# Partial Fractions

**Exercise A**

**1** Express the following as partial fractions:

**a** $\dfrac{6x - 2}{(x - 2)(x + 3)}$
        
**b** $\dfrac{2x + 11}{(x + 1)(x + 4)}$
        
**c** $\dfrac{-7x - 12}{2x(x - 4)}$

**d** $\dfrac{2x - 13}{(2x + 1)(x - 3)}$
        
**e** $\dfrac{6x + 6}{x^2 - 9}$
        
**Hint**   First factorise the denominator.

**f** $\dfrac{7 - 3x}{x^2 - 3x - 4}$
        
**g** $\dfrac{8 - x}{x^2 + 4x}$
        
**h** $\dfrac{2x - 14}{x^2 + 2x - 15}$

**2** Show that $\dfrac{-2x - 5}{(4 + x)(2 - x)}$ can be written in the form $\dfrac{A}{4 + x} + \dfrac{B}{2 - x}$ where $A$ and $B$ are constants to be found.            **(3 marks)**

**3** The expression $\dfrac{A}{(x - 4)(x + 8)}$ can be written in partial fractions as $\dfrac{2}{x - 4} + \dfrac{B}{x + 8}$

Find the values of the constants $A$ and $B$.

**4** $h(x) = \dfrac{2x^2 - 12x - 26}{(x + 1)(x - 2)(x + 5)}, x > 2$

Given that $h(x)$ can be expressed in the form $\dfrac{A}{x + 1} + \dfrac{B}{x - 2} + \dfrac{C}{x + 5}$, find the values of $A$, $B$ and $C$.           **(4 marks)**

**5** Given that, for $x < -1$, $\dfrac{-10x^2 - 8x + 2}{x(2x + 1)(3x - 2)} \equiv \dfrac{D}{x} + \dfrac{E}{2x + 1} + \dfrac{F}{3x - 2}$, where $D$, $E$ and $F$ are constants. Find the values of $D$, $E$ and $F$.           **(4 marks)**

**6** Express the following as partial fractions:

$\dfrac{-5x^2 - 19x - 32}{(x + 1)(x + 2)(x - 5)}$

**7** Express the following as partial fractions:

**a** $\dfrac{6x^2 + 7x - 3}{x^3 - x}$
        
**b** $\dfrac{8x + 9}{10x^2 + 3x - 4}$
        
**Hint**   First factorise the denominator.

**Challenge**

Express $\dfrac{5x^2 - 15x - 8}{x^3 - 4x^2 + x + 6}$ as a sum of fractions with linear denominators.

**Exercise B**

1  $f(x) = \dfrac{3x^2 + x + 1}{x^2(x + 1)}, x \neq 0, x \neq -1$

Given that $f(x)$ can be expressed in the form $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x + 1}$, find the values of
$A$, $B$ and $C$.  **(4 marks)**

2  $g(x) = \dfrac{-x^2 - 10x - 5}{(x + 1)^2(x - 1)}, x \neq -1, x \neq 1$

Find the values of the constants $D$, $E$ and $F$ such that $g(x) = \dfrac{D}{x + 1} + \dfrac{E}{(x + 1)^2} + \dfrac{F}{x - 1}$  **(4 marks)**

3  Given that, for $x < 0$, $\dfrac{2x^2 + 2x - 18}{x(x - 3)^2} \equiv \dfrac{P}{x} + \dfrac{Q}{x - 3} + \dfrac{R}{(x - 3)^2}$, where $P$, $Q$ and $R$ are constants,
find the values of $P$, $Q$ and $R$.  **(4 marks)**

4  Show that $\dfrac{5x^2 - 2x - 1}{x^3 - x^2}$ can be written in the form $\dfrac{C}{x} + \dfrac{D}{x^2} + \dfrac{E}{x - 1}$ where $C$, $D$ and $E$
are constants to be found.  **(4 marks)**

5  $p(x) = \dfrac{2x}{(x + 2)^2}, x \neq -2.$

Find the values of the constants $A$ and $B$ such that $p(x) = \dfrac{A}{x + 2} + \dfrac{B}{(x + 2)^2}$  **(4 marks)**

6  $\dfrac{10x^2 - 10x + 17}{(2x + 1)(x - 3)^2} \equiv \dfrac{A}{2x + 1} + \dfrac{B}{x - 3} + \dfrac{C}{(x - 3)^2}, x > 3$
Find the values of the constants $A$, $B$ and $C$.  **(4 marks)**

7  Show that $\dfrac{39x^2 + 2x + 59}{(x + 5)(3x - 1)^2}$ can be written in the form $\dfrac{A}{x + 5} + \dfrac{B}{3x - 1} + \dfrac{C}{(3x - 1)^2}$ where
$A$, $B$ and $C$ are constants to be found.  **(4 marks)**

8  Express the following as partial fractions:

  a  $\dfrac{4x + 1}{x^2 + 10x + 25}$   b  $\dfrac{6x^2 - x + 2}{4x^3 - 4x^2 + x}$

**(Exercise C is on the next page.)**

**Exercise C**

1   $\dfrac{x^3 + 2x^2 + 3x - 4}{x + 1} \equiv Ax^2 + Bx + C + \dfrac{D}{x + 1}$

Find the values of the constants $A$, $B$, $C$ and $D$. **(4 marks)**

2   Given that $\dfrac{2x^3 + 3x^2 - 4x + 5}{x + 3} \equiv ax^2 + bx + c + \dfrac{d}{x + 3}$ find the values of $a$, $b$, $c$ and $d$. **(4 marks)**

3   $f(x) = \dfrac{x^3 - 8}{x - 2}$

Show that $f(x)$ can be written in the form $px^2 + qx + r$ and find the values of $p$, $q$ and $r$. **(4 marks)**

4   Given that $\dfrac{2x^2 + 4x + 5}{x^2 - 1} \equiv m + \dfrac{nx + p}{x^2 - 1}$ find the values of $m$, $n$ and $p$. **(4 marks)**

5   Find the values of the constants $A$, $B$, $C$ and $D$ in the following identity:

$8x^3 + 2x^2 + 5 \equiv (Ax + B)(2x^2 + 2) + Cx + D$ **(4 marks)**

6   $\dfrac{4x^3 - 5x^2 + 3x - 14}{x^2 + 2x - 1} \equiv Ax + B + \dfrac{Cx + D}{x^2 + 2x - 1}$

Find the values of the constants $A$, $B$, $C$ and $D$. **(4 marks)**

7   $g(x) = \dfrac{x^4 + 3x^2 - 4}{x^2 + 1}$. Show that $g(x)$ can be written in the form $px^2 + qx + r + \dfrac{sx + t}{x^2 + 1}$

and find the values of $p$, $q$, $r$, $s$ and $t$. **(4 marks)**

8   Given that $\dfrac{2x^4 + 3x^3 - 2x^2 + 4x - 6}{x^2 + x - 2} \equiv ax^2 + bx + c + \dfrac{dx + e}{x^2 + x - 2}$ find the values

of $a$, $b$, $c$, $d$ and $e$. **(5 marks)**

9   Find the values of the constants $A$, $B$, $C$, $D$ and $E$ in the following identity:

$3x^4 - 4x^3 - 8x^2 + 16x - 2 \equiv (Ax^2 + Bx + C)(x^2 - 3) + Dx + E$ **(5 marks)**

10  **a**  Fully factorise the expression $x^4 - 1$. **(2 marks)**

**b**  Hence, or otherwise, write the algebraic fraction $\dfrac{x^4 - 1}{x + 1}$ in the form

$(ax + b)(cx^2 + dx + e)$ and find the values of $a$, $b$, $c$, $d$ and $e$. **(4 marks)**