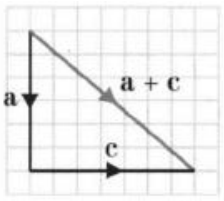
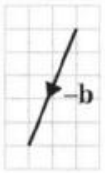
Vectors – Year 12 - Answers

Exercise A

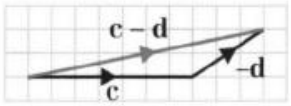
1 **a**



b



c



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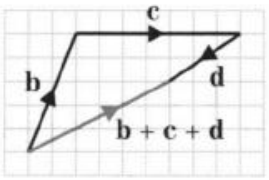

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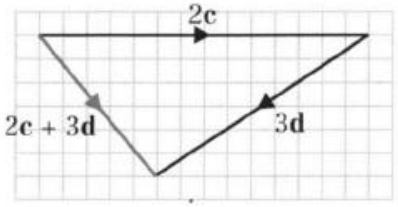

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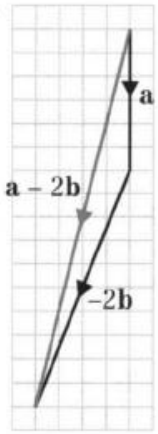

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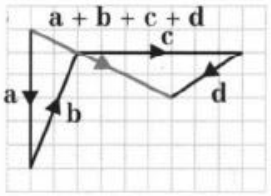

2 **a** 2b **b** d **c** b
d 2b **e** d + b **f** d + b
g –2d **h** –b **i** 2d + b
j –b + 2d **k** –b + d **l** –d – b

3 **a** 2m **b** 2p **c** m
d m **e** p + m **f** p + m
g p + 2m **h** p – m **i** –m –p
j –2m + p **k** –2p + m **l** –m – 2p

4 **a** d – a **b** a + b + c
c a + b – d **d** a + b + c – d

5 **a** 2a + 2b **b** a + b **c** b – a

6 **a** b **b** b – 3a **c** a – b
d 2a – b

7 **a** $\overrightarrow{OB} = \mathbf{a} + \mathbf{b}$ **b** $\overrightarrow{OP} = \frac{5}{8}(\mathbf{a} + \mathbf{b})$ **c** $\overrightarrow{AP} = \frac{5}{8}\mathbf{b} - \frac{3}{8}\mathbf{a}$

8 **a** Yes ($\lambda = 2$) **b** Yes ($\lambda = 4$) **c** No
d Yes ($\lambda = -1$) **e** Yes ($\lambda = -3$) **f** No

9 **a** **i** b – a **ii** $\frac{1}{2}$a **iii** $\frac{1}{2}$b **iv** $\frac{1}{2}$b – $\frac{1}{2}$a

b $\overrightarrow{BC} = \mathbf{b} - \mathbf{a}$, $\overrightarrow{PQ} = \frac{1}{2}(\mathbf{b} - \mathbf{a})$ so PQ is parallel to BC.

10 **a** **i** $2\mathbf{b}$ **ii** $\mathbf{a} - \mathbf{b}$

b $\overrightarrow{AB} = 2\mathbf{b}$, $\overrightarrow{OC} = 3\mathbf{b}$ so AB is parallel to OC.

11 1.2

Exercise B

1 $\mathbf{v}_1$: $8\mathbf{i}$, $\begin{pmatrix}8\\0\end{pmatrix}$ $\mathbf{v}_2$: $9\mathbf{i} + 3\mathbf{j}$, $\begin{pmatrix}9\\3\end{pmatrix}$ $\mathbf{v}_3$: $-4\mathbf{i} + 2\mathbf{j}$, $\begin{pmatrix}-4\\2\end{pmatrix}$

$\mathbf{v}_4$: $3\mathbf{i} + 5\mathbf{j}$, $\begin{pmatrix}3\\5\end{pmatrix}$ $\mathbf{v}_5$: $-3\mathbf{i} - 2\mathbf{j}$, $\begin{pmatrix}-3\\-2\end{pmatrix}$ $\mathbf{v}_6$: $-5\mathbf{j}$, $\begin{pmatrix}0\\-5\end{pmatrix}$

2 **a** $8\mathbf{i} + 12\mathbf{j}$ **b** $\mathbf{i} + 1.5\mathbf{j}$ **c** $-4\mathbf{i} + \mathbf{j}$

d $10\mathbf{i} + \mathbf{j}$ **e** $-2\mathbf{i} + 11\mathbf{j}$ **f** $-2\mathbf{i} - 10\mathbf{j}$

g $14\mathbf{i} - 7\mathbf{j}$ **h** $-8\mathbf{i} + 9\mathbf{j}$

3 **a** $\begin{pmatrix}45\\35\end{pmatrix}$ **b** $\begin{pmatrix}4\\0.5\end{pmatrix}$ **c** $\begin{pmatrix}12\\3\end{pmatrix}$

d $\begin{pmatrix}-1\\16\end{pmatrix}$ **e** $\begin{pmatrix}-21\\-29\end{pmatrix}$ **f** $\begin{pmatrix}10\\2\end{pmatrix}$

4 **a** $\lambda = 5$ **b** $\mu = -\frac{3}{2}$

5 **a** $\lambda = \frac{1}{3}$ **b** $\mu = -1$

c $s = -1$ **d** $t = -\frac{1}{17}$

6 $\mathbf{i} - \mathbf{j}$

7 **a** $\overrightarrow{AC} = 5\mathbf{i} - 4\mathbf{j} = \begin{pmatrix}5\\-4\end{pmatrix}$ **b** $\overrightarrow{AP} = 3\mathbf{i} - \frac{12}{5}\mathbf{j} = \begin{pmatrix}3\\-\frac{12}{5}\end{pmatrix}$

c $\overrightarrow{OP} = 5\mathbf{i} + \frac{8}{5}\mathbf{j} = \begin{pmatrix}5\\\frac{8}{5}\end{pmatrix}$

8 $j = 4, k = 11$

9 $p = 3, q = 2$

10 **a** $p = 5$ **b** $8\mathbf{i} - 12\mathbf{j}$

Exercise C

1 **a** 5 **b** 10 **c** 13

d 4.47 (3 s.f.) **e** 5.83 (3 s.f.) **f** 8.06 (3 s.f.)

g 5.83 (3 s.f.) **h** 4.12 (3 s.f.)

2 **a** $\sqrt{26}$ **b** $5\sqrt{2}$ **c** $\sqrt{101}$

3 **a** $\frac{1}{5}\begin{pmatrix}4\\3\end{pmatrix}$ **b** $\frac{1}{13}\begin{pmatrix}5\\-12\end{pmatrix}$

c $\frac{1}{25}\begin{pmatrix}-7\\24\end{pmatrix}$ **d** $\frac{1}{\sqrt{10}}\begin{pmatrix}1\\-3\end{pmatrix}$

4 **a** 53.1° above **b** 53.1° below

c 67.4° above **d** 63.4° above

5 **a** 149° to the right **b** 29.7° to the right

c 31.0° to the left **d** 104° to the left

6 **a** $\frac{15\sqrt{2}}{2}\mathbf{i} + \frac{15\sqrt{2}}{2}\mathbf{j}$, $\begin{pmatrix}\frac{15\sqrt{2}}{2}\\\frac{15\sqrt{2}}{2}\end{pmatrix}$ **b** $7.52\mathbf{i} + 2.74\mathbf{j}$, $\begin{pmatrix}7.52\\2.74\end{pmatrix}$

c $18.1\mathbf{i} - 8.45\mathbf{j}$, $\begin{pmatrix}18.1\\-8.45\end{pmatrix}$ **d** $\frac{5\sqrt{3}}{2}\mathbf{i} - 2.5\mathbf{j}$, $\begin{pmatrix}\frac{5\sqrt{3}}{2}\\-2.5\end{pmatrix}$

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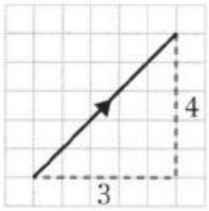
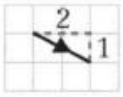
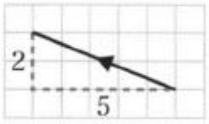
7 **a** $|3\mathbf{i} + 4\mathbf{j}| = 5$, 53.1° above



b $|2\mathbf{i} - \mathbf{j}| = \sqrt{5}$, 26.6° below



c $|-5\mathbf{i} + 2\mathbf{j}| = \sqrt{29}$, 158.2° above



8 $k = \pm 6$

9 $p = \pm 8$, $q = 6$

10 **a** 36.9° **b** 33.7° **c** 70.6°

11 **a** 67.2° **b** 19.0

Exercise D

1 **a** **i** $\overrightarrow{OA} = 3\mathbf{i} - \mathbf{j}$, $\overrightarrow{OB} = 4\mathbf{i} + 5\mathbf{j}$, $\overrightarrow{OC} = -2\mathbf{i} + 6\mathbf{j}$
ii $\mathbf{i} + 6\mathbf{j}$ **iii** $-5\mathbf{i} + 7\mathbf{j}$

b **i** $\sqrt{40} = 2\sqrt{10}$ **ii** $\sqrt{37}$ **iii** $\sqrt{74}$

2 **a** $-\mathbf{i} + 5\mathbf{j}$ or $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$

b **i** 5 **ii** $\sqrt{13}$ **iii** $\sqrt{26}$

3 **a** $-\mathbf{i} - 9\mathbf{j}$ or $\begin{pmatrix} -1 \\ -9 \end{pmatrix}$

b **i** $\sqrt{82}$ **ii** 5 **iii** $\sqrt{61}$

4 **a** $-2\mathbf{a} + 2\mathbf{b}$ **b** $-3\mathbf{a} + 2\mathbf{b}$ **c** $-2\mathbf{a} + \mathbf{b}$

5 $\begin{pmatrix} 7 \\ 9 \end{pmatrix}$ or $\begin{pmatrix} 9 \\ 3 \end{pmatrix}$

6 **a** $2\mathbf{i} + 8\mathbf{j}$ **b** $2\sqrt{17}$

7 $\frac{3\sqrt{5}}{5}$

Exercise E

1 $\overrightarrow{XY} = \mathbf{b} - \mathbf{a}$ and $\overrightarrow{YZ} = \mathbf{c} - \mathbf{b}$, so $\mathbf{b} - \mathbf{a} = \mathbf{c} - \mathbf{b}$.
Hence $\mathbf{a} + \mathbf{c} = 2\mathbf{b}$.

2 **a** **i** $2\mathbf{r}$ **ii** $\mathbf{r}$

b Sides of triangle OAB are twice the length of sides of triangle PAQ and angle A is common to both SAS.

3 **a** $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

b $\overrightarrow{AN} = \frac{1}{3}(\mathbf{b} - \mathbf{a})$, $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$, $\overrightarrow{NB} = \frac{2}{3}(\mathbf{b} - \mathbf{a})$
so $AN : NB = 1 : 2$.

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4 **a** $\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{c}$

b $\overrightarrow{AP} = -\mathbf{a} + \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{c} = \frac{2}{5}(\mathbf{c} - \mathbf{a})$,

$\overrightarrow{PC} = \mathbf{c} - (\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{c}) = \frac{3}{5}(\mathbf{c} - \mathbf{a})$ so $AP : PC = 2 : 3$

5 **a** $\sqrt{26}$ **b** $2\sqrt{2}$ **c** $3\sqrt{2}$

d $\angle BAC = 56°$, $\angle ABC = 34°$, $\angle ACB = 90°$

6 **a** $\overrightarrow{OR} = \mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a}) = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$,

$\overrightarrow{OS} = 3\overrightarrow{OR} = 3(\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}) = 2\mathbf{a} + \mathbf{b}$

b $\overrightarrow{TP} = \overrightarrow{TO} + \overrightarrow{OP} = \mathbf{a} + \mathbf{b}$, $\overrightarrow{PS} = \overrightarrow{PO} + \overrightarrow{OS} = -\mathbf{a} + 2\mathbf{a} + \mathbf{b}$
$= \mathbf{a} + \mathbf{b}$

$\overrightarrow{TP}$ is parallel (and equal) to $\overrightarrow{PS}$ and they have a point, P, in common so T, P and S lie on a straight line.

Exercise F

1 **a** $5\,\text{m s}^{-1}$ **b** $25\,\text{km h}^{-1}$

c $5.39\,\text{m s}^{-1}$ **d** $8.06\,\text{cm s}^{-1}$

2 **a** 50 km **b** 51.0 m

c 4.74 km **d** 967 cm

3 **a** $5\,\text{m s}^{-1}$, 75 m **b** $5.39\,\text{m s}^{-1}$, 16.2 m

c $5.39\,\text{km h}^{-1}$, 16.2 km **d** $13\,\text{km h}^{-1}$, 6.5 km

4 $(2.8\mathbf{i} - 1.6\mathbf{j})\,\text{m s}^{-2}$

5 **a** 54.5° **b** $0.3\sqrt{74} = 2.58\,\text{N}$

6 **a** 26.6° below **i**

b $\mathbf{R} = (3 + p)\mathbf{i} + (q - 4)\mathbf{j}$, $3 + p = 2\lambda$ and
$q - 4 = -\lambda \Rightarrow \lambda = 4 - q$
$3 + p = 2(4 - q) \Rightarrow 3 + p = 8 - 2q$ so $p + 2q = 5$

c $|\mathbf{R}| = 2\sqrt{5}$ newtons

7 **a** $10\mathbf{i} - 100\mathbf{j}$ metres **b** 109.4°

c $1700\,\text{m}^2$

8 **a** $\sqrt{41} = 64.0\,\text{km}$ **b** 321.3°

c $\overrightarrow{AB} = 4\mathbf{i} - 5\mathbf{j}$, $\mathbf{v} = 2(4\mathbf{i} - 5\mathbf{j})$ so the boat is travelling directly towards the buoy.

d $2\sqrt{41} = 12.8\,\text{km h}^{-1}$ **e** 30 minutes
