Revision Paper 5
(Time Allowed: 1 hour and 30 minutes)

(Please skip Questions 2, 3 and 9 (i) as they have now been moved to Year 13.)

1 **(i)** Find $\frac{dy}{dx}$ when $y = 6\sqrt{x}$. **[2]**

(ii) Find $\int \frac{12}{x^2} dx$. **[3]**

2 A sequence is defined as follows.

$u_1 = a$, where $a > 0$

To obtain u_{r+1}

- find the remainder when u_r is divided by 3,
- multiply the remainder by 5,
- the result is u_{r+1}.

Find $\sum_{r=2}^{4} u_r$ in each of the following cases.

(i) $a = 5$

(ii) $a = 6$ **[3]**

3 An arithmetic progression (AP) and a geometric progression (GP) have the same first and fourth terms as each other. The first term of both is 1.5 and the fourth term of both is 12. Calculate the difference between the tenth terms of the AP and the GP. **[5]**

4

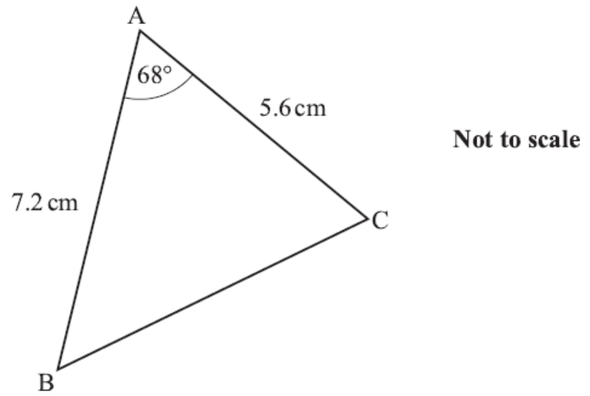


Fig. 4

Fig. 4 shows triangle ABC, where AB = 7.2 cm, AC = 5.6 cm and angle BAC = 68°.

Calculate the size of angle ACB. **[5]**

5 **(i)** Fig. 5 shows the graph of a sine function.

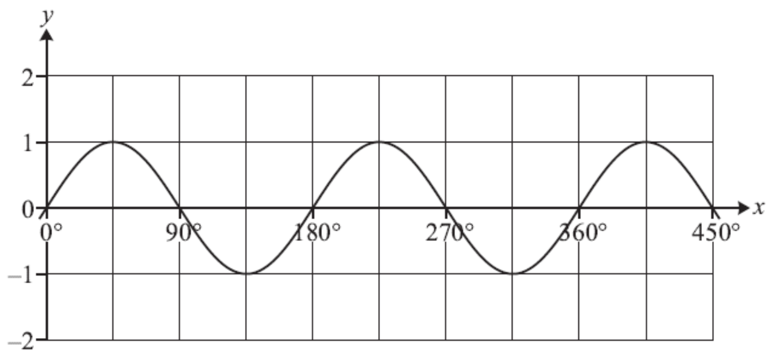
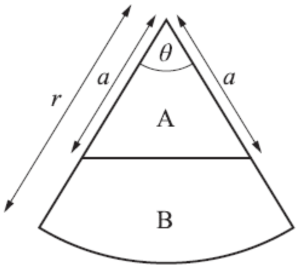


Fig. 5

State the equation of this curve. **[2]**

(ii) Sketch the graph of $y = \sin x - 3$ for $0° \leqslant x \leqslant 450°$. **[2]**

6 A sector of a circle has radius r cm and sector angle θ radians. It is divided into two regions, A and B. Region A is an isosceles triangle with the equal sides being of length a cm, as shown in Fig. 6.



Not to scale

Fig. 6

(i) Express the area of B in terms of a, r and θ. **[2]**

(ii) Given that $r = 12$ and $\theta = 0.8$, find the value of a for which the areas of A and B are equal. Give your answer correct to 3 significant figures. **[2]**

7 **(i)** Show that, when x is an acute angle, $\tan x\sqrt{1-\sin^2 x} = \sin x$. **[2]**

(ii) Solve $4\sin^2 y = \sin y$ for $0° \leqslant y \leqslant 360°$. **[3]**

8 **(i)** Simplify $\log_a 1 - \log_a (a^m)^3$. **[2]**

(ii) Use logarithms to solve the equation $3^{2x+1} = 1000$. Give your answer correct to 3 significant figures. **[3]**

Question 9 is on the next page.

9 Fig. 9 shows the cross-section of a straight, horizontal tunnel. The x-axis from 0 to 6 represents the floor of the tunnel.

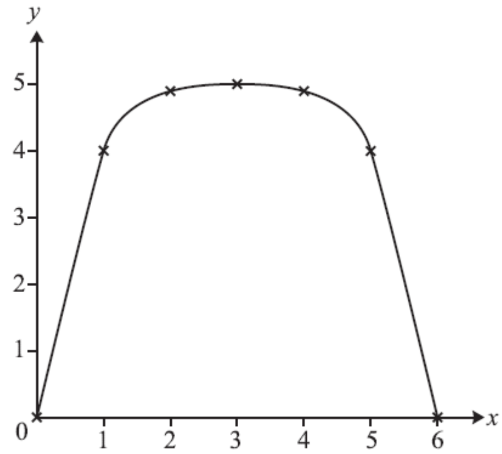


Fig. 9

With axes as shown, and units in metres, the roof of the tunnel passes through the points shown in the table.

x	0	1	2	3	4	5	6
y	0	4.0	4.9	5.0	4.9	4.0	0

The length of the tunnel is 50 m.

(i) Use the trapezium rule with 6 strips to estimate the area of cross-section of the tunnel. Hence estimate the volume of earth removed in digging the tunnel. **[4]**

(ii) An engineer models the height of the roof of the tunnel using the curve $y = \frac{5}{81}(108x - 54x^2 + 12x^3 - x^4)$. This curve is symmetrical about $x = 3$.

(*A*) Show that, according to this model, a vehicle of rectangular cross-section which is 3.6 m wide and 4.4 m high would not be able to pass through the tunnel. **[2]**

(*B*) Use integration to calculate the area of the cross-section given by this model. Hence obtain another estimate of the volume of earth removed in digging the tunnel. **[5]**

10 **(i)** Calculate the gradient of the chord of the curve $y = x^2 - 2x$ joining the points at which the values of x are 5 and 5.1. **[2]**

(ii) Given that $f(x) = x^2 - 2x$, find and simplify $\dfrac{f(5+h) - f(5)}{h}$. **[4]**

(iii) Use your result in part (ii) to find the gradient of the curve $y = x^2 - 2x$ at the point where $x = 5$, showing your reasoning. **[2]**

(iv) Find the equation of the tangent to the curve $y = x^2 - 2x$ at the point where $x = 5$.

Find the area of the triangle formed by this tangent and the coordinate axes. **[5]**

Question 11 is on the next page.

11 There are many different flu viruses. The numbers of flu viruses detected in the first few weeks of the 2012–2013 flu epidemic in the UK were as follows.

Week	1	2	3	4	5	6	7	8	9	10
Number of flu viruses	7	10	24	32	40	38	63	96	234	480

These data may be modelled by an equation of the form $y = a \times 10^{bt}$, where y is the number of flu viruses detected in week t of the epidemic, and a and b are constants to be determined.

(i) Explain why this model leads to a straight-line graph of $\log_{10} y$ against t. State the gradient and intercept of this graph in terms of a and b. **[3]**

(ii) Complete the values of $\log_{10} y$ in the table, draw the graph of $\log_{10} y$ against t, and draw by eye a line of best fit for the data.

Hence determine the values of a and b and the equation for y in terms of t for this model. **[8]**

During the decline of the epidemic, an appropriate model was

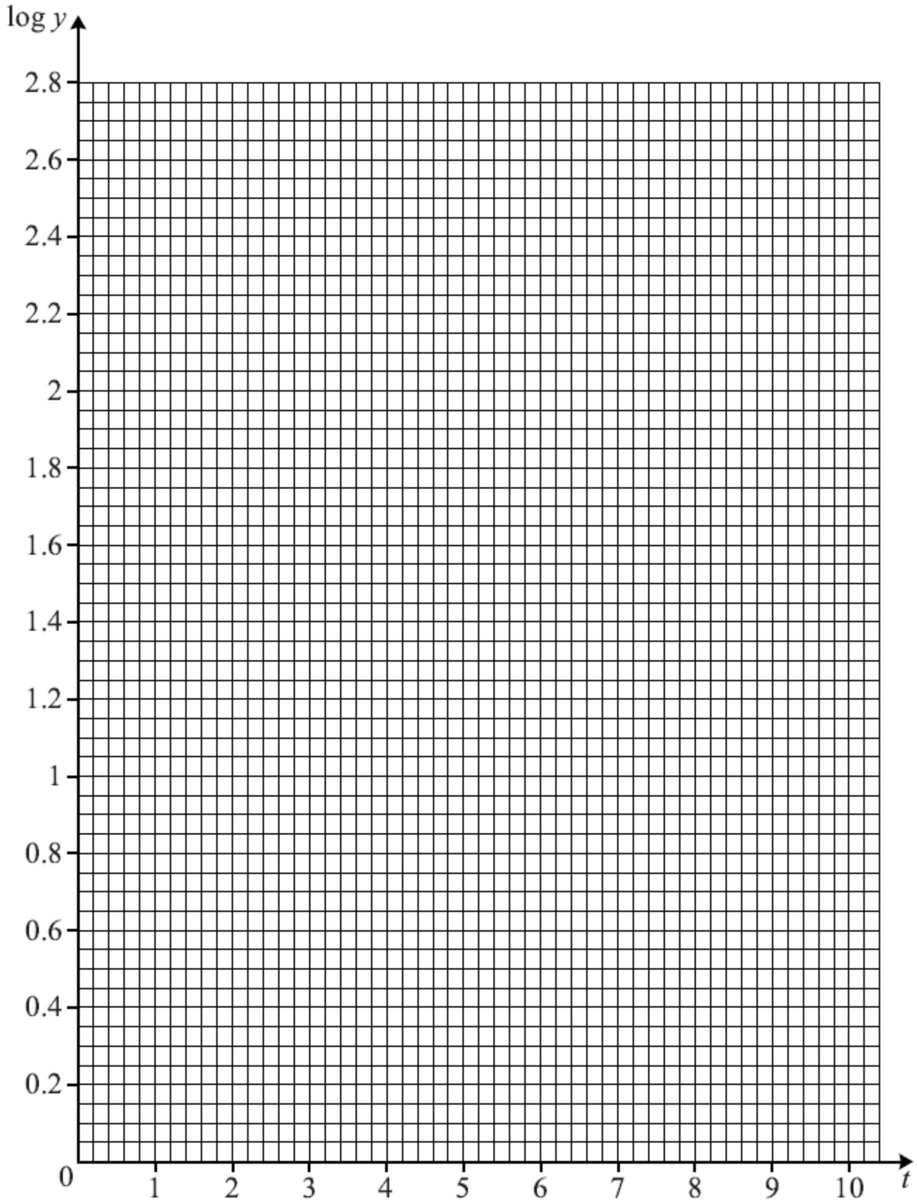
$$y = 921 \times 10^{-0.137w},$$

where y is the number of flu viruses detected in week w of the decline.

(iii) Use this to find the number of viruses detected in week 4 of the decline. **[1]**

t	1	2	3	4	5	6	7	8	9	10
y	7	10	24	32	40	38	63	96	234	480
$\log_{10} y$	0.85	1	1.38	1.51	1.60					

The grid for the graph is on the next page.


log y
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
9
10
t