Binomial Distribution

1 The random variable X has a binomial distribution with $n = 6$ and $p = 0.2$. Calculate

(a) $P(X = 3)$, (b) $P(X = 4)$, (c) $P(X = 6)$.

2 Given that $Y \sim B\left(7, \frac{2}{3}\right)$, calculate

(a) $P(Y = 4)$, (b) $P(Y = 6)$, (c) $P(Y = 0)$.

3 Given that $Z \sim B(9, 0.45)$, calculate

(a) $P(Z = 3)$, (b) $P(Z = 4 \text{ or } 5)$, (c) $P(Z \geqslant 7)$.

4 Given that $D \sim B(12, 0.7)$, calculate

(a) $P(D < 4)$, (b) the smallest value of d such that $P(D > d) < 0.95$.

5 A fair coin is thrown 9 times. Calculate the probability that the number of heads is

(a) exactly 5, (b) 5 or 6, (c) at least 8, (d) more than 2.

6 A fair cubical dice is rolled 7 times. Find the probability that the number of sixes is

(a) exactly 3, (b) at least 4.

7 In a certain school, 30% of the students are members of the Sixth Form.

(a) Ten students are chosen at random. What is the probability that fewer than 4 of them are in the Sixth Form?

(b) If the ten students were chosen by picking ten who were sitting together at lunch, explain why a binomial distribution might no longer have been suitable.

8 A factory makes large quantities of coloured sweets, and it is known that on average 20% of the sweets are coloured green. A packet contains twenty sweets. Assuming that the packet forms a random sample of the sweets made by the factory, calculate the probability that exactly 7 of the sweets are green.

If you knew that, in fact, the sweets could have been green, red, orange or brown, would it have invalidated your calculation?

9 Eggs produced at a farm are packaged in boxes of six. Assume that, for any egg, the probability that it is broken when it reaches the retail outlet is 0.1, independent of all other eggs. A box is said to be bad if it contains at least two broken eggs. Calculate the probability that a randomly selected box is bad.

Ten boxes are chosen at random. Find the probability that just two of these boxes are bad.

It is known that, in fact, breakages are more likely to occur after the eggs have been packed into boxes, and while they are being transported to the retail outlet. Explain why this fact is likely to invalidate the calculation.

10 On a particular tropical island, the probability that there is a hurricane in any given month can be taken to be 0.08. Use a binomial distribution to calculate the probability that there is a hurricane in more than two months of the year. State two assumptions needed for a binomial distribution to be a good model, and give a reason why one of the assumptions may not be valid.

11 The random variable X has a binomial distribution with $n = 5$ and $p = 0.35$. Use tables to find

(a) $P(X \leqslant 2)$, (b) $P(X = 3)$, (c) $P(X < 5)$, (d) $P(X > 2)$.

12 The random variable Y has the distribution $B(10, 0.4)$. Use tables to find

(a) $P(Y \leqslant 5)$, (b) $P(Y = 2)$, (c) $P(Y < 4)$, (d) $P(Y \geqslant 6)$.

13 The random variable Z has the distribution $B(10, 0.7)$. Use tables to find

(a) $P(Z \leqslant 5)$, (b) $P(Z \geqslant 4)$, (c) $P(Z < 8)$.

14 The random variable X has the distribution $B(n, 0.4)$.

(a) Given that $n = 10$, find the smallest value of x such that $P(X > x) < 0.05$.

(b) Find the smallest value of n such that $P(X = 0)$ is less than 0.001.

(c) Find the smallest value of n such that $P(X < 2)$ is less than 0.01, using trial and improvement or another suitable numerical method.

15 It is given that, at a stated time of day, 35% of the adults in the country are wearing jeans. At that time, a sample of twelve adults is selected. Use a binomial distribution to calculate the probability that exactly five out of these twelve are wearing jeans. Explain carefully two assumptions that must be made for your calculation to be valid. (If you say 'sample is random' you must explain what this means in the context of the question.)