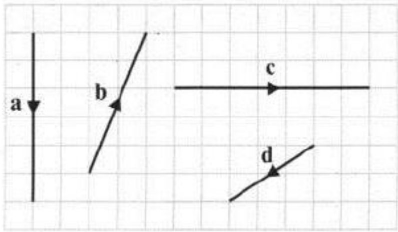
Vectors – Year 12

Exercise A

1 The diagram shows the vectors **a**, **b**, **c** and **d**.

Draw a diagram to illustrate these vectors:



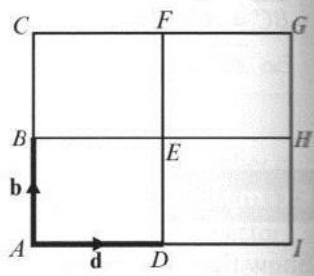
a $\mathbf{a} + \mathbf{c}$ **b** $-\mathbf{b}$

c $\mathbf{c} - \mathbf{d}$ **d** $\mathbf{b} + \mathbf{c} + \mathbf{d}$

e $2\mathbf{c} + 3\mathbf{d}$ **f** $\mathbf{a} - 2\mathbf{b}$

g $\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d}$

2 *ACGI* is a square, *B* is the midpoint of *AC*, *F* is the midpoint of *CG*, *H* is the midpoint of *GI*, *D* is the midpoint of *AI*. $\overrightarrow{AB} = \mathbf{b}$ and $\overrightarrow{AD} = \mathbf{d}$. Find, in terms of **b** and **d**:



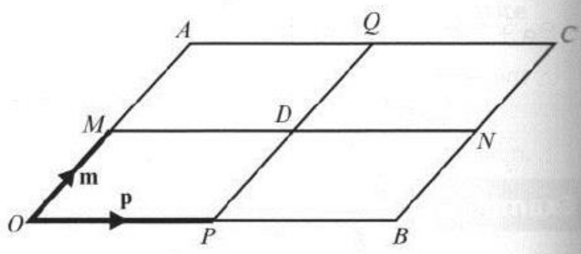
a $\overrightarrow{AC}$ **b** $\overrightarrow{BE}$ **c** $\overrightarrow{HG}$ **d** $\overrightarrow{DF}$

e $\overrightarrow{AE}$ **f** $\overrightarrow{DH}$ **g** $\overrightarrow{HB}$ **h** $\overrightarrow{FE}$

i $\overrightarrow{AH}$ **j** $\overrightarrow{BI}$ **k** $\overrightarrow{EI}$ **l** $\overrightarrow{FB}$

3 *OACB* is a parallelogram. *M*, *Q*, *N* and *P* are the midpoints of *OA*, *AC*, *BC* and *OB* respectively.

Vectors **p** and **m** are equal to $\overrightarrow{OP}$ and $\overrightarrow{OM}$ respectively. Express in terms of **p** and **m**.

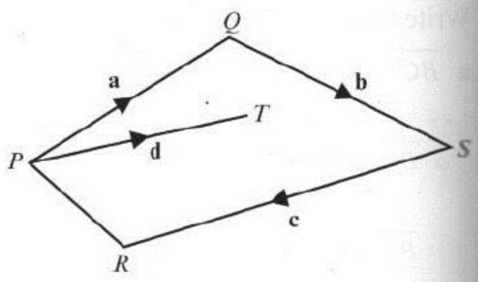


a $\overrightarrow{OA}$ **b** $\overrightarrow{OB}$ **c** $\overrightarrow{BN}$ **d** $\overrightarrow{DQ}$

e $\overrightarrow{OD}$ **f** $\overrightarrow{MQ}$ **g** $\overrightarrow{OQ}$ **h** $\overrightarrow{AD}$

i $\overrightarrow{CD}$ **j** $\overrightarrow{AP}$ **k** $\overrightarrow{BM}$ **l** $\overrightarrow{NO}$

4 In the diagram, $\overrightarrow{PQ} = \mathbf{a}$, $\overrightarrow{QS} = \mathbf{b}$, $\overrightarrow{SR} = \mathbf{c}$ and $\overrightarrow{PT} = \mathbf{d}$. Find in terms of **a**, **b**, **c** and **d**:



a $\overrightarrow{QT}$ **b** $\overrightarrow{PR}$

c $\overrightarrow{TS}$ **d** $\overrightarrow{TR}$

5 In the triangle *PQR*, $PQ = 2\mathbf{a}$ and $QR = 2\mathbf{b}$. The midpoint of *PR* is *M*. Find, in terms of **a** and **b**:

a $\overrightarrow{PR}$ **b** $\overrightarrow{PM}$ **c** $\overrightarrow{QM}$

6 *ABCD* is a trapezium with *AB* parallel to *DC* and $DC = 3AB$. *M* divides *DC* such that $DM : MC = 2 : 1$. $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{b}$. Find, in terms of **a** and **b**:

a $\overrightarrow{AM}$ **b** $\overrightarrow{BD}$ **c** $\overrightarrow{MB}$ **d** $\overrightarrow{DA}$

Problem-solving

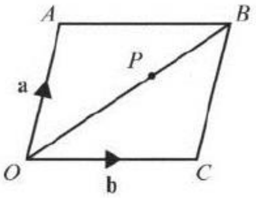
Draw a sketch to show the information given in the question.

7 $OABC$ is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{b}$.

The point P divides OB in the ratio 5:3.

Find, in terms of **a** and **b**:

a $\overrightarrow{OB}$ **b** $\overrightarrow{OP}$ **c** $\overrightarrow{AP}$



8 State with a reason whether each of these vectors is parallel to the vector $\mathbf{a} - 3\mathbf{b}$:

a $2\mathbf{a} - 6\mathbf{b}$ **b** $4\mathbf{a} - 12\mathbf{b}$ **c** $\mathbf{a} + 3\mathbf{b}$ **d** $3\mathbf{b} - \mathbf{a}$ **e** $9\mathbf{b} - 3\mathbf{a}$ **f** $\frac{1}{2}\mathbf{a} - \frac{2}{3}\mathbf{b}$

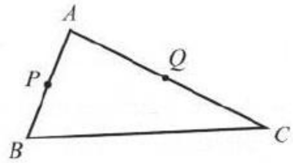
9 In triangle ABC, $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AC} = \mathbf{b}$.

P is the midpoint of AB and Q is the midpoint of AC.

a Write in terms of **a** and **b**:

i $\overrightarrow{BC}$ **ii** $\overrightarrow{AP}$ **iii** $\overrightarrow{AQ}$ **iv** $\overrightarrow{PQ}$

b Show that PQ is parallel to BC.

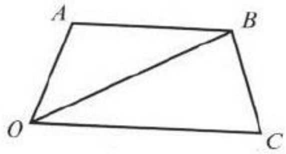


10 $OABC$ is a quadrilateral. $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OC} = 3\mathbf{b}$ and $\overrightarrow{OB} = \mathbf{a} + 2\mathbf{b}$.

a Find, in terms of **a** and **b**:

i $\overrightarrow{AB}$ **ii** $\overrightarrow{CB}$

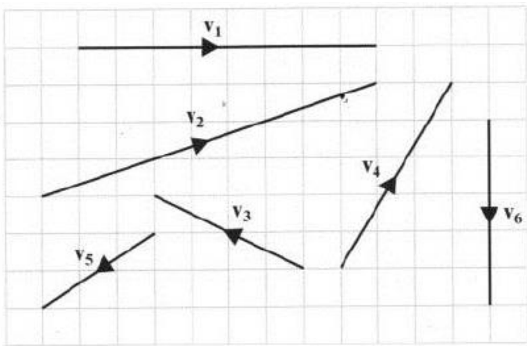
b Show that AB is parallel to OC.



11 The vectors $2\mathbf{a} + k\mathbf{b}$ and $5\mathbf{a} + 3\mathbf{b}$ are parallel. Find the value of k.

Exercise B

1 These vectors are drawn on a grid of unit squares. Express the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$, $\mathbf{v}_5$ and $\mathbf{v}_6$ in **i**, **j** notation and column vector form.



2 Given that $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = 4\mathbf{i} - \mathbf{j}$, find these vectors in terms of **i** and **j**.

a $4\mathbf{a}$ **b** $\frac{1}{2}\mathbf{a}$ **c** $-\mathbf{b}$ **d** $2\mathbf{b} + \mathbf{a}$

e $3\mathbf{a} - 2\mathbf{b}$ **f** $\mathbf{b} - 3\mathbf{a}$ **g** $4\mathbf{b} - \mathbf{a}$ **h** $2\mathbf{a} - 3\mathbf{b}$

3 Given that $\mathbf{a} = \begin{pmatrix} 9 \\ 7 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 11 \\ -3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -8 \\ -1 \end{pmatrix}$ find:

a $5\mathbf{a}$ **b** $-\frac{1}{2}\mathbf{c}$ **c** $\mathbf{a} + \mathbf{b} + \mathbf{c}$ **d** $2\mathbf{a} - \mathbf{b} + \mathbf{c}$

e $2\mathbf{b} + 2\mathbf{c} - 3\mathbf{a}$ **f** $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

4 Given that $\mathbf{a} = 2\mathbf{i} + 5\mathbf{j}$ and $\mathbf{b} = 3\mathbf{i} - \mathbf{j}$, find:

a λ if $\mathbf{a} + \lambda\mathbf{b}$ is parallel to the vector $\mathbf{i}$ **b** μ if $\mu\mathbf{a} + \mathbf{b}$ is parallel to the vector $\mathbf{j}$

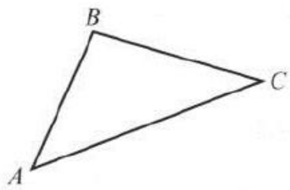
5 Given that $\mathbf{c} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{d} = \mathbf{i} - 2\mathbf{j}$, find:

a λ if $\mathbf{c} + \lambda\mathbf{d}$ is parallel to $\mathbf{i} + \mathbf{j}$ **b** μ if $\mu\mathbf{c} + \mathbf{d}$ is parallel to $\mathbf{i} + 3\mathbf{j}$

c s if $\mathbf{c} - s\mathbf{d}$ is parallel to $2\mathbf{i} + \mathbf{j}$ **d** t if $\mathbf{d} - t\mathbf{c}$ is parallel to $-2\mathbf{i} + 3\mathbf{j}$

6 In triangle ABC, $\overrightarrow{AB} = 4\mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{AC} = 5\mathbf{i} + 2\mathbf{j}$. Find $\overrightarrow{BC}$.

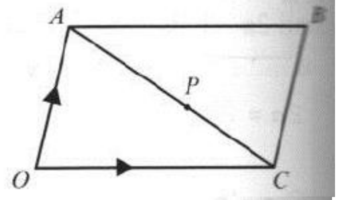
(2 marks)



7 $OABC$ is a parallelogram. P divides AC in the ratio $3:2$. $\overrightarrow{OA} = 2\mathbf{i} + 4\mathbf{j}$, $\overrightarrow{OC} = 7\mathbf{i}$.

Find in $\mathbf{i}$, $\mathbf{j}$ format and column vector format:

a $\overrightarrow{AC}$ **b** $\overrightarrow{AP}$ **c** $\overrightarrow{OP}$



8 $\mathbf{a} = \begin{pmatrix} j \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 10 \\ k \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

Given that $\mathbf{b} - 2\mathbf{a} = \mathbf{c}$, find the values of j and k.

(2 marks)

Problem-solving

You can consider $\mathbf{b} - 2\mathbf{a} = \mathbf{c}$ as two linear equations. One for the x-components and one for the y-components.

9 $\mathbf{a} = \begin{pmatrix} p \\ -q \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} q \\ p \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$

Given that $\mathbf{a} + 2\mathbf{b} = \mathbf{c}$, find the values of p and q. **(2 marks)**

10 The resultant of the vectors $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{b} = p\mathbf{i} - 2p\mathbf{j}$ is parallel to the vector $\mathbf{c} = 2\mathbf{i} - 3\mathbf{j}$.

Find:

a the value of p **(4 marks)**

b the resultant of vectors $\mathbf{a}$ and $\mathbf{b}$. **(1 mark)**

Exercise C

1 Find the magnitude of each of these vectors.

a $3\mathbf{i} + 4\mathbf{j}$ **b** $6\mathbf{i} - 8\mathbf{j}$ **c** $5\mathbf{i} + 12\mathbf{j}$ **d** $2\mathbf{i} + 4\mathbf{j}$

e $3\mathbf{i} - 5\mathbf{j}$ **f** $4\mathbf{i} + 7\mathbf{j}$ **g** $-3\mathbf{i} + 5\mathbf{j}$ **h** $-4\mathbf{i} - \mathbf{j}$

2 $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$, $\mathbf{b} = 3\mathbf{i} - 4\mathbf{j}$ and $\mathbf{c} = 5\mathbf{i} - \mathbf{j}$. Find the exact value of the magnitude of:

a $\mathbf{a} + \mathbf{b}$ **b** $2\mathbf{a} - \mathbf{c}$ **c** $3\mathbf{b} - 2\mathbf{c}$

3 For each of the following vectors, find the unit vector in the same direction.

a $\mathbf{a} = 4\mathbf{i} + 3\mathbf{j}$ **b** $\mathbf{b} = 5\mathbf{i} - 12\mathbf{j}$ **c** $\mathbf{c} = -7\mathbf{i} + 24\mathbf{j}$ **d** $\mathbf{d} = \mathbf{i} - 3\mathbf{j}$

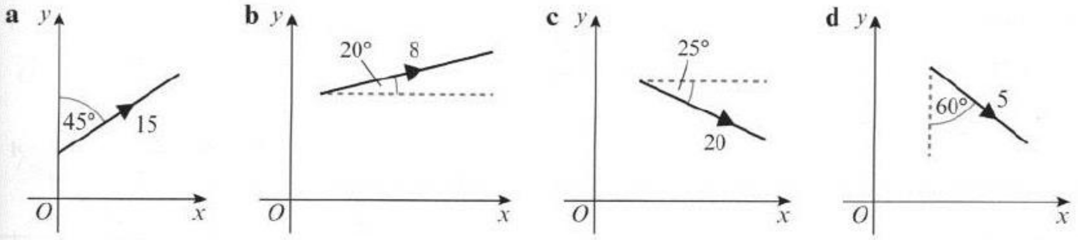
4 Find the angle that each of these vectors makes with the positive x-axis.

a $3\mathbf{i} + 4\mathbf{j}$ **b** $6\mathbf{i} - 8\mathbf{j}$ **c** $5\mathbf{i} + 12\mathbf{j}$ **d** $2\mathbf{i} + 4\mathbf{j}$

5 Find the angle that each of these vectors makes with $\mathbf{j}$.

a $3\mathbf{i} - 5\mathbf{j}$ **b** $4\mathbf{i} + 7\mathbf{j}$ **c** $-3\mathbf{i} + 5\mathbf{j}$ **d** $-4\mathbf{i} - \mathbf{j}$

6 Write these vectors in $\mathbf{i}$, $\mathbf{j}$ and column vector form.



7 Draw a sketch for each vector and work out the exact value of its magnitude and the angle it makes with the positive x-axis to one decimal place.

a $3\mathbf{i} + 4\mathbf{j}$ **b** $2\mathbf{i} - \mathbf{j}$ **c** $-5\mathbf{i} + 2\mathbf{j}$

8 Given that $|2\mathbf{i} - k\mathbf{j}| = 2\sqrt{10}$, find the exact value of k. **(3 marks)**

9 Vector $\mathbf{a} = p\mathbf{i} + q\mathbf{j}$ has magnitude 10 and makes an angle θ with the positive x-axis where $\sin\theta = \frac{3}{5}$. Find the possible values of p and q.

(4 marks)

Problem-solving

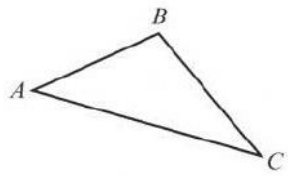
Make sure you consider all the possible cases.

10 In triangle ABC, $\overrightarrow{AB} = 4\mathbf{i} + 3\mathbf{j}$, $\overrightarrow{AC} = 6\mathbf{i} - 4\mathbf{j}$.

a Find the angle between $\overrightarrow{AB}$ and $\mathbf{i}$.

b Find the angle between $\overrightarrow{AC}$ and $\mathbf{i}$.

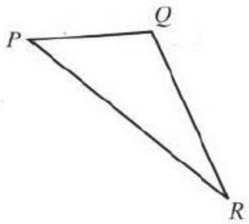
c Hence find the size of $\angle BAC$, in degrees, to one decimal place.



11 In triangle PQR, $\overrightarrow{PQ} = 4\mathbf{i} + \mathbf{j}$, $\overrightarrow{PR} = 6\mathbf{i} - 8\mathbf{j}$.

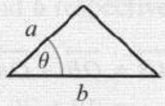
a Find the size of $\angle QPR$, in degrees, to one decimal place. **(5 marks)**

b Find the area of triangle PQR. **(2 marks)**



Hint The area of a triangle is $\frac{1}{2}ab\sin\theta$.

← Section 9.3



Exercise D

1 The points A, B and C have coordinates $(3, -1)$, $(4, 5)$ and $(-2, 6)$ respectively, and O is the origin. Find, in terms of $\mathbf{i}$ and $\mathbf{j}$:

a **i** the position vectors of A, B and C **ii** $\overrightarrow{AB}$ **iii** $\overrightarrow{AC}$

b Find, in surd form: **i** $|\overrightarrow{OC}|$ **ii** $|\overrightarrow{AB}|$ **iii** $|\overrightarrow{AC}|$

2 $\overrightarrow{OP} = 4\mathbf{i} - 3\mathbf{j}$, $\overrightarrow{OQ} = 3\mathbf{i} + 2\mathbf{j}$

a Find $\overrightarrow{PQ}$

b Find, in surd form: **i** $|\overrightarrow{OP}|$ **ii** $|\overrightarrow{OQ}|$ **iii** $|\overrightarrow{PQ}|$

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3 $\overrightarrow{OQ} = 4\mathbf{i} - 3\mathbf{j}$, $\overrightarrow{PQ} = 5\mathbf{i} + 6\mathbf{j}$

a Find $\overrightarrow{OP}$

b Find, in surd form: i $|\overrightarrow{OP}|$ **ii** $|\overrightarrow{OQ}|$ **iii** $|\overrightarrow{PQ}|$

4 *OABCDE* is a regular hexagon. The points *A* and *B* have position vectors **a** and **b** respectively, where *O* is the origin.

Find, in terms of **a** and **b**, the position vectors of

a *C* **b** *D* **c** *E*.

5 The position vectors of 3 vertices of a parallelogram are $\begin{pmatrix}4\\2\end{pmatrix}$, $\begin{pmatrix}3\\5\end{pmatrix}$ and $\begin{pmatrix}8\\6\end{pmatrix}$.

Find the possible position vectors of the fourth vertex.

Problem-solving

Use a sketch to check that you have considered all the possible positions for the fourth vertex.

6 Given that the point *A* has position vector $4\mathbf{i} - 5\mathbf{j}$ and the point *B* has position vector $6\mathbf{i} + 3\mathbf{j}$,

a find the vector $\overrightarrow{AB}$. **(2 marks)**

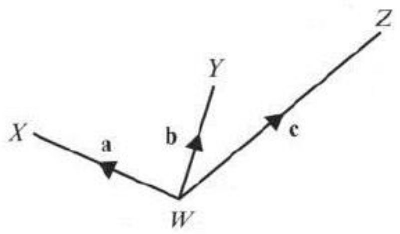
b find $|\overrightarrow{AB}|$ giving your answer as a simplified surd. **(2 marks)**

7 The point *A* lies on the circle with equation $x^2 + y^2 = 9$. Given that $\overrightarrow{OA} = 2k\mathbf{i} + k\mathbf{j}$, find the exact value of *k*. **(3 marks)**

Exercise E

1 In the diagram, $\overrightarrow{WX} = \mathbf{a}$, $\overrightarrow{WY} = \mathbf{b}$ and $\overrightarrow{WZ} = \mathbf{c}$. It is given that $\overrightarrow{XY} = \overrightarrow{YZ}$.

Prove that $\mathbf{a} + \mathbf{c} = 2\mathbf{b}$.

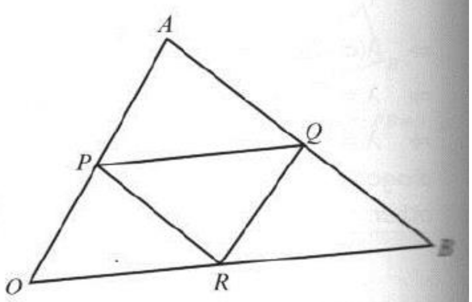


2 *OAB* is a triangle. *P*, *Q* and *R* are the midpoints of *OA*, *AB* and *OB* respectively.

OP and *OR* are equal to **p** and **r** respectively.

a Find i $\overrightarrow{OB}$ **ii** $\overrightarrow{PQ}$

b Hence prove that triangle *PAQ* is similar to triangle *OAB*.



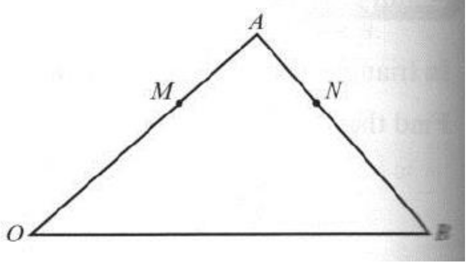
3 *OAB* is a triangle. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

The point *M* divides *OA* in the ratio 2 : 1.

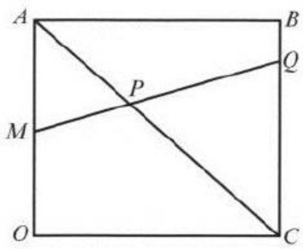
MN is parallel to *OB*.

a Express the vector $\overrightarrow{ON}$ in terms of **a** and **b**.

b Show that *AN* : *NB* = 1 : 2



4 $OABC$ is a square. M is the midpoint of OA, and Q divides BC in the ratio 1 : 3.
AC and MQ meet at P.



a If $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$, express $\overrightarrow{OP}$ in terms of **a** and **c**.

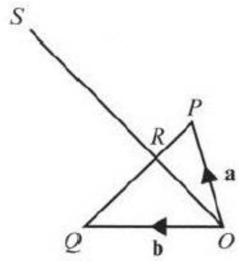
b Show that P divides AC in the ratio 2 : 3.

5 In triangle ABC the position vectors of the vertices A, B and C are $\begin{pmatrix}5\\8\end{pmatrix}$, $\begin{pmatrix}4\\3\end{pmatrix}$ and $\begin{pmatrix}7\\6\end{pmatrix}$. Find:

a $|\overrightarrow{AB}|$ **b** $|\overrightarrow{AC}|$ **c** $|\overrightarrow{BC}|$

d the size of $\angle BAC$, $\angle ABC$ and $\angle ACB$ to the nearest degree.

6 OPQ is a triangle.
$2\overrightarrow{PR} = \overrightarrow{RQ}$ and $3\overrightarrow{OR} = \overrightarrow{OS}$
$\overrightarrow{OP} = \mathbf{a}$ and $\overrightarrow{OQ} = \mathbf{b}$.



a Show that $\overrightarrow{OS} = 2\mathbf{a} + \mathbf{b}$.

b Point T is added to the diagram such that $\overrightarrow{OT} = -\mathbf{b}$.
Prove that points T, P and S lie on a straight line.

Problem-solving
To show that T, P and S lie on the same straight line you need to show that any **two** of the vectors $\overrightarrow{TP}$, $\overrightarrow{TS}$ or $\overrightarrow{PS}$ are parallel.

Exercise F

1 Find the speed of a particle moving with these velocities:

a $(3\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$ **b** $(24\mathbf{i} - 7\mathbf{j})\,\text{km h}^{-1}$

c $(5\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$ **d** $(-7\mathbf{i} + 4\mathbf{j})\,\text{cm s}^{-1}$

Hint Speed is the magnitude of the velocity vector.

2 Find the distance moved by a particle which travels for:

a 5 hours at velocity $(8\mathbf{i} + 6\mathbf{j})\,\text{km h}^{-1}$

b 10 seconds at velocity $(5\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$

c 45 minutes at velocity $(6\mathbf{i} + 2\mathbf{j})\,\text{km h}^{-1}$

d 2 minutes at velocity $(-4\mathbf{i} - 7\mathbf{j})\,\text{cm s}^{-1}$.

Hint Find the speed in each case then use:
Distance travelled = speed × time

3 Find the speed and the distance travelled by a particle moving in a straight line with:

a velocity $(-3\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$ for 15 seconds **b** velocity $(2\mathbf{i} + 5\mathbf{j})\,\text{m s}^{-1}$ for 3 seconds

c velocity $(5\mathbf{i} - 2\mathbf{j})\,\text{km h}^{-1}$ for 3 hours **d** velocity $(12\mathbf{i} - 5\mathbf{j})\,\text{km h}^{-1}$ for 30 minutes.

4 A particle P is accelerating at a constant speed.
When $t = 0$, P has velocity $\mathbf{u} = (2\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$ and at time $t = 5$ s, P has velocity $\mathbf{v} = (16\mathbf{i} - 5\mathbf{j})\,\text{m s}^{-1}$.

Hint The units of acceleration will be m/s² or m s^{-2}.

The acceleration vector of the particle is given by the formula: $\mathbf{a} = \dfrac{\mathbf{v} - \mathbf{u}}{t}$

Find the acceleration of P in terms of **i** and **j**.

5 A particle P of mass $m = 0.3$ kg moves under the action of a single constant force $\mathbf{F}$ newtons. The acceleration of P is $\mathbf{a} = (5\mathbf{i} + 7\mathbf{j})\text{ m s}^{-2}$.

a Find the angle between the acceleration and $\mathbf{i}$. **(2 marks)**

Force, mass and acceleration are related by the formula $\mathbf{F} = m\mathbf{a}$.

b Find the magnitude of $\mathbf{F}$. **(3 marks)**

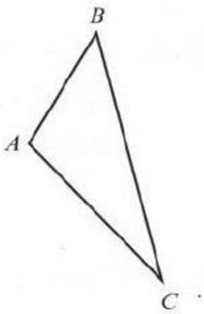
6 Two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, are given by the vectors $\mathbf{F}_1 = (3\mathbf{i} - 4\mathbf{j})$ N and $\mathbf{F}_2 = (p\mathbf{i} + q\mathbf{j})$ N. The resultant force, $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$ acts in a direction which is parallel to the vector $(2\mathbf{i} - \mathbf{j})$.

a Find the angle between $\mathbf{R}$ and the vector $\mathbf{i}$. **(2 marks)**

b Show that $p + 2q = 5$. **(3 marks)**

c Given that $p = 1$, find the magnitude of $\mathbf{R}$. **(3 marks)**

7 The diagram shows a sketch of a field in the shape of a triangle ABC. Given $\overrightarrow{AB} = 30\mathbf{i} + 40\mathbf{j}$ metres and $\overrightarrow{AC} = 40\mathbf{i} - 60\mathbf{j}$ metres,



a find $\overrightarrow{BC}$ **(2 marks)**

b find the size of $\angle BAC$, in degrees, to one decimal place **(4 marks)**

c find the area of the field in square metres. **(3 marks)**

8 A boat A has a position vector of $(2\mathbf{i} + \mathbf{j})$ km and a buoy B has a position vector of $(6\mathbf{i} - 4\mathbf{j})$ km, relative to a fixed origin O.

a Find the distance of the boat from the buoy.

b Find the bearing of the boat from the buoy.

The boat travels with constant velocity $(8\mathbf{i} - 10\mathbf{j})$ km/h.

c Verify that the boat is travelling directly towards the buoy

d Find the speed of the boat.

e Work out how long it will take the boat to reach the buoy.

Problem-solving

Draw a sketch showing the initial positions of the boat, the buoy and the origin.