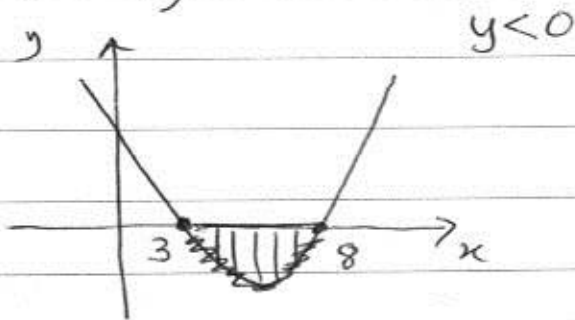


# Quadratic Inequalities

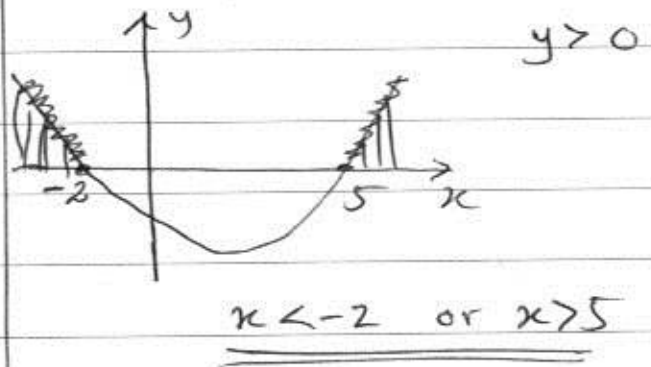
## - Answers

① (a)  $x^2 - 11x + 24 < 0$   
 (Sketch  $y = x^2 - 11x + 24$ )  
 Critical values:  
 $x^2 - 11x + 24 = 0$   
 $(x - 8)(x - 3) = 0$   
 $x = 3, 8$



$3 < x < 8$

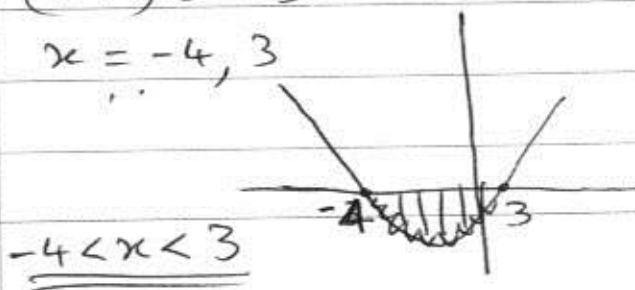
(c)  $x^2 - 3x - 10 > 0$   
 Critical values:  
 $x^2 - 3x - 10 = 0$   
 $(x - 5)(x + 2) = 0$   
 $x = 5, -2$



$x < -2$  or  $x > 5$

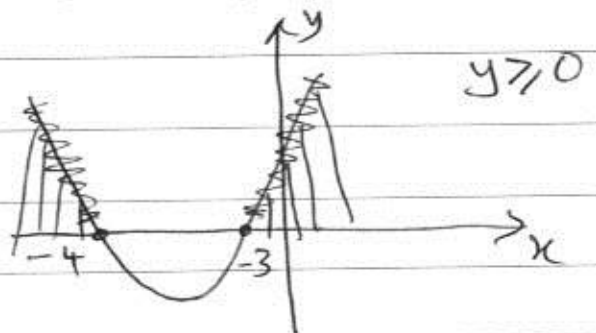
(b) ~~12 - x - x^2 > 0~~  
 $(x - 1)$   
 $-12 + x + x^2 < 0$   
 $x^2 + x - 12 < 0$

Critical values:  
 $x^2 + x - 12 = 0$   
 $(x + 4)(x - 3) = 0$   
 $x = -4, 3$



$-4 < x < 3$

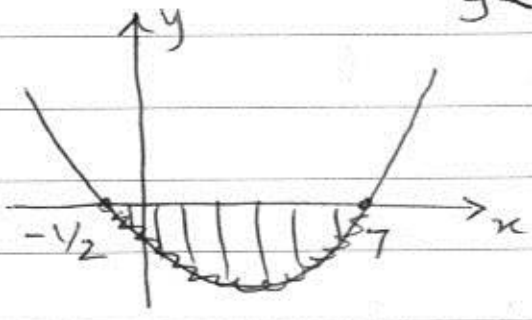
(d)  $x^2 + 7x + 12 \geq 0$   
 Critical values:  
 $x^2 + 7x + 12 = 0$   
 $(x + 4)(x + 3) = 0$   
 $x = -4, -3$



$x \leq -4$  or  $x \geq -3$

(e)  $7 + 13x - 2x^2 > 0$   
 $(x - 1)$   
 $-7 - 13x + 2x^2 < 0$   
 $2x^2 - 13x - 7 < 0$

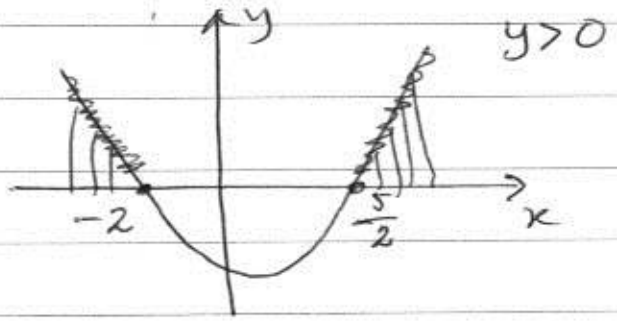
Critical values:  
 $2x^2 - 13x - 7 = 0$   
 $2x^2 - 14x + x - 7 = 0$   
 $2x(x - 7) + 1(x - 7) = 0$   
 $(x - 7)(2x + 1) = 0$   
 $x = 7, -\frac{1}{2}$



$-\frac{1}{2} < x < 7$

$y < 0$

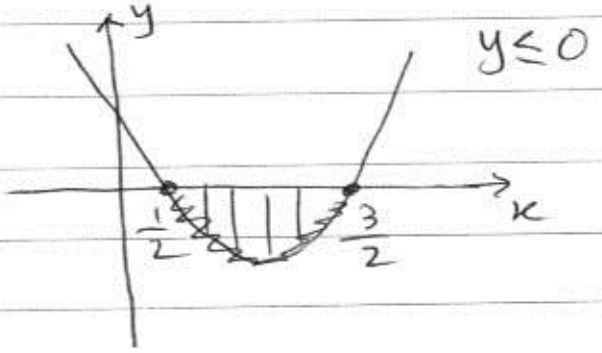
$x(2x - 5) + 2(2x - 5) = 0$   
 $(2x - 5)(x + 2) = 0$   
 $x = 5/2, x = -2$



$x < -2$  or  $x > \frac{5}{2}$

(g)  $4x^2 - 8x + 3 \leq 0$

Critical values:  
 $4x^2 - 8x + 3 = 0$   
 $4x^2 - 6x - 2x + 3 = 0$   
 $2x(2x - 3) - 1(2x - 3) = 0$   
 $(2x - 3)(2x - 1) = 0$   
 $x = \frac{3}{2}, x = \frac{1}{2}$



$\frac{1}{2} \leq x \leq \frac{3}{2}$

(f)  $10 + x - 2x^2 < 0$   
 $(x - 1)$   
 $-10 - x + 2x^2 > 0$   
 $2x^2 - x - 10 > 0$

Critical values:  
 $2x^2 - x - 10 = 0$   
 $2x^2 - 5x + 4x - 10 = 0$

(f)

(h)  $-2 + 7x - 3x^2 < 0$

$(x - 1)$

$2 - 7x + 3x^2 > 0$

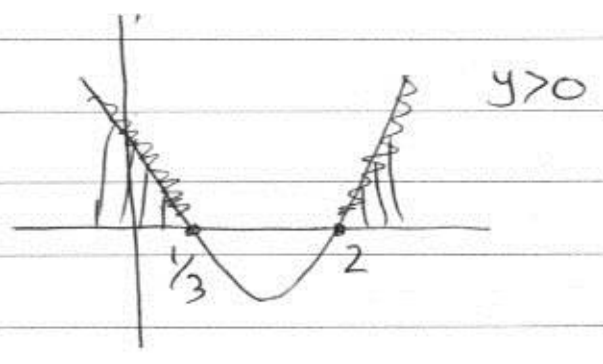
$3x^2 - 7x + 2 > 0$

Critical values:

$3x^2 - 7x + 2 = 0$

(Solve using calculator)

$x = 2, \frac{1}{3}$



$x < \frac{1}{3} \text{ or } x > 2$

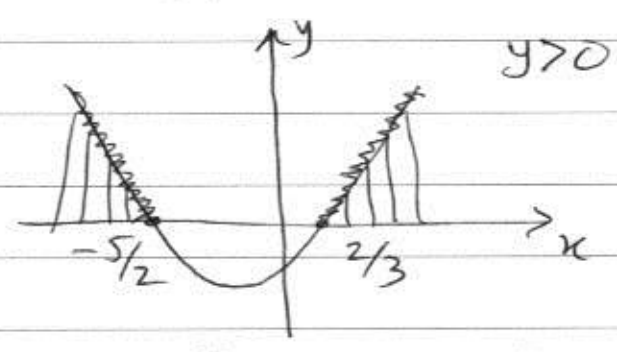
(j)  $6x^2 + 11x - 10 > 0$

Critical values:

$6x^2 + 11x - 10 = 0$

Using calculator,

$x = \frac{2}{3}, -\frac{5}{2}$



$x < -\frac{5}{2} \text{ or } x > \frac{2}{3}$

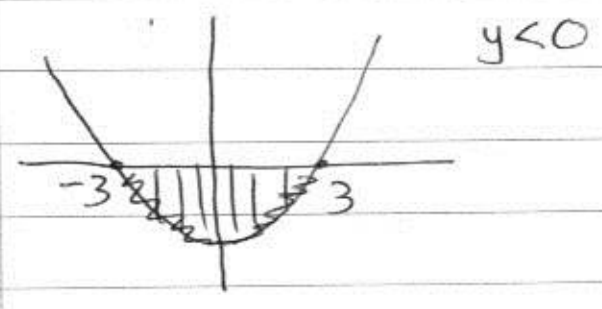
(i)  $x^2 - 9 < 0$

Critical values:

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$



$-3 < x < 3$

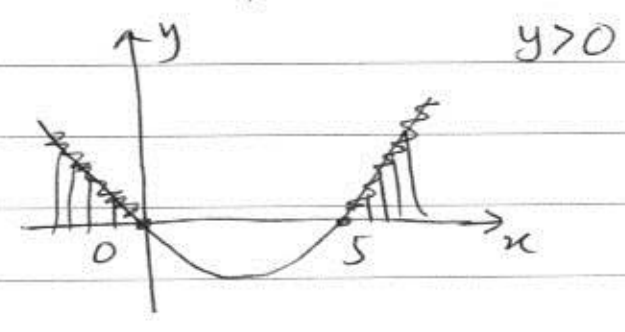
(k)  $x^2 - 5x > 0$

Critical values:

$x^2 - 5x = 0$

$x(x - 5) = 0$

$x = 0, 5$



$x < 0 \text{ or } x > 5$

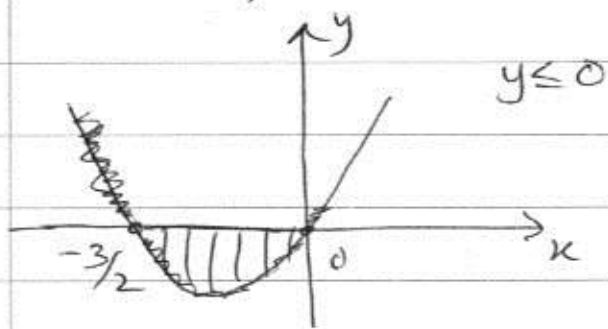
(1)  $2x^2 + 3x \leq 0$

Critical values:

$$2x^2 + 3x = 0$$

$$x(2x + 3) = 0$$

$$x = 0, -3/2$$



$-3/2 \leq x \leq 0$

(b)

$$11 < x^2 + 10$$

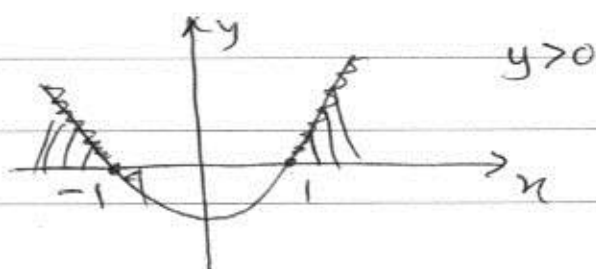
$$x^2 > 1$$

$$\boxed{x^2 - 1 > 0}$$

Critical values:

$$x^2 - 1 = 0$$

$$x = \pm 1$$



$x < -1$  or  $x > 1$

2 (a)  $x^2 < 10 - 3x$

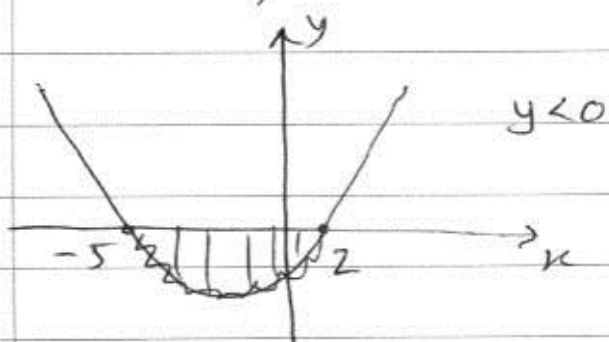
$$\boxed{x^2 + 3x - 10 < 0}$$

Critical values:

$$x^2 + 3x - 10 = 0$$

Using calculator,

$$x = 2, -5$$



$-5 < x < 2$

(c)  $x(3 - 2x) > 1$

$$3x - 2x^2 - 1 > 0$$

(x - 1)

$$-3x + 2x^2 + 1 < 0$$

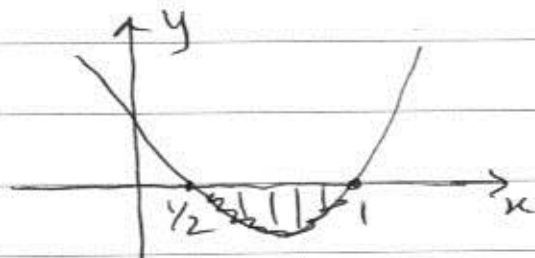
$$\boxed{2x^2 - 3x + 1 < 0}$$

Critical values:

$$2x^2 - 3x + 1 = 0$$

Using calculator,

$$x = 1, 1/2$$



$1/2 < x < 1$

(d)  $x(x+11) < 3(1-x^2)$

$x^2 + 11x < 3 - 3x^2$

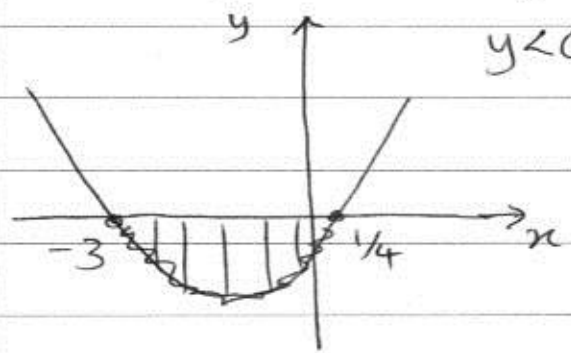
$4x^2 + 11x - 3 < 0$

Critical values:

$4x^2 + 11x - 3 = 0$

Using calculator,

$x = \frac{1}{4}, -3$

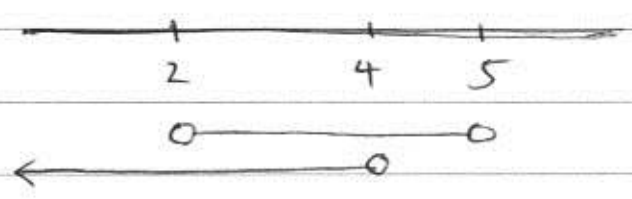


$-3 < x < \frac{1}{4}$

$3x + 5 < 17$

$3x < 12$

$x < 4$



Common range:

$2 < x < 4$

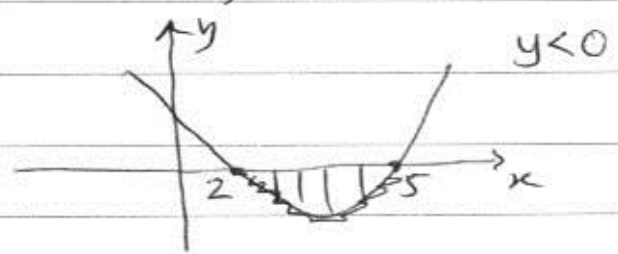
3

(a)  $x^2 - 7x + 10 < 0$

Critical values:

$x^2 - 7x + 10 = 0$

$x = 2, 5$



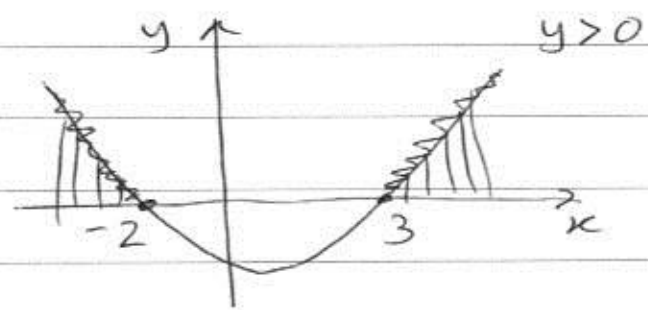
$2 < x < 5$

(b)  $x^2 - x - 6 > 0$

Critical values:

$x^2 - x - 6 = 0$

$x = 3, -2$



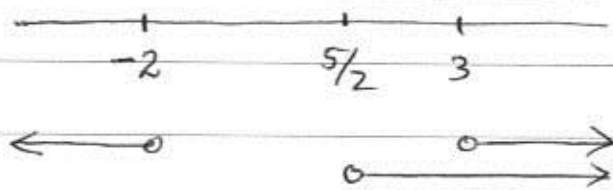
$x < -2$  or  $x > 3$

$10 - 2x < 5$

$10 < 2x + 5$

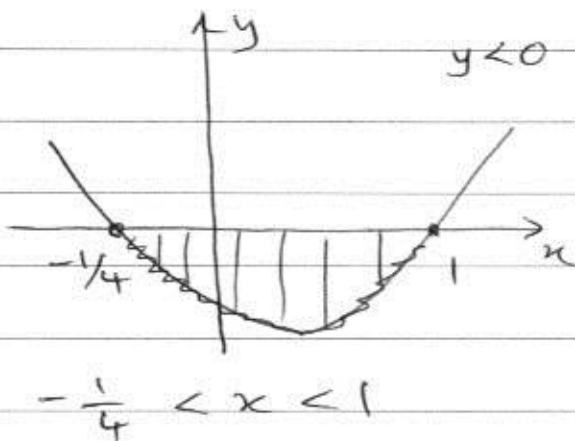
$5 < 2x$

$x > \frac{5}{2}$

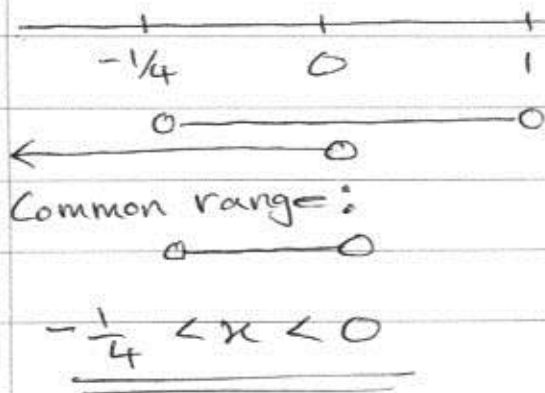


Common range:  
 $x > 3$

(c)  $4x^2 - 3x - 1 < 0$   
 Critical values:  
 $4x^2 - 3x - 1 = 0$   
 $x = 1, -1/4$



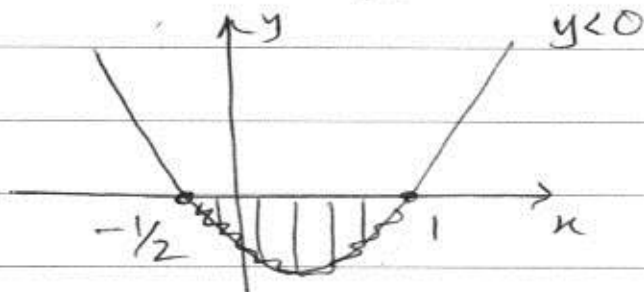
$4(x+2) < 15 - (x+7)$   
 $4x + 8 < 15 - x - 7$   
 $5x < 0$   
 $x < 0$



Common range:  
 $-1/4 < x < 0$

(d)  $2x^2 - x - 1 < 0$

Critical values:  
 $2x^2 - x - 1 = 0$   
 $x = 1, -1/2$

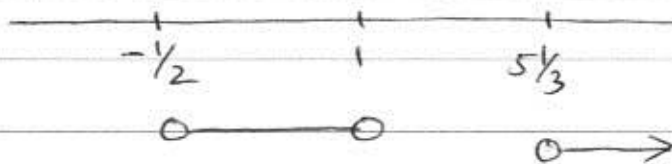


$-1/2 < x < 1$

$14 < 3x - 2$

$16 < 3x$

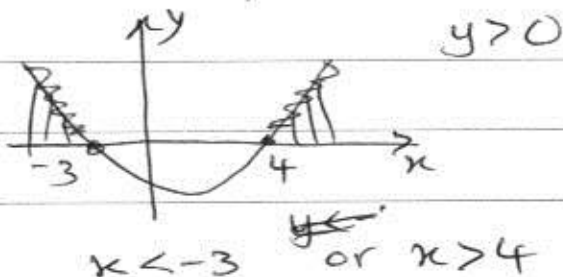
$x > 5 1/3$



No common range.  
 $\therefore$  No solution.

(e)  $x^2 - x - 12 > 0$

Critical values:  
 $x^2 - x - 12 = 0$   
 $x = 4, -3$

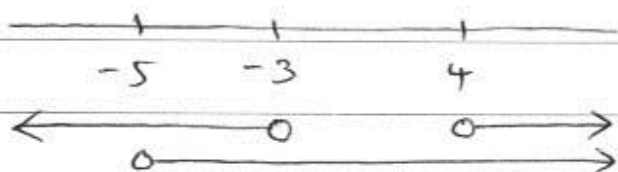


$x < -3$  or  $x > 4$

$$3x + 17 > 2$$

$$3x > -15$$

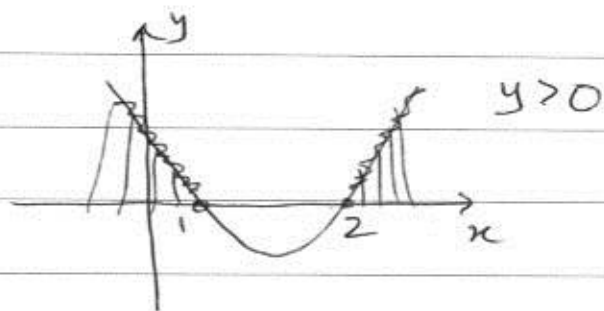
$$x > -5$$



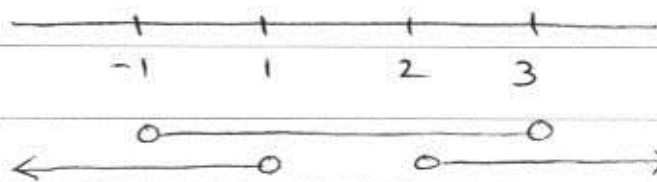
Common range:



$$\underline{\underline{-5 < x < -3 \text{ or } x > 4}}$$



$$x < 1 \text{ or } x > 2$$



Common range:



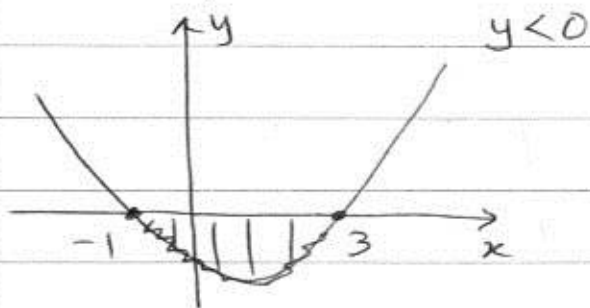
$$\underline{\underline{-1 < x < 1 \text{ or } 2 < x < 3}}$$

(f)  $x^2 - 2x - 3 < 0$

Critical values:

$$x^2 - 2x - 3 = 0$$

$$x = 3, -1$$



$$-1 < x < 3$$

$$x^2 - 3x + 2 > 0$$

Critical values:

$$x^2 - 3x + 2 = 0$$

$$x = 2, 1$$

(4)

(a)

$$x^2 - kx + (k+3) = 0$$

$$a = 1, b = -k, c = k+3$$

If it has real roots,

$$b^2 - 4ac \geq 0$$

(This is because, real roots means, there could be two distinct real roots or one repeated real root)

$$(-k)^2 - 4(1)(k+3) \geq 0$$

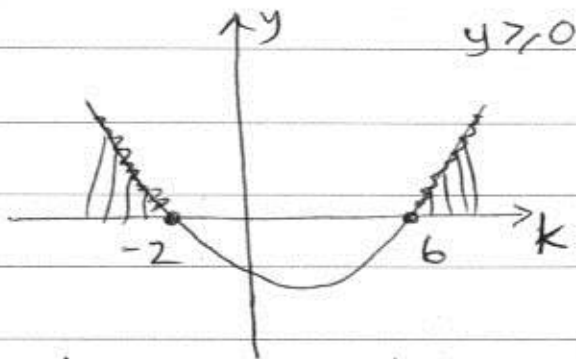
$$k^2 - 4k - 12 \geq 0$$

$$k^2 - 4k - 12 \geq 0$$

Critical values:

$$k^2 - 4k - 12 = 0$$

$$k = 6, -2$$



$$k \leq -2 \text{ or } k \geq 6$$

(b)  $px^2 + px - 2 = 0$

If the roots are real,

$$b^2 - 4ac \geq 0$$

$$p^2 - 4(p)(-2) \geq 0$$

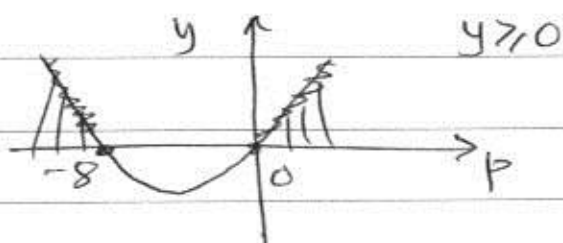
$$p^2 + 8p \geq 0$$

Critical values:

$$p^2 + 8p = 0$$

$$p(p+8) = 0$$

$$p = 0, p = -8$$



$$\underline{p \leq -8 \text{ or } p \geq 0}$$