Binomial Expansions - Answers

Exercise A

1 a
$$16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4$$

b $p^5 - 5p^4q + 10p^3q^2 - 10p^2q^3 + 5pq^4 - q^5$
c $1 + 8x + 24x^2 + 32x^3 + 16x^4$
d $81 + 108x + 54x^2 + 12x^3 + x^4$
e $1 - 2x + \frac{3}{2}x^2 - \frac{1}{2}x^3 + \frac{1}{16}x^4$
f $256 - 256x + 96x^2 - 16x^3 + x^4$
g $32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$
h $x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + 64$

2 a
$$90x^3$$
 b $80x^3y^2$ c $-20x^3$
d $720x^3$ e $120x^3$ f $-4320x^3$
g $1140x^3$ h $-241920x^3$

3 a
$$1 + 10x + 45x^2 + 120x^3$$

b $1 - 10x + 40x^2 - 80x^3$
c $1 + 18x + 135x^2 + 540x^3$
d $256 - 1024x + 1792x^2 - 1792x^3$
e $1024 - 2560x + 2880x^2 - 1920x^3$
f $2187 - 5103x + 5103x^2 - 2835x^3$
g $x^8 + 16x^7y + 112x^6y^2 + 448x^5y^3$
h $512x^9 - 6912x^8y + 41472x^7y^2 - 145152x^6y^3$

4
$$a = \pm \frac{1}{2}$$

5 $b = -2$
6 $1, \frac{5 \pm \sqrt{105}}{8}$

7
$$1 - 0.6x + 0.15x^2 - 0.02x^3$$
, 0.941 48, accurate to 5 dp
8 $1024 + 1024x + 460.8x^2 + 122.88x^3$, 1666.56, accurate to 3 sf

Exercise B

1 a
$$1 + 8x + 28x^2 + 56x^3$$

b $1 - 12x + 60x^2 - 160x^3$
c $1 + 5x + \frac{45}{4}x^2 + 15x^3$
d $1 - 15x + 90x^2 - 270x^3$
e $128 + 448x + 672x^2 + 560x^3$
f $27 - 54x + 36x^2 - 8x^3$
g $64 - 576x + 2160x^2 - 4320x^3$
h $256 + 256x + 96x^2 + 16x^3$
i $128 + 2240x + 16800x^2 + 70000x^3$
3 $a = 162, b = 135, c = 0$
4 a $p = 5$ b -10 c -80
5 $1 + 16x + 112x^2 + 448x^3, 1.171648$, accurate to 4 sf

Exercise C

1 **a**
$$p = 16$$
 b 270 **c** -1890
2 **a** $A = 8192$, $B = -53248$, $C = 159744$
3 **a** $1 - 20x + 180x^2 - 960x^3$
b 0.81704 , $x = 0.01$
4 **a** $1024 - 15360x + 103680x^2 - 414720x^3$
b 880.35
5 **a** $81 + 216x + 216x^2 + 96x^3 + 16x^4$
b $81 - 216x + 216x^2 - 96x^3 + 16x^4$
c 1154
6 **a** $n = 8$ **b** $\frac{35}{8}$
7 **a** $81 + 1080x + 5400x^2 + 12000x^3 + 10000x^4$
b 1012054108081 , $x = 100$
8 **a** $1 + 24x + 264x^2 + 1760x^3$
b 1.26816
c 1.268241795
d 0.00645% (3 sf)
9 $x^5 - 5x^3 + 10x - \frac{10}{x} + \frac{5}{x^3} - \frac{1}{x^5}$
10 b $\frac{4096}{729} + \frac{2048}{81}x + \frac{1280}{27}x^2 + \frac{1280}{27}x^3$
11 **a** $64 + 192x + 240x^2 + 160x^3 + 60x^4 + 12x^5 + x^6$
b $k = 1560$
12 **a** $k = 1.25$ **b** 3500
13 **a** $A = 64$, $B = 160$, $C = 20$ **b** $x = \pm \sqrt{\frac{3}{2}}$
14 **a** $p = 1.5$ **b** 50.625