

Hypothesis Test Using Binomial Distribution – Year 12

Exercise A

1.

Mrs da Silva is running for President. She claims to have 60% of the population supporting her.

She is suspected of overestimating her support and a random sample of 20 people are asked whom they support. Only nine say Mrs da Silva.

Test, at the 5% significance level, the hypothesis that she has overestimated her support.

2.

On average 70% of people pass their driving test first time. There are complaints that Mr McTaggart is too harsh and so, unknown to himself, his work is monitored. It is found that he fails 10 out of 20 candidates. Are the complaints justified at the 5% significance level?

3.

A machine makes bottles. In normal running 5% of the bottles are expected to be cracked, but if the machine needs servicing this proportion will increase. As part of a routine check, 50 bottles are inspected and 5 are found to be unsatisfactory. Does this provide evidence, at the 5% significance level, that the machine needs servicing?

4.

A group of 18 students decides to investigate the truth of the saying that if you drop a piece of toast it is more likely to land butter-side down. They each take one piece of toast, butter it on one side and throw it in the air. Eleven land butter-side down, the rest butter-side up. Use their results to carry out a hypothesis test at the 10% significance level, stating clearly your null and alternative hypotheses.

5.

The success rate of the standard treatment for patients suffering from a particular skin disease is claimed to be 68%.

a In a sample of n patients, X is the number for which the treatment is successful.

Write down a suitable distribution to model X . Give reasons for your choice of model.

A random sample of 10 patients receives the standard treatment and in only 3 cases was the treatment successful. It is thought that the standard treatment was not as effective as it is claimed.

b Test the claim at the 5% level of significance.

6.

A multinational accountancy firm receives a large number of job applications from graduates each year. On average 20% of applicants are successful.

A researcher in the human resources department of the firm selects a random sample of 17 graduate applicants.

(i) Find the probability that at least 4 of the 17 applicants are successful. [3]

(ii) Find the expected number of successful applicants in the sample. [2]

(iii) Find the most likely number of successful applicants in the sample, justifying your answer. [3]

It is suggested that mathematics graduates are more likely to be successful than those from other fields. In order to test this suggestion, the researcher decides to select a new random sample of 17 mathematics graduate applicants. The researcher then carries out a hypothesis test at the 5% significance level.

(iv) (A) Write down suitable null and alternative hypotheses for the test.

(B) Give a reason for your choice of the alternative hypothesis. [4]

(C) Among the 17 applicants, 10 are found to be successful. Determine whether there is sufficient evidence to believe that mathematics graduates are more likely to be successful.

7.

A die used in playing a board game is suspected of not giving the number 6 often enough. During a particular game it was rolled 12 times and only one 6 appeared. Does this represent significant evidence, at the 5% level of significance, that the probability of a 6 on this die is less than $\frac{1}{6}$?

8.

A psychology student is investigating memory. In an experiment, volunteers are given 30 seconds to try to memorise a number of items. The items are then removed and the volunteers have to try to name all of them. It has been found that the probability that a volunteer names all of the items is 0.35. The student believes that this probability may be increased if the volunteers listen to the same piece of music while memorising the items and while trying to name them.

The student selects 15 volunteers at random to do the experiment while listening to music. Of these volunteers, 8 name all of the items.

(i) Write down suitable hypotheses for a test to determine whether there is any evidence to support the student's belief, giving a reason for your choice of alternative hypothesis. [4]

(ii) Carry out the test at the 5% significance level. [4]

Exercise B

- 1 To test the claim that a coin is biased, it is tossed 20 times. It comes down heads 7 times. Test at the 10% significance level whether this claim is justified.
- 2 A biologist discovers a colony of a previously unknown type of bird nesting in a cave. Out of the 16 chicks which hatch during his period of investigation, 13 are female. Test at the 5% significance level whether this supports the view that the sex ratio for the chicks differs from 1.
- 3 People entering an exhibition have to choose whether to turn left or right. Out of the first twelve people, nine turn left and three right. Test at the 5% significance level whether people are more likely to turn one way than another.
- 4 Weather records for a certain seaside resort show that on average one day in four in April is wet, but local people write to their newspaper complaining that the climate is changing.

A reporter on the paper records the weather for the next 20 days in April and finds that 10 of them are wet.

Do you think the complaint is justified? (Assume a 10% significance level.)

Exercise C

For each of the questions in Exercise A and Exercise B, find the critical region for the hypothesis test and complete the test.
