Year 12 Mathematics

Test 1

Time Allowed: 1 Hour

Total Marks: 50

31 October 2021

Calculator Allowed

Full Name of Student: …………………………………………………………

1.

$$x^2 + 2x + 3 \equiv (x + a)^2 + b.$$

(a) Find the values of the constants a and b.

(2)

(b) Sketch the graph of $y = x^2 + 2x + 3$, indicating clearly the coordinates of any intersections with the coordinate axes.

(3)

(c) Find the value of the discriminant of $x^2 + 2x + 3$. Explain how the sign of the discriminant relates to your sketch in part (b).

(2)

The equation $x^2 + kx + 3 = 0$, where k is a constant, has no real roots.

(d) Find the set of possible values of k, giving your answer in surd form.

(4)

[Total for Question 1 = 11 marks]

2.

The polynomial p(x) is given by

$$\mathrm{p}(x) = x^3 + x^2 - 10x + 8$$

(a) (i) Using the factor theorem, show that $x - 2$ is a factor of p(x). *(2 marks)*

  (ii) Hence express p(x) as the product of three linear factors. *(3 marks)*

(b) Sketch the curve with equation $y = x^3 + x^2 - 10x + 8$, showing the coordinates of the points where the curve cuts the axes.

(4 marks)

[Total for Question 2 = 9 marks]

3.

Solve the equation $x^{\frac{2}{3}} + 3x^{\frac{1}{3}} - 10 = 0$.

[3]

[Total for Question 3 = 3 marks]

4.

Find the set of values of x for which

(a) $3(2x + 1) > 5 - 2x$,

(2)

(b) $2x^2 - 7x + 3 > 0$,

(4)

(c) **both** $3(2x + 1) > 5 - 2x$ **and** $2x^2 - 7x + 3 > 0$.

(2)

[Total for Question 4 = 8 marks]

5.

(i) Find the first four terms of the expansion, in ascending powers of x, of $(1 - 2x)^{12}$.

[4]

(ii) Hence find the coefficient of x^2 in the expansion of

$$(1 + 3x)\,(1 - 2x)^{12}.$$

[3]

(iii) Use your expansions in part (i) to find an estimate for the value of 0.98^{12}.

[2]

[Total for Question 5 = 9 marks]

6.

(a) **(i)** Show that the equation

$$3\cos^2\theta = \sin\theta + 1$$

can be expressed in the form

$$3\sin^2\theta + \sin\theta - 2 = 0.$$

[2]

(ii) Hence solve the equation

$$3\cos^2\theta = \sin\theta + 1,$$

giving all values of θ between $0°$ and $360°$.

[5]

(b) Solve the equation $\sin(\theta - 30°) = 0.7$, giving your answers to the nearest $0.1°$ in the interval $0° \leqslant \theta \leqslant 360°$.

[3]

[Total for Question 6 = 10 marks]

- End of Test -