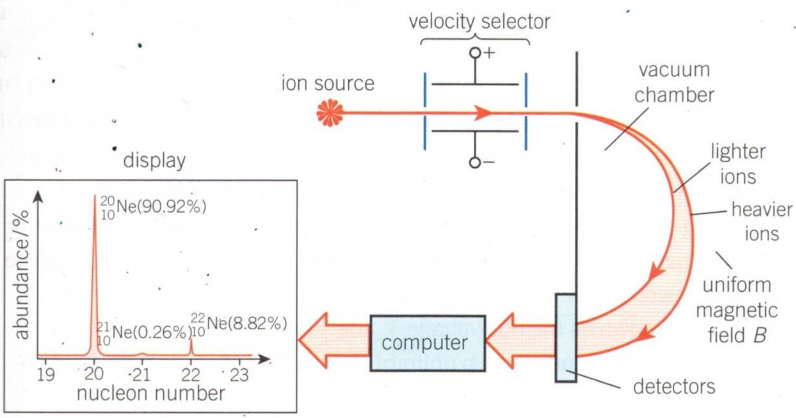
When E is $4.0 \times 10^5\,\mathrm{V\,m^{-1}}$ and $B = 0.10\,\mathrm{T}$, only particles with a speed of $4.0 \times 10^6\,\mathrm{m\,s^{-1}}$ will emerge from the slit Z.



▲ **Figure 5** *A velocity selector*

Mass spectrometers

Mass spectrometers measure the masses and relative concentrations of atoms and molecules. They are used for all kinds of chemical analyses, from detecting the age of ancient rocks to examining pharmaceuticals. Figure 6 shows the basic structure of a mass spectrometer.



▲ **Figure 6** *A mass spectrometer*

Atoms from a sample are ionised and accelerated through a potential difference. They pass through a velocity selector and emerge with the same speed v before entering a uniform magnetic field of flux density B. The radius r of curvature of each ion is given by the equation $r = \frac{mv}{BQ}$, where Q is the charge on the ion and m is its mass . For a singly ionised atom, $Q = e$.

Since $r \propto m$, each different ion is deflected by a different amount onto the detector. The detector is connected to a computer programmed to show the relative abundance of each type of ion. Modern mass spectrometers are capable of identifying relative abundances as small as 1 in 10^{14} ions.

1. Suggest why it is important that all the ions have the same speed in a mass spectrometer.
2. Calculate the radius of curvature for a singly ionised carbon-13 ion travelling at a speed of $8.00 \times 10^4\,\mathrm{m\,s^{-1}}$ in a mass spectrometer with a field of flux density 0.750 T. The mass of the ion is 2.16×10^{-26} kg.
3. Estimate the radius of curvature of a singly ionised carbon-14 ion in the same mass spectrometer.

Synoptic link

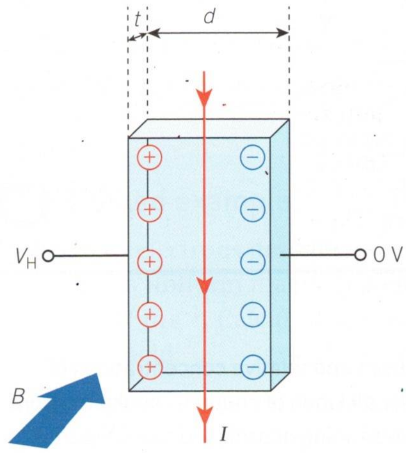
You first met number density and the equation $I = Anev$ in Topic 8.4, Mean drift velocity.

Summary questions

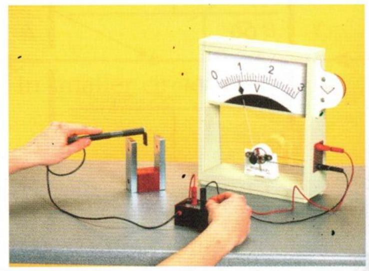
1. Explain why a stationary charged particle in a magnetic field does not experience a magnetic force. *(1 mark)*
2. Calculate the maximum magnetic force experienced by an electron travelling at a speed of $6.0 \times 10^5\,\mathrm{m\,s^{-1}}$ in a uniform field of flux density 0.20 T. *(2 marks)*
3. A particle of charge +2e describes a circle of radius 2.5 cm in a uniform magnetic field of flux density 130 mT. Calculate its momentum. *(3 marks)*
4. The parallel plates of a velocity selector are connected to a 1.3 kV supply and have a separation of 2.5 cm. It selects particles of speed $4.0 \times 10^5\,\mathrm{m\,s^{-1}}$. Calculate the magnetic flux density of the magnetic field used in the velocity selector. *(3 marks)*
5. A proton travelling at $4.0 \times 10^6\,\mathrm{m\,s^{-1}}$ describes a circular path in a uniform magnetic field of flux density 800 mT. Calculate:
   - **a** the radius of the path; *(3 marks)*
   - **b** the period of the proton. *(2 marks)*
6. Show that the period of the proton in question 5 is independent of its speed. *(3 marks)*

The Hall probe

A Hall probe is a device used to measure magnetic flux density directly.



▲ **Figure 7** *Generation of a Hall voltage*



▲ **Figure 8** *Use of a Hall probe – the probe is being held between the poles of a U-shaped magnet, and a voltmeter. It is being used to detect the Hall voltage. The effect is strongest when the main current is aligned at right angles to the magnetic field, so rotating the probe causes the voltage to drop*

Figure 7 shows a thin slice of a semiconductor. It has thickness t and width d and carries a current I in the direction shown. An external magnetic field of flux density B is applied at right angles to the direction of the current. According to Fleming's left-hand rule, the electrons will be deflected towards the right-hand surface, where they accumulate, leaving the left-hand surface of the semiconductor with fewer electrons. As a result, a small potential difference, known as the Hall voltage, V_H, develops across the slice. The accumulated electrons create a uniform electric field of magnitude E, where

$$E = \frac{V_H}{d}$$

The Hall voltage V_H is given by the equation

$$V_H = \frac{BI}{nte}$$

where e is the elementary charge and n is the number density of the electrons within the semiconductor.

1. The internal electric field and the external magnetic field make the electrons travel undeflected through the semiconductor. The current is given by the equation $I = Anev$. Use this equation and the principles of a velocity selector to derive the equation
$$V_H = \frac{BI}{nte}$$
2. Suggest why semiconductors are preferable to metals in a Hall probe.
3. A flux density of 60 mT produces a Hall voltage of 14 mV. Calculate the Hall voltage when the flux density is 1.2 T.