Simple Harmonic Motion-1

Exercise A

1.

An object suspended from the lower end of a vertical spring is displaced downwards from equilibrium. It takes 9.6 s to undergo 20 complete cycles of oscillation. Calculate:

a its time period,

b its frequency of oscillation.

2.

Two identical pendulums X and Y each consist of a small metal sphere attached to a thread of a certain length. Each pendulum makes 20 complete cycles of oscillation in 16 s. State the phase difference, in radians, between the motion of X and that of Y if

a X passes through equilibrium 0.2 s after Y passes through equilibrium in the same direction,

b X reaches maximum displacement at the same time as Y reaches maximum displacement in the opposite direction.

Exercise B

1 A small object attached to the end of a vertical spring oscillates with an amplitude of 25 mm and a time period of 2.0 s. The object passes through equilibrium moving upwards at time $t = 0$. What is the displacement and direction of motion of the object:

a $\frac{1}{4}$ cycle later, b $\frac{1}{2}$ cycle later,

c $\frac{3}{4}$ cycle later, d 1 cycle later?

2 For the oscillations in Q1, calculate:

a the frequency,

b the acceleration of the object when its displacement is

i +25 mm, ii 0, iii −25 mm.

3 A simple pendulum consists of a small weight on the end of a thread. The weight is displaced from equilibrium and released. It oscillates with an amplitude of 32 mm, taking 20 s to execute 10 oscillations. Calculate:

a its frequency, b its initial acceleration.

4 For the oscillations in Q3, the object is released at time $t = 0$. State the displacement and calculate the acceleration when:

a $t = 1.0$ s, b $t = 1.5$ s.

Exercise C

1 An object oscillates in simple harmonic motion with a time period of 3.0 s and an amplitude of 58 mm. Calculate:

a its frequency, b its maximum acceleration.

2 The displacement of an object oscillating in simple harmonic motion varies with time according to the equation x (mm) = 12 cos 10t, where t is the time in seconds after the object's displacement was at its maximum positive value.

a Determine:

i the amplitude, ii the time period.

b Calculate the displacement of the object at $t = 0.1$ s.

3 An object on a spring oscillates with a time period of 0.48 s and a maximum acceleration of 9.8 m s^{-2}. Calculate:

a its frequency, b its amplitude.

4 An object oscillates in simple harmonic motion with an amplitude of 12 mm and a time period of 0.27 s. Calculate:

a its frequency,

b its displacement and its direction of motion

i 0.10 s, ii 0.20 s after its displacement was +12 mm.

Exercise D

1 An object was suspended from the end of a vertical spring and set into oscillating motion along a vertical line. The amplitude of its oscillations was 20 mm, and it took 6.5 s to perform 20 oscillations. Calculate:

a i its time period, ii its frequency,

b its acceleration when its displacement was

i 0 mm, ii 10 mm, iii 20 mm.

2 In the arrangement described in Q1, the object was replaced by an object of different mass. When the second object was oscillating vertically, its acceleration a at displacement x was given by $a = -360x$.

a Calculate:

i the frequency,

ii the time period of the oscillations.

b By comparing the frequency of the oscillations of the second object with that of the first, discuss whether the mass of the second object is greater than or less than the mass of the first object.

3 The upper end of a vertical spring of natural length 250 mm is attached to a fixed point. When a small object of mass 0.15 kg is attached to the lower end of the spring, the spring stretches to an equilibrium length of 320 mm.

a Calculate:

i the extension of the spring at equilibrium,

ii the spring constant.

b The object is displaced vertically from its equilibrium position and released. Show that it oscillates at a frequency of 1.9 Hz and calculate its period of oscillation.

4 A mass of 0.50 kg is attached to the lower end of a vertical spring that has a spring constant of $25\ N\ m^{-1}$. The mass is displaced downwards by a distance of 50 mm then released.

a Calculate:

i the force on the object at a displacement of 50 mm,

ii the acceleration of the object at the instant it was released.

b i Show that the acceleration a at displacement x is given by $a = -50x$.

ii Calculate the frequency of the oscillations and the displacement of the mass 0.050 s after it was released.

5 Calculate the time period of a simple pendulum:

a of length

i 1.0 m, ii 0.25 m.

b of length 1.0 m on the surface of the Moon, where $g = 1.6\ m\ s^{-2}$.

6 A simple pendulum and a mass suspended on a vertical spring have equal time periods on the Earth. Discuss whether or not they would have the same time periods on the surface of the Moon, where $g = 1.6\ m\ s^{-2}$.

Exercise E

$g = 9.81\ m\ s^{-2}$

1 A simple pendulum consists of a small metal sphere of mass 0.30 kg attached to a thread. The sphere is displaced through a height of 10 mm with the thread taut then released. It takes 15.0 s to make 10 complete cycles of oscillation.

a Calculate:

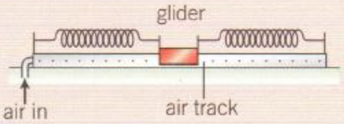
i the time period of the pendulum,

ii the length of the pendulum,

iii the initial potential energy of the pendulum relative to its equilibrium position.

b Sketch graphs on the same axes to show how the potential energy and the kinetic energy of the pendulum vary with its displacement from equilibrium.

2 A glider of mass 0.45 kg on a frictionless air track is attached to two stretched springs at either end, as shown in Figure 5. A force of 3.0 N is needed to displace the glider from equilibrium and hold it at a displacement of 50 mm. The glider is then released, and it oscillates freely on the air track.



▲ **Figure 5**

Calculate:

a i the spring constant k for the system,

ii the time period of the oscillations,

b i the initial potential energy of the system when the glider is held at a displacement of 50 mm,

ii the maximum kinetic energy of the glider,

iii the speed of the glider at a displacement of 25 mm.

3 a State whether the damping in each of the following examples is light, critical, or heavy:

i a child on a swing displaced from equilibrium then released,

ii oil in a U-shaped tube displaced from equilibrium then released.

b Discuss how effective a car suspension damper would be, if the oil in the damper was replaced by oil that was much more viscous.

4 The amplitude of an oscillating mass on a spring decreases by 4% each cycle from an initial amplitude of 100 mm. Calculate the amplitude after:

a 5 cycles of oscillation,

b 20 cycles of oscillation.

Exercise F

1 a A mass suspended on a vertical spring is made to oscillate by applying a periodic force of natural frequency f_0.

i Define resonance.

ii Explain why the frequency of the periodic force needs to be f_0 to cause resonance.

b With reference to the mass–spring system shown in Figure 1, state and explain what the effect would be on the resonant frequency of

i increasing the mass,

ii replacing the springs with stiffer springs.

2 A 0.12 kg mass suspended on a vertical spring is made to resonate by applying a periodic force of frequency 2.4 Hz to it. Calculate:

a the spring constant of the system,

b the frequency at which the system would resonate if the mass were doubled.

3 The panel of a washing machine vibrates loudly when the drum rotates at a particular frequency. Explain why this happens only when the drum rotates at this frequency.

4 A vehicle of mass 850 kg has a suspension system that is lightly damped. When it is driven without extra load by a driver of mass 50 kg over speed bumps spaced 15 m apart at a speed of 3.0 m s^{-1}, the vehicle resonates.

a Explain why this effect happens.

b Calculate the speed that resonance would occur at over the same speed bumps if the vehicle had also been carrying an extra load of 130 kg.