19.6 Analysing starlight

Specification reference: 5.5.2

Colours on an optical disc

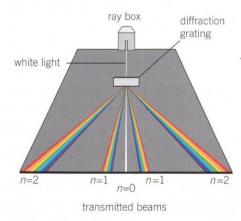
When white light shines onto an optical disc, it splits into beams of different colours (Figure 1) – try it by reflecting sunlight off the back of a DVD onto a wall. The disc has millions of equally spaced lines of microscopic pits on its surface that diffract the light to form an interference pattern.

A diffraction grating is an optical component with regularly spaced slits or lines that diffract and split light into beams of different colour travelling in different directions. These beams can be analysed to determine the wavelengths of spectral lines in the laboratory or from starlight.

The diffraction grating

The fringes produced by passing light through a double slit are not very sharp, so it can be difficult to determine the position of the centre of each maximum. To overcome this limitation, a transmission diffraction grating can be used in place of the double slit. The grating consists of a large number of lines ruled on a glass or plastic slide. There can be as many as 1000 lines in a millimetre. Each line diffracts light like a slit. Using a large number of lines produces a clearer and brighter interference pattern.

When light passes through a diffraction grating it is split into a series of narrow beams. The direction of these beams depends on the spacing of the lines, or slits, of the grating and the wavelength of the light. Therefore, when white light is passed through a diffraction grating it splits into its component colours, making gratings especially useful in spectroscopy.



▲ Figure 2 Use of a diffraction grating

Forming maxima

Consider monochromatic light incident normally at a diffraction grating. The light is diffracted at each slit, and the interference pattern is the result of superposition of the diffracted waves in the space beyond the grating. Just as with the interference pattern created by a double slit, the formation of a maximum at a particular point depends on the path difference and the phase difference of the waves from all the slits.

Learning outcomes

Demonstrate knowledge, understanding, and application of:

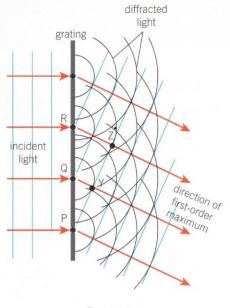
- use of a transmission diffraction grating to determine the wavelength of light
- ⇒ the condition for maxima $d \sin \theta = n\lambda$, where d is the grating spacing.

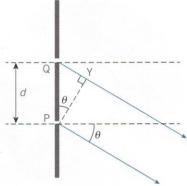


▲ Figure 1 The small pits and grooves found on optical discs act like a diffraction grating, splitting light into beams of different colours

Synoptic link

You saw in Topic 12.3, the Young double-slit experiment, that when light passes through a double slit it produces an interference pattern as a series of bright and dark fringes.





▲ Figure 3 Formation of the first-order

Study tip

The largest possible angle θ = 90°. This makes $\sin\theta$ = 1. The largest possible order number n that you can observe is therefore given by the equation $n_{\rm max}\lambda$ = d.

The zero-order maximum, n = 0, is formed when the path difference is zero, that is, at an angle $\theta = 0$. The angle θ is measured relative to the normal to the grating or to the direction of the incident light. Figure 3 shows the formation of one of the two first-order maxima, n = 1. The waves from adjacent slits P and Q have a path difference of exactly one whole wavelength λ . The same is true for waves from any two adjacent slits on the grating. Therefore, the distance QY is λ , distance RZ is 2λ and so on.

For the two nth-order maxima, the path difference QY at an angle θ will be equal to $n\lambda$ (in Figure 3, n=1). The distance PQ is the separation between adjacent lines or slits on the grating. This distance is called the **grating spacing** d. From the triangle PQY, you can see that

$$\sin\theta = \frac{QY}{QP} = \frac{n\lambda}{d} \quad \text{or}$$
$$d\sin\theta = n\lambda$$

where n is an integer with values 0, 1, 2, etc. The equation is known as the **grating equation** and can be used to accurately determine the wavelength of monochromatic light.



Worked example: Finding the grating spacing

Monochromatic light from a laser of wavelength 532 nm is incident normally at a diffraction grating. The angle between the second-order maximum and the zero-order maximum is measured to be 32° . Calculate the grating spacing d.

Step 1: Rearrange $d \sin \theta = n\lambda$ for d.

$$d = \frac{n\lambda}{\sin \theta}$$

Step 2: Since the angle between the second-order maximum and the zero-order maximum is used, n = 2.

Therefore
$$d = \frac{2 \times 532 \times 10^{-9}}{\sin 32^{\circ}} = 2.0 \times 10^{-6} \,\text{m} \ (2 \text{ s.f.})$$



Using a diffraction grating to determine the wavelength of light

Like the double-slit experiment, a diffraction grating can be used to determine the wavelength of monochromatic light. Measuring the angle between several maxima and the zero-order maximum and then plotting a graph of $\sin\theta$ against n will produce a straight line through the origin with a gradient of $\frac{\lambda}{d}$.

Many diffraction gratings are not labelled with the value of the grating spacing but with the numbers of lines (slits) per mm. Since grating spacing = $(\frac{1}{\text{lines per metre}})$, a grating with 600 lines/mm has 600 000 lines/m and so a grating spacing of 1.67×10^{-6} m.

- A laser emitting red light shines through a diffraction grating with 200 lines/mm. The angles between the zero-order maximum and the first six maxima are measured (Table 1).
 - ▼ Table 1 Angle measurement for maxima in a diffraction pattern

n	1	2	3	4	5	6
θ/°	7.2	14.7	22.2	30.6	39.0	49.2

- a Plot a graph of $\sin \theta$ against n.
- Use your graph to determine the wavelength of the light emitted by the laser.
- Calculate the largest number of orders that can be observed in this experiment.
- On your graph sketch two other lines to show the relationship between n and θ if:
 - a diffraction grating with twice as many lines/mm is used;
 - a laser with a shorter wavelength is used.

Study tip

Remember the largest number of maxima you can see is rounded down. For example, if you calculate n = 5.7, maxima for n = 0 to n = 5are visible - a total of 11 maxima.

Summary questions

- Suggest why the maxima produced from a diffraction grating are brighter than those produced via the double-slit experiment. (2 marks)
- Explain why the highest order maxima visible through a diffraction grating is given by $\frac{a}{2}$. (2 marks)
- 3 A diffraction grating with grating spacing of 3.3×10^{-6} m is used to observe light from a star. The spectral line produces a first-order image at a diffraction angle of 8.6°. Calculate the wavelength of this spectral line.

(3 marks)

Calculate the angle of the third-order maximum when light of wavelength 450 nm is incident on a diffraction grating with 350 lines/mm.

(4 marks)

5 Calculate the maximum number of orders that can be observed with the arrangement in question 4. (2 marks)

6 A spectral line from a distant star is analysed using a diffraction grating with a grating spacing of 2.5×10^{-6} m. An absorption line in the first-order spectrum is observed at an angle of 13.4°. Calculate the energy in eV of the photons responsible for this spectral line. (4 marks)



▲ Figure 4 The diffraction pattern from a monochromatic light source obtained using a diffraction grating