Answers – Hypothesis Test Using Binomial Distribution (Year 12)

Exercise A

(We did Questions 1 to 3 during our lessons.)

4.

Let p be the probability that a random toast lands butter-side down when dropped.

$H_0 : p = 0.5$

$H_1 : p > 0.5$

Success: A toast landing butter-side down when dropped
X : Number of successes

Assuming that H_0 is true,

$X \sim B(18, 0.5)$

<u>Binomial CD</u>
X : 10
N : 18
p : 0.5

$P(X \geq 11) = 1 - P(X \leq 10)$

$= 1 - 0.7597$

$= 0.2403 > 0.1$

Therefore we do not reject H_0.

There is insufficient evidence at 10% siginificance level to say that a toast is more likely to land butter-side down when dropped.

5.

(a) $X \sim B(n, 0.68)$

Reasons:
- There is a fixed number of trials, which is n
- The probability of successful treatment remains constant at 0.68
- The outcome of the treatment for a patient is independent of that for the other pateints

(b)

Let p be the probability that the treatment is successful for a random patient.

$H_0 : p = 0.68$

$H_1 : p < 0.68$

Success: The treatment is successful for a partient
X : Number of successes

Assuming that H_0 is true,

X ~ B(10, 0.68)

<u>Binomial CD</u>
X : 3
N : 10
p : 0.68

$P(X \leq 3) = 0.0155 < 0.05$

Therefore we reject H_0.

There is sufficient evidence at 5% siginificance level to say that the treatment is not as effective as it is claimed.

6.

Success : An applicant is successful.
Let X be the number of successes.

X ~ B (17, 0.2)

(i) $P(X \geq 4) = 1 - P(X \leq 3)$

$= 1 - 0.5489$

$= 0.451$ (3 s.f.)

<u>Binomial CD</u>

X : 3
N : 17
p : 0.2

(ii) $np = 17 \times 0.2 = 3.4$

(iii) <u>Binomial PD (List)</u>

N : 17
p : 0.2

The highest value among all possible probabilities of the form P(X = r) is 0.2392 when r = 3.

Therefore the most likely number of successful applicants = 3

(iv) (A) Let p be the probability that a random mathematics graduate applicant is successful

$H_0 : p = 0.2$

$H_1 : p > 0.2$

(B) We are testing the suggestion that mathemetics graduates are more likely to be successful than those from other fields. Hence, the alternative hypothesis takes the form shown above.

(C) Success: A mathematics graduate applicant is successful.
X : Number of successes

Assuming that H_0 is true,

X ~ B(17, 0.2)

$P(X \geq 10) = 1 - P(X \leq 9)$

$= 1 - 0.9995$

$= 0.0005 < 0.05$

Binomial CD

X : 9
N : 17
p : 0.2

Therefore we reject H_0.

There is sufficient evidence at 5% siginificance level to say that mathematics graduates are more likely to be successful than those from other fields.

7.

Let p be the probability that the die lands on 6, when rolled.

$H_0 : p = 1/6$

$H_1 : p < 1/6$

Success: The die lands on 6, when rolled.
X : Number of successes

Assuming that H_0 is true,

$X \sim B(12, 1/6)$

Binomial CD
X : 1
N : 12
p : 1/6

$P(X \leq 1) = 0.3813 > 0.05$

Therefore we do not reject H_0.

There is insufficient evidence at 5% siginificance level to say that the probability fo a 6 on the die is less than 1/6.

8.

(i) Let p be the probability that a random volunteer names all of the items.

$H_0 : p = 0.35$

$H_1 : p > 0.35$

The alternative hypothesis takes the form given above because we are testing the student's belief that the probability of a volunteer naming all of the items would be higher if the volunteer listens to the same piece of music while memorising the items and while trying to name them.

(ii)

Success: A volunteer names all of the items.

X : Number of successes

Assuming that H_0 is true,

$X \sim B(15, 0.35)$

$P(X \geq 8) = 1 - P(X \leq 7)$

$= 1 - 0.8868$

$= 0.1132 > 0.05$

Binomial CD
X : 7
N : 15
p : 0.35

Therefore we do not reject H_0.

There is insufficient evidence at 5% siginificance level to say that the probability of a volunteer naming all of the items would be higher if the volunteer listens to the same piece of music while memorising the items and while trying to name them.

Exercise B

(We did Questions 1 and 2 during our lessons.)

3.

Let p be the probability that a random person turns left.

$H_0 : p = 0.5$

$H_1 : p \neq 0.5$

Success: A person turns left.

X : Number of successes

Assuming that H_0 is true,

$X \sim B(12, 0.5)$

$np = 12 \times 0.5 = 6 < 9$

$P(X \geq 9) = 1 - P(X \leq 8)$

$= 1 - 0.9270$

$= 0.073 > 0.025$

Binomial CD
X : 8
N : 12
p : 0.5

Therefore we do not reject H_0.

There is insufficient evidence at 5% siginificance level to say that people are more likely to turn one way than another.

4.

Let p be the probability that a random day in April is wet.

$H_0 : p = 0.25$

$H_1 : p \neq 0.25$

Success: A day in April is wet.

X : Number of successes

Assuming that H_0 is true,

$X \sim B(20, 0.25)$

$np = 20 \times 0.25 = 5 < 10$

$P(X \geq 10) = 1 - P(X \leq 9)$

$= 1 - 0.9861$

$= 0.0139 < 0.05$

<u>Binomial CD</u>
X : 9
N : 20
p : 0.25

Therefore we reject H_0.

There is sufficient evidence at 10% siginificance level to say that the climate is changing and therefore the complaint is justified.

Exercise C – Exercise A

(We did Questions 1 to 3 during our lessons.)

4.

Let p be the probability that a random toast lands butter-side down when dropped.

$H_0 : p = 0.5$

$H_1 : p > 0.5$

Success: A toast landing butter-side down when dropped
X : Number of successes

Assuming that H_0 is true,

$X \sim B(18, 0.5)$

Let $X \geq C$ be the critical region.

Then, $P(X \geq C) < 0.1$

$1 - P(X \leq C - 1) < 0.1$

$1 - 0.1 < P(X \leq C - 1)$

$P(X \leq C - 1) > 0.9$

Binomial CD (List)

N: 18
P : 0.5

P (X ≤ 11) = 0.881 < 0.9

P (X ≤ 12) = 0.9518 > 0.9

Therefore, C – 1 = 12

C = 13

Hence the crotican region is X ≥ 13.

Since the obeserved outcome X = 11 doesn't fall into the critical region, we do not reject H_0.

There is insufficient evidence at 10% siginificance level to say that a toast is more likely to land butter-side down when dropped.

5.

(a) Refer to the answer given for Ex A/Q5 (a) on page 1.

(b)

Let p be the probability that the treatment is successful for a random patient.

H_0 : p = 0.68

H_1 : p < 0.68

Success: The treatment is successful for a partient
X : Number of successes

Assuming that H_0 is true,

X ~ B(10, 0.68)

Let X ≤ C be the critical region.

Then, P (X ≤ C) < 0.05

Binomial CD (List)
N : 10
p : 0.68

P (X ≤ 3) = 0.0155 < 0.05

P (X ≤ 4) = 0.0637 > 0.05

Therefore C = 3.

Hence the crtical region is, X ≤ 3.

Since the obeserved outcome X = 3 falls into the critical region, we reject H_0.

There is sufficient evidence at 5% siginificance level to say that the treatment is not as effective as it is claimed.

6.

For parts (i), (ii) and (iii), please refer to the answers for Ex A/Q6 given on page 2.

(iv) (A) Let p be the probability that a random mathematics graduate applicant is successful

$H_0 : p = 0.2$

$H_1 : p > 0.2$

(B) We are testing the suggestion that mathemetics graduates are more likely to be successful than those from other fields. Hence, the alternative hypothesis takes the form shown above.

(C) Success: A mathematics graduate applicant is successful.
X : Number of successes

Assuming that H_0 is true,

$X \sim B(17, 0.2)$

Let $X \geq C$ be the critical region.

Then, $P(X \geq C) < 0.05$

$1 - P(X \leq C - 1) < 0.05$

$1 - 0.05 < P(X \leq C - 1)$

$P(X \leq C - 1) > 0.95$

<u>Binomial CD (List)</u>

N: 17
P : 0.2

$P(X \leq 5) = 0.8942 < 0.95$

$P(X \leq 6) = 0.9623 > 0.95$

Therefore, $C - 1 = 6$

$C = 7$

Hence the crotican region is $X \geq 7$

Since the obeserved outcome $X = 10$ falls into the critical region, we reject H_0.

There is sufficient evidence at 5% siginificance level to say that mathematics graduates are more likely to be successful than those from other fields.

7.

Let p be the probability that the die lands on 6, when rolled.

$H_0 : p = 1/6$

$H_1 : p < 1/6$

Success: The die lands on 6, when rolled.

X : Number of successes

Assuming that H_0 is true,

$X \sim B(12, 1/6)$

Let $X \leq C$ be the critical region.

Then, $P(X \leq C) < 0.05$

<u>Binomial CD (List)</u>
N : 12
p : 1/6

There is no value of r for which $P(X \leq r) < 0.05$.

This means there is no critical region.

In other words, regardless of the observed outcome, H_0 will never be rejected in this hypothesis test.

Therefore this test cannot provide evidence at 5% significance level to say that the probability of 6 on this die is less than 1/6.

8.

(i) Let p be the probability that a random volunteer names all of the items.

$H_0 : p = 0.35$

$H_1 : p > 0.35$

The alternative hypothesis takes the form given above because we are testing the student's belief that the probability of a volunteer naming all of the items would be higher if the volunteer listens to the same piece of music while memorising the items and while trying to name them.

(ii)

Success: A volunteer names all of the items.

X : Number of successes

Assuming that H_0 is true,

$X \sim B(15, 0.35)$

Let $X \geq C$ be the critical region.

Then, $P(X \geq C) < 0.05$

$1 - P(X \leq C - 1) < 0.05$

$1 - 0.05 < P(X \leq C - 1)$

$P(X \leq C - 1) > 0.95$

Binomial CD (List)

N: 15
P : 0.35

$P (X \leq 7) = 0.8867 < 0.95$

$P (X \leq 8) = 0.9578 > 0.95$

Therefore, $C - 1 = 8$

$C = 9$

Hence the crotican region is $X \geq 9$

Since the obeserved outcome $X = 8$ does not fall into the critical region, we do not reject H_0.

There is insufficient evidence at 5% siginificance level to say that the probability of a volunteer naming all of the items would be higher if the volunteer listens to the same piece of music while memorising the items and while trying to name them.

Exercise C – Exercise B

(We did Questions 1 and 3 during our lessons.)

2.

Let p be the probability that a random chick beloging to this unknown type of bird being female.

$H_0 : p = 0.5$

$H_1 : p \neq 0.5$

Success: A chick being a female.

X : Number of successes

Assuming that H_0 is true,

$X \sim B(16, 0.5)$

Let the critical region be, $X \leq C_1$ or $X \geq C_2$.

Then, $P (X \leq C_1) < 0.025$ and $P (X \geq C_2) < 0.025$.

For $P (X \leq C_1) < 0.025$

Binomial CD (List)
N : 16
p : 0.5

$P (X \leq 3) = 0.0106 < 0.025$

$P (X \leq 4) = 0.0384 > 0.025$

Therefore $C_1 = 3$.

For $P(X \geq C_2) < 0.025$

$P(X \geq C_2) < 0.025$

$1 - P(X \leq C_2 - 1) < 0.025$

$1 - 0.025 < P(X \leq C_2 - 1)$

$P(X \leq C_2 - 1) > 0.975$

Binomial CD (List)
N : 16
p : 0.5

$P(X \leq 11) = 0.9615 < 0.975$

$P(X \leq 12) = 0.9893 > 0.975$

Therefore $C_2 - 1 = 12$
$C_2 = 13$

Hence the crtical region is, $X \leq 3$ or $X \geq 13$

Since the obeserved outcome $X = 13$ falls into the critical region, we reject H_0.

There is sufficient evidence at 5% siginificance level to support the view that the sex ratio for the chicks differs from 1.

4.

Let p be the probability that a random day in April is wet.

$H_0 : p = 0.25$

$H_1 : p \neq 0.25$

Success: A day in April is wet.

X : Number of successes

Assuming that H_0 is true,

$X \sim B(20, 0.25)$

Let the critical region be, $X \leq C_1$ or $X \geq C_2$.

Then, $P(X \leq C_1) < 0.05$ and $P(X \geq C_2) < 0.05$.

For $P(X \leq C_1) < 0.05$

Binomial CD (List)
N : 20
p : 0.25

$P(X \leq 1) = 0.0243 < 0.05$

$P(X \leq 2) = 0.0912 > 0.05$

Therefore $C_1 = 1$.

<u>For $P(X \geq C_2) < 0.05$</u>

$P(X \geq C_2) < 0.05$

$1 - P(X \leq C_2 - 1) < 0.05$

$1 - 0.05 < P(X \leq C_2 - 1)$

$P(X \leq C_2 - 1) > 0.95$

<u>Binomial CD (List)</u>
N : 20
p : 0.25

$P(X \leq 7) = 0.8981 < 0.95$

$P(X \leq 8) = 0.959 > 0.95$

Therefore $C_2 - 1 = 8$
$C_2 = 9$

Hence the crtical region is, $X \leq 1$ or $X \geq 9$

Since the obeserved outcome $X = 10$ falls into the critical region, we reject H_0.

There is sufficient evidence at 10% siginificance level to say that the climate is changing and therefore the complaint is justified.