Errors are not mistakes

▲ **Figure 1** *An error made when you use a calculator is a mistake that can be corrected but measurement errors are more difficult to deal with*

Making measurements and using instruments is a key part of scientific activity. No measurement can ever be perfect. When you think about errors you may think about mistakes. If you make a mistake in an experiment, you may be may be able to do the experiment again without the mistake. However, an experiment will still contain errors. An error in a scientific sense is the difference between the result you get and the correct result. Errors are usually caused by measuring devices, even if they are used correctly, or by the design of the experiment itself.

Measurement errors

A **true value** is the value that would be obtained in an ideal measurement. True values may be values found in a data book, or the values you expect to get in your experiments. A **measurement error** is the difference between a measured value and the true value for the quantity being measured. Remember that mistakes are not counted as errors here.

Random errors

Random errors can happen when any measurement is being made. They are measurement errors in which measurements vary unpredictably. There can be many reasons for this, including

- factors that are not controlled in the experiment
- difficulty in deciding on the reading given by a measuring device.

Random errors cannot be corrected. All you can hope to do is reduce their effect by making more measurements and reporting the mean value.

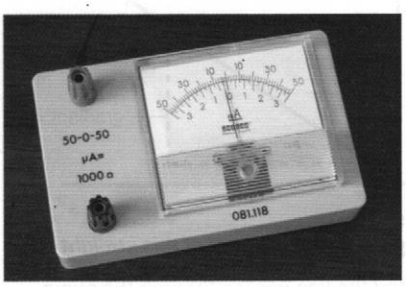
Systematic errors

Systematic errors are measurement errors in which the measurements differ from the true values by a consistent amount each time a measurement is made. Reasons for this include

- the way in which measurements are taken
- faulty measuring devices.

For example, poor contact between a thermometer and the object whose temperature is being measured will cause systematic errors. A faulty measuring device may give readings that are consistently too high or too low. This may be because it has not been calibrated correctly. A faulty device may give a **zero error**, in which the reading is not zero when the quantity being measured is zero (Figure 2).

Unlike random errors, it may be possible to correct for systematic errors.



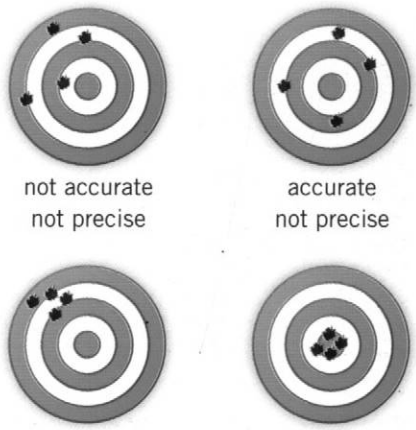
▲ **Figure 2** *Zero error on an ammeter*

Precision and accuracy

In everyday life, people often use the words *precise* and *accurate* to mean the same thing – this is not the case in physics.

- **Accuracy** is to do with how close a measurement result is to the true value – the closer it is, the more **accurate** it is.
- **Precision** is to do with how close repeated measurements are to each other – the closer they are to each other, the more **precise** the measurement is.

Figure 3 is a visual way of appreciating the terms accuracy and precision using a dartboard as an example. You are aiming for bullseye – it represents the true value.



▲ **Figure 3** *Accuracy and precision*

Uncertainty

Random and systematic errors mean that you rarely obtain the same value for a particular measurement. Consider using a micrometer to measure the diameter of a supposedly uniform copper wire. The readings below show the readings along the length of the copper wire:

$$0.53\,\text{mm},\ 0.49\,\text{mm},\ 0.52\,\text{mm},\ 0.51\,\text{mm}$$

The smallest scale division of this instrument is 0.01 mm but the readings above show a spread much greater than this.

The **mean** value for the diameter can be calculated by adding together the values for each repeat reading, then dividing by the number of readings. The mean diameter of the copper wire is

$$\frac{0.53\,\text{mm} + 0.49\,\text{mm} + 0.52\,\text{mm} + 0.51\,\text{mm}}{4} = 0.51\,\text{mm}.$$

The **range** of the measurements is 0.04 mm. This is the difference between the smallest and largest readings (0.53 mm – 0.49 mm).

The **uncertainty** in the measurement is an interval within which the true value can be expected to lie. The **absolute uncertainty** in the mean value of a measurement can be approximated as half the range. This is often expressed as ± value. In this example, you can write the diameter as its mean value ± absolute uncertainty

$$\text{diameter} = 0.51 \pm 0.02\,\text{mm}$$

The **percentage uncertainty** in the diameter can be calculated from its absolute uncertainty and mean value as follows:

$$\%\ \text{uncertainty in diameter} = \frac{\text{absolute uncertainty}}{\text{mean value}} \times 100$$

$$= \frac{0.02}{0.51} \times 100 = 3.9\%$$

Finally, what do you do when repeat measurements give identical values or you have just taken a single measurement? In this situation you can approximate the absolute uncertainty to be equal to the **resolution** of the measuring instrument. This is the smallest change in the measured quantity that the instrument can show. The micrometer readings for the copper wire are written to 2 decimal places, with ±0.01 mm being the smallest change it could show.

Therefore, if all the readings were 0.53 mm, or you just had a single reading of 0.53 mm, the diameter of the copper wire may be written as

$$\text{diameter} = 0.53 \pm 0.01\ \text{mm}$$

Analysing uncertainties

Uncertainties can help you identify where the greatest errors in an experiment are, giving you the chance to improve it using a different method or measuring instrument.

The final uncertainty in an answer depends on how quantities are combined. Here are three important rules about the way uncertainties propagate.

1 **Adding or subtracting quantities**

When you add or subtract quantities in an equation, you add the absolute uncertainties for each value.

What is the extension?

The original length of a spring is 2.5 ± 0.1 cm and the final length is 15.0 ± 0.2 cm. Calculate the extension of the spring and the absolute uncertainty.

Step 1: Calculate the extension by subtracting the lengths.

$$\text{extension} = 15.0 - 2.5 = 12.5\ \text{cm}$$

Step 2: Add the absolute uncertainties.

$$\text{absolute uncertainty} = 0.1 + 0.2 = 0.3$$

Step 3: Write the answer in the normal convention.

$$\text{extension} = 12.5 \pm 0.3\ \text{mm}$$

2 **Multiplying or dividing quantities**

When you multiply or divide quantities, you add the percentage uncertainties for each value.

What is the resistance?

The current I in a resistor is 1.60 ± 0.02 A and the potential difference V across the resistor is 6.00 ± 0.20 V. Calculate the resistance and the absolute uncertainty.

Step 1: Calculate the resistance R of the resistor.

$$R = \frac{V}{I} = \frac{6.00}{1.60} = 3.75\,\Omega$$

Step 2: Calculate the percentage uncertainty in each measurement.

$$\%\ \text{uncertainty in}\ I = \frac{0.02}{1.60} \times 100 = 1.25\%$$

$$\%\ \text{uncertainty in}\ V = \frac{0.20}{6.00} \times 100 = 3.33\%$$

Step 3: Add the percentage uncertainties.

% uncertainty in $R = 1.25 + 3.33 = 4.58\%$

Step 4: Calculate the absolute uncertainty in R.

absolute uncertainty in $R = 0.0458 \times 3.75 = 0.17\,\Omega$

Step 5: The values of V and I are quoted to 3 significant figures, therefore the final answer for the resistance must also be written to 3 significant figures.

$R = 3.75 \pm 0.17\,\Omega$ (3 s.f.)

3 **Raising a quantity to a power**

When a measurement in a calculation is raised to a power n, your percentage uncertainty is increased n times. The power n can be an integer or a fraction.

Cross-sectional area of a wire

The diameter of a wire is recorded as 0.51 ± 0.02 mm. Calculate the cross-sectional area of the wire and the absolute uncertainty.

Step 1: Calculate the cross-sectional area A of the wire.

$$A = \frac{\pi d^2}{4} = \frac{\pi \times (0.51 \times 10^{-3})^2}{4} = 2.04 \times 10^{-7}\,\text{m}^2$$

Step 2: The percentage uncertainty in A is equal to 2 times the percentage uncertainty in d.

(The π and the 4 are numbers and therefore have no uncertainty associated with them.)

$$\%\text{ uncertainty in } A = 2 \times \left(\frac{0.02}{0.51} \times 100\right) = 7.84\%$$

Step 3: Calculate the absolute uncertainty in A.

absolute uncertainty in $A = 0.0784 \times 2.04 \times 10^{-7} = 0.16 \times 10^{-7}\,\text{m}^2$

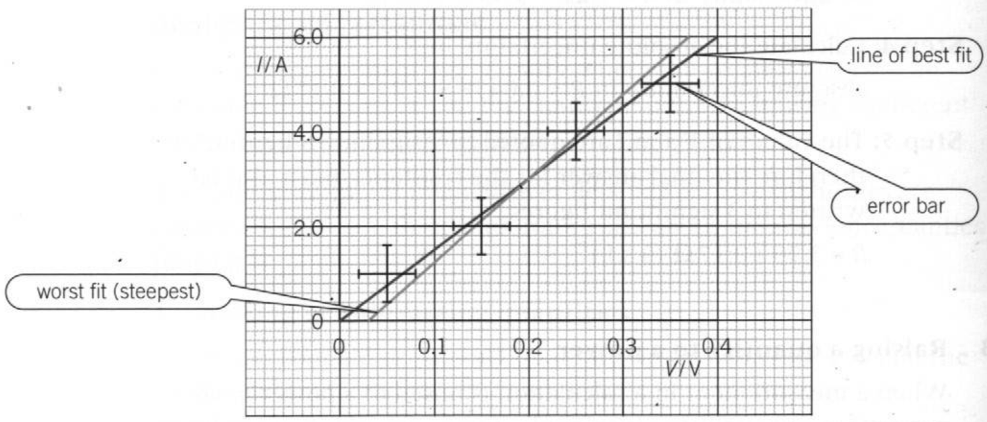
Step 4: The diameter of the wire is quoted to 2 significant figures, therefore the final answer for the cross-sectional must also be written to 2 significant figures.

$A = (2.0 \pm 0.2) \times 10^{-7}\text{m}^2$ (2 s.f.)

Graphs

Straight line graphs are important in physics because you can use them to formulate relationships between physical quantities. As indicated in Appendix A2, Recording results, you plot points using small crosses. If the points appear to lie on a straight line, then you can draw your straight line of best fit using a long ruler. You must ignore a point that is much further than any other point from the best fit line. This point is referred to as being **anomalous**.

The uncertainty in a measurement can be used to give a small range or **error bar** for each measurement. Instead of plotting just the points on a graph, you can plot an error bar for all of your measurements.



▲ **Figure 4** *Error bars are useful when you draw the line of best fit and the worst line for your measurements*

Your straight best fit line must pass through all the error bars (Figure 4). You would use this line to determine the value of the gradient. How can you determine an approximate value for the uncertainty in the gradient? You would draw the **line of worst fit** – the least acceptable straight line through the data points – this can either be the steepest or the shallowest line.

The absolute uncertainty in the gradient is the positive difference between the gradient of the line of best fit and the gradient of the line of worst fit.

The percentage uncertainty in the gradient can be calculated as

$$\%\text{ uncertainty in gradient} = \frac{\text{absolute uncertainty}}{\text{gradient best fit line}} \times 100\%$$