

Exercise A

- 1** **a** $U_1 = 5$ $U_2 = 8$ $U_3 = 11$ $U_{10} = 32$
b $U_1 = 7$ $U_2 = 4$ $U_3 = 1$ $U_{10} = -20$
c $U_1 = 6$ $U_2 = 9$ $U_3 = 14$ $U_{10} = 105$
d $U_1 = 4$ $U_2 = 1$ $U_3 = 0$ $U_{10} = 49$
e $U_1 = -2$ $U_2 = 4$ $U_3 = -8$ $U_{10} = 1024$
f $U_1 = \frac{1}{3}$ $U_2 = \frac{1}{2}$ $U_3 = \frac{3}{5}$ $U_{10} = \frac{5}{6}$
g $U_1 = -\frac{1}{3}$ $U_2 = \frac{1}{2}$ $U_3 = -\frac{3}{5}$ $U_{10} = \frac{5}{6}$
h $U_1 = -1$ $U_2 = 0$ $U_3 = 1$ $U_{10} = 512$
- 2** **a** 14 **b** 9 **c** 11 **d** 9
e 6 **f** 9 **g** 8 **h** 14
i 4 **j** 5
- 3** $U_n = 4n^2 + 4n = 4(n^2 + n)$ which is a multiple of 4
4 $U_n = (n-5)^2 + 2 > 0$ U_n is smallest when $n = 5$ ($U_n = 2$)
5 $a = 12, b = -22$
6 $a = 1, b = 3, c = 0$
7 $p = \frac{1}{2}, q = 5\frac{1}{2}$

Exercise B

- 1** **a** 1, 4, 7, 10 **b** 9, 4, -1, -6
c 3, 6, 12, 24 **d** 2, 5, 11, 23
e 10, 5, 2.5, 1.25 **f** 2, 3, 8, 63
g 3, 5, 13, 31
- 2** **a** $U_{k+1} = U_k + 2, U_1 = 3$
b $U_{k+1} = U_k - 3, U_1 = 20$
c $U_{k+1} = 2U_k, U_1 = 1$
d $U_{k+1} = U_k/4, U_1 = 100$
e $U_{k+1} = -1 \times U_k, U_1 = 1$
f $U_{k+1} = 2U_k + 1, U_1 = 3$
g $U_{k+1} = (U_k)^2 + 1, U_1 = 0$
h $U_{k+1} = \frac{U_k + 2}{2}, U_1 = 26$
i $U_{k+2} = U_{k+1} + U_k, U_1 = 1, U_2 = 1$
j $U_{k+1} = 2U_k + 2(-1)^{k+1}, U_1 = 4$
- 3** **a** $U_{k+1} = U_k + 2, U_1 = 1$
b $U_{k+1} = U_k + 3, U_1 = 5$
c $U_{k+1} = U_k + 1, U_1 = 3$
d $U_{k+1} = U_k + \frac{1}{2}, U_1 = 1$
e $U_{k+1} = U_k + 2k + 1, U_1 = 1$
f $U_{k+1} = U_k - (-1)^k(2k + 1), U_1 = -1$
- 4** **a** $3k + 2$ **b** $3k^2 + 2k + 2$ **c** $\frac{10}{3}, -4$
- 5** **a** $4 - 2p$ **b** $4 - 6p$ **c** $p = -2$

Exercise C

- 1** Arithmetic sequences are **a, b, c, h, i**
2 **a** $23, 2n + 3$ **b** $32, 3n + 2$
c $-3, 27 - 3n$ **d** $35, 4n - 5$
e $10x, nx$ **f** $a + 9d, a + (n - 1)d$
3 **a** £5800 **b** £(3800 + 200m)
4 **a** 22 **b** 40 **c** 39
d 46 **e** 18 **f** n
-

Exercise D

- 1** **a** $78, 4n - 2$ **b** $42, 2n + 2$
c $23, 83 - 3n$ **d** $39, 2n - 1$
e $-27, 33 - 3n$ **f** $59, 3n - 1$
g $39p, (2n - 1)p$ **h** $-71x, (9 - 4n)x$
2 **a** 30 **b** 29 **c** 32
d 31 **e** 221 **f** 77
3 $d = 6$
4 $a = 36, d = -3, 14\text{th term}$
5 24
6 $x = 5; 25, 20, 15$
7 $x = \frac{1}{2}, x = 8$
-

Exercise E

- 1** **a** 820 **b** 450 **c** -1140
d -294 **e** 1440 **f** 1425
g -1155 **h** $21(11x + 1)$
2 **a** 20 **b** 25
c 65 **d** 4 or 14 (2 answers)
3 2450
4 **i** £222 500 **ii** £347 500
5 1683, 3267 **6** £9.03, 141 days
7 $d = -\frac{1}{2}, -5.5$
-

Exercise F

- 1** **a** $\sum_{r=1}^{10} (3r + 1)$ **b** $\sum_{r=1}^{30} (3r - 1)$
c $\sum_{r=1}^{11} 4(11 - r)$ **d** $\sum_{r=1}^{16} 6r$
2 **a** 210 **b** 1010 **c** -60 **d** 147
3 19 **4** 49
-

Exercise G

- 1 (a) 59 048 (b) -29 524
(c) 1.9922 (d) 0.6641
(e) 12 285 (f) 8.9998
 - 2 (a) 2047 (b) 683
(c) 262 143 (d) $\frac{1023}{512}$
(e) $\frac{14762}{19683}$ (f) 19.843 75
(g) $\frac{341}{1024}$ (h) $2 - \left(\frac{1}{2}\right)^n$
(i) $\frac{1}{3} \left(64 - \left(\frac{1}{4}\right)^n\right)$ (j) $\frac{1}{4} \left(243 + \left(-\frac{1}{3}\right)^n\right)$
 - 3 (a) 93 (b) -10 (c) $31\frac{7}{8}$ (d) 21 844
 - 4 $2^{64} - 1 \approx 1.84 \times 10^{19}$
 - 5 £2 684 354.55
 - 6 £32 289.76
 - 7 0.979 litres
 - 8 $\left(\frac{3}{4}\right)^{n-1} - \frac{1}{3}$
 - 9 (a) $\frac{x(1-x^n)}{1-x}$ (b) $\frac{x(1-(-x)^n)}{1+x}$
(c) $\frac{x^{2n}-1}{x^{2n-3}(x^2-1)}$ (d) $\frac{x^{2n}-(-1)^n}{x^{2n-2}(x^2+1)}$
-

Exercise H

- 1 (a) 2 (b) $\frac{3}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{9}$
(e) $\frac{3}{4}$ (f) $\frac{1}{6}$ (g) 3 (h) $\frac{1}{3}$
(i) $\frac{20}{3}$ (j) 62.5 (k) $\frac{x}{1-x}$ (l) $\frac{1}{1+x^2}$
(m) $\frac{x}{x-1}$ (n) $\frac{x^3}{x+1}$
- 2 (a) $\frac{4}{11}$ (b) $\frac{41}{333}$ (c) $\frac{5}{9}$ (d) $\frac{157}{333}$
(e) $\frac{1}{7}$ (f) $\frac{2}{7}$ (g) $\frac{5}{7}$ (h) $\frac{6}{7}$
- 3 $\frac{1}{6}$
- 4 $-\frac{5}{6}$
- 5 3
- 6 19.2
- 7 (a) $\left(\frac{1}{3}\right)^{i-1}, \frac{2\,441\,406}{1\,953\,125}, 1\frac{1}{4}$
(b) $2^i, 2046$, not convergent
(c) $8 \times 2^{-i}, \frac{1023}{128}, 8$
(d) $\left(-\frac{1}{10}\right)^{i-1}, \frac{909\,090\,909}{1\,000\,000\,000}, \frac{10}{11}$

8.

0.375 m east of O, 1.5 m

9.

19 m