

Exponential Models

1.

A hot drink is made and left to cool. The table shows its temperature at ten-minute intervals after it is made.

Time (minutes)	10	20	30	40	50
Temperature (°C)	68	53	42	36	31

The room temperature is 22 °C. The difference between the temperature of the drink and room temperature at time t minutes is z °C. The relationship between z and t is modelled by

$$z = z_0 10^{-kt},$$

where z_0 and k are positive constants.

(i) Give a physical interpretation for the constant z_0 . [2]

(ii) Show that $\log_{10} z = -kt + \log_{10} z_0$. [2]

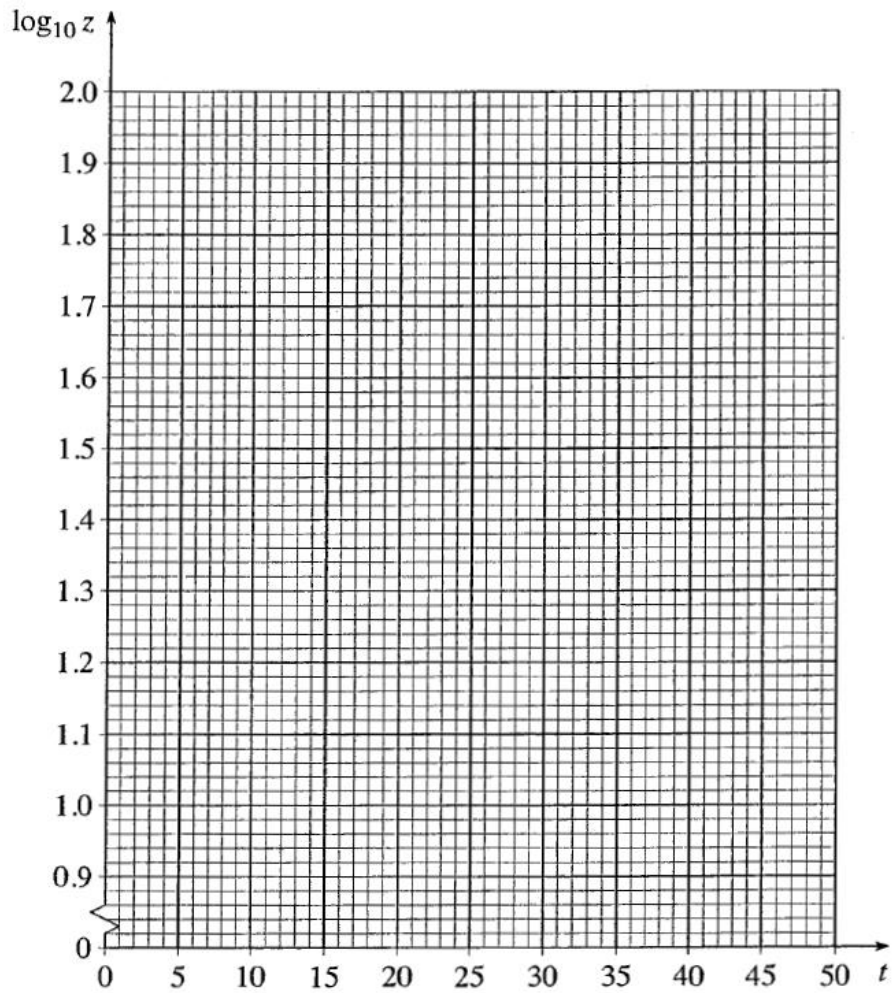
(iii) On the insert, complete the table and draw the graph of $\log_{10} z$ against t .

Use your graph to estimate the values of k and z_0 .

Hence estimate the temperature of the drink 70 minutes after it is made. [9]

(iii)

t	10	20	30	40	50
z	46				
$\log_{10} z$					



2.

A colony of bats is increasing. The population, P , is modelled by $P = a \times 10^{bt}$, where t is the time in years after 2000.

(i) Show that, according to this model, the graph of $\log_{10} P$ against t should be a straight line of gradient b . State, in terms of a , the intercept on the vertical axis. [3]

(ii) The table gives the data for the population from 2001 to 2005.

Year	2001	2002	2003	2004	2005
t	1	2	3	4	5
P	7900	8800	10000	11 300	12 800

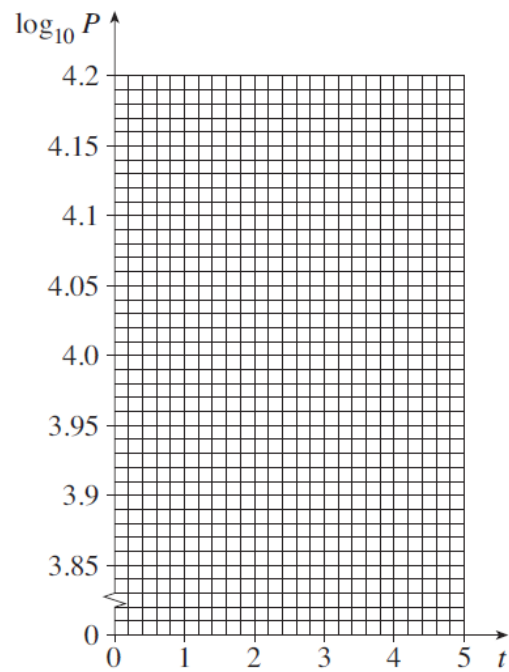
Complete the table of values on the insert, and plot $\log_{10} P$ against t . Draw a line of best fit for the data. [3]

(iii) Use your graph to find the equation for P in terms of t . [4]

(iv) Predict the population in 2008 according to this model. [2]

(ii)

Year	2001	2002	2003	2004	2005
t	1	2	3	4	5
P	7900	8800	10 000	11 300	12 800
$\log_{10} P$					



3.

The table gives a firm's monthly profits for the first few months after the start of its business, rounded to the nearest £100.

Number of months after start-up (x)	1	2	3	4	5	6
Profit for this month (£ y)	500	800	1200	1900	3000	4800

The firm's profits, £ y , for the x th month after start-up are modelled by

$$y = k \times 10^{ax}$$

where a and k are constants.

(i) Show that, according to this model, a graph of $\log_{10} y$ against x gives a straight line of gradient a and intercept $\log_{10} k$. [2]

(ii) **On the insert**, complete the table and plot $\log_{10} y$ against x , drawing by eye a line of best fit. [3]

(iii) Use your graph to find an equation for y in terms of x for this model. [3]

(iv) For which month after start-up does this model predict profits of about £75 000? [3]

(v) State one way in which this model is unrealistic. [1]

(ii)

Number of months after start-up (x)	1	2	3	4	5	6
Profit for this month (£ y)	500	800	1200	1900	3000	4800
$\log_{10} y$	2.70					

