Revision Paper 6

Please skip Q1 and Q2 as they are not in the year 12 specification.

1. A geometric series has first term a and common ratio $r = \frac{3}{4}$

The sum of the first 4 terms of this series is 175

(a) Show that a = 64

(2)

(b) Find the sum to infinity of the series.

(2)

(c) Find the difference between the 9th and 10th terms of the series. Give your answer to 3 decimal places.

(3)

2. The curve C has equation

$$y = 8 - 2^{x-1}, \qquad 0 \leqslant x \leqslant 4$$

(a) Complete the table below with the value of y corresponding to x = 1

x	0	1	2	3	4
y	7.5		6	4	0

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for $\int_0^4 (8-2^{x-1}) dx$

(3)

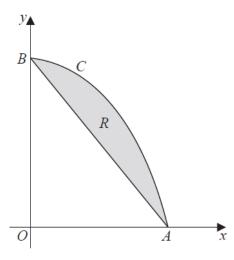


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = 8 - 2^{x-1}$, $0 \le x \le 4$

The curve C meets the x-axis at the point A and meets the y-axis at the point B.

The region R, shown shaded in Figure 1, is bounded by the curve C and the straight line through A and B.

(c) Use your answer to part (b) to find an approximate value for the area of R.

(2)

3.

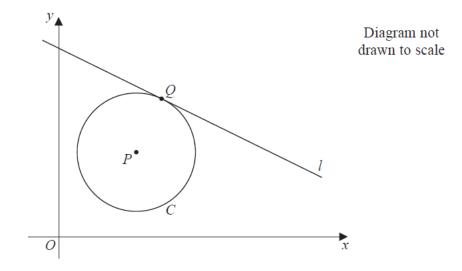


Figure 2

The circle C has centre P(7, 8) and passes through the point Q(10, 13), as shown in Figure 2.

(a) Find the length PQ, giving your answer as an exact value.

(2)

(b) Hence write down an equation for C.

(2)

The line l is a tangent to C at the point Q, as shown in Figure 2.

(c) Find an equation for l, giving your answer in the form ax + by + c = 0, where a, b and c are integers.

(4)

You may do part (a) of Q4 by Algebraic division.

4. $f(x) = 6x^3 + 13x^2 - 4$

(a) Use the remainder theorem to find the remainder when f(x) is divided by (2x + 3).

(b) Use the factor theorem to show that (x + 2) is a factor of f(x).

(2)

(c) Factorise f(x) completely.

(4)

5. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2-9x)^4$$

giving each term in its simplest form.

(4)

$$f(x) = (1 + kx)(2 - 9x)^4$$
, where k is a constant

The expansion, in ascending powers of x, of f(x) up to and including the term in x^2 is

$$A - 232x + Bx^2$$

where A and B are constants.

(b) Write down the value of A.

(1)

(c) Find the value of k.

(2)

(d) Hence find the value of B.

(2)

6. (i) Solve, for $-\pi < \theta \leqslant \pi$,

$$1 - 2\cos\left(\theta - \frac{\pi}{5}\right) = 0$$

giving your answers in terms of π .

(3)

(ii) Solve, for $0 \le x < 360^\circ$,

$$4\cos^2 x + 7\sin x - 2 = 0$$

giving your answers to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

Question 7 is on the next page.

7.

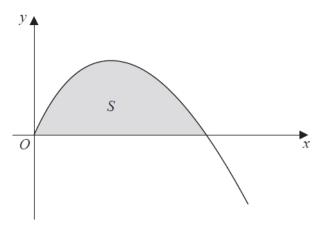


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 3x - x^{\frac{3}{2}}, \qquad x \geqslant 0$$

The finite region S, bounded by the x-axis and the curve, is shown shaded in Figure 3.

(a) Find

$$\int \left(3x - x^{\frac{3}{2}}\right) \mathrm{d}x \tag{3}$$

(b) Hence find the area of S.

(3)

8. (i) Given that

$$\log_3(3b+1) - \log_3(a-2) = -1, \quad a > 2$$

express b in terms of a.

(3)

(ii) Solve the equation

$$2^{2x+5} - 7(2^x) = 0$$

giving your answer to 2 decimal places.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(4)

Question 9 is on the next page.

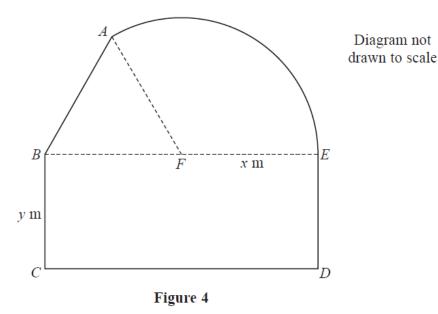


Figure 4 shows a plan view of a sheep enclosure.

The enclosure ABCDEA, as shown in Figure 4, consists of a rectangle BCDE joined to an equilateral triangle BFA and a sector FEA of a circle with radius x metres and centre F.

The points B, F and E lie on a straight line with FE = x metres and $10 \le x \le 25$

(a) Find, in m^2 , the exact area of the sector FEA, giving your answer in terms of x, in its simplest form.

Given that BC = y metres, where y > 0, and the area of the enclosure is 1000 m²,

(b) show that

$$y = \frac{500}{x} - \frac{x}{24} \left(4\pi + 3\sqrt{3} \right) \tag{3}$$

(c) Hence show that the perimeter P metres of the enclosure is given by

$$P = \frac{1000}{x} + \frac{x}{12} \left(4\pi + 36 - 3\sqrt{3} \right) \tag{3}$$

- (d) Use calculus to find the minimum value of P, giving your answer to the nearest metre. (5)
- (e) Justify, by further differentiation, that the value of P you have found is a minimum. (2)

(2)