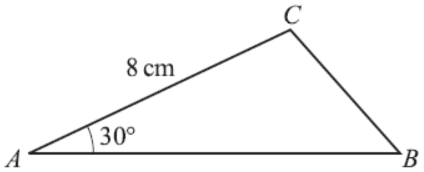
Revision Paper 4
(1 hour and 30 minutes)

1

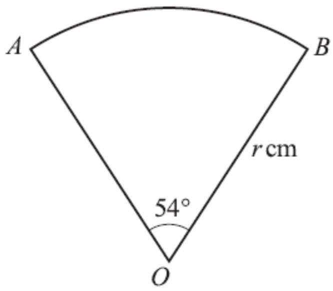


The diagram shows triangle ABC, with $AC = 8$ cm and angle $CAB = 30°$.

(i) Given that the area of the triangle is 20 cm^2, find the length of AB. **[2]**

(ii) Find the length of BC, giving your answer correct to 3 significant figures. **[2]**

2



The diagram shows a sector AOB of a circle with centre O and radius r cm. The angle AOB is 54°. The perimeter of the sector is 60 cm.

(i) Express 54° exactly in radians, simplifying your answer. **[2]**

(ii) Find the value of r, giving your answer correct to 3 significant figures. **[3]**

3 **(i)** Find the binomial expansion of $(3 + kx)^3$, simplifying the terms. **[4]**

(ii) It is given that, in the expansion of $(3 + kx)^3$, the coefficient of x^2 is equal to the constant term. Find the possible values of k, giving your answers in an exact form. **[2]**

4 **(i)** Express $2\log_3 x - \log_3(x + 4)$ as a single logarithm. **[2]**

(ii) Hence solve the equation $2\log_3 x - \log_3(x + 4) = 2$. **[4]**

(Question 5 is on the next page)

5 **(a)** Find $\int (x^2+2)(2x-3)\,\mathrm{d}x$. **[3]**

(b) **(i)** Find, in terms of a, the value of $\int_1^a (6x^{-2}-4x^{-3})\,\mathrm{d}x$, where a is a constant greater than 1. **[4]**

(ii) Deduce the value of $\int_1^{\infty} (6x^{-2}-4x^{-3})\,\mathrm{d}x$. **[1]**

6 An arithmetic progression $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 5$ and $u_{n+1} = u_n + 1.5$ for $n \geqslant 1$.

(i) Given that $u_k = 140$, find the value of k. **[3]**

A geometric progression $w_1, w_2, w_3, \ldots$ is defined by $w_n = 120 \times (0.9)^{n-1}$ for $n \geqslant 1$.

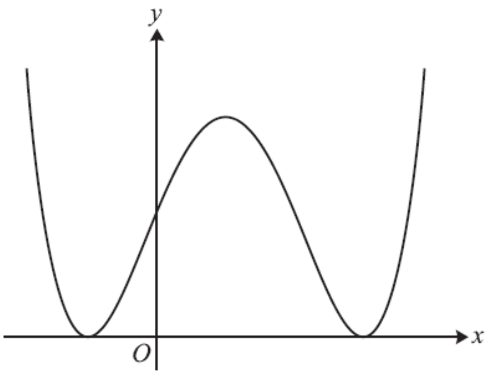
(ii) Find the sum of the first 16 terms of this geometric progression, giving your answer correct to 3 significant figures. **[2]**

(iii) Use an algebraic method to find the smallest value of N such that $\sum_{n=1}^{N} u_n > \sum_{n=1}^{\infty} w_n$. **[6]**

7 The cubic polynomial f(x) is defined by $\mathrm{f}(x) = x^3 - 3x^2 - x + 3$.

(i) Find the quotient and remainder when f(x) is divided by $(x + 1)$. **[3]**

(ii) Hence find the three roots of the equation $\mathrm{f}(x) = 0$. **[3]**



The diagram shows the curve C with equation $y = x^4 - 4x^3 - 2x^2 + 12x + 9$.

(iii) Show that the x-coordinates of the stationary points on C are given by $x^3 - 3x^2 - x + 3 = 0$. **[2]**

(iv) Use integration to find the exact area of the region enclosed by C and the x-axis. **[4]**

(Question 8 is on the next page)

8 **(i)** The curve $y = 3^x$ can be transformed to the curve $y = 3^{x-2}$ by a translation. Give details of the translation. **[2]**

(ii) Alternatively, the curve $y = 3^x$ can be transformed to the curve $y = 3^{x-2}$ by a stretch. Give details of the stretch. **[2]**

(iii) Sketch the curve $y = 3^{x-2}$, stating the coordinates of any points of intersection with the axes. **[2]**

(iv) The point P on the curve $y = 3^{x-2}$ has y-coordinate equal to 180. Use logarithms to find the x-coordinate of P, correct to 3 significant figures. **[3]**

(v) Use the trapezium rule, with 2 strips each of width 1.5, to find an estimate for $\int_1^4 3^{x-2}\,dx$. Give your answer correct to 3 significant figures. **[3]**

9 A curve has equation $y = \sin(ax)$, where a is a positive constant and x is in radians.

(i) State the period of $y = \sin(ax)$, giving your answer in an exact form in terms of a. **[1]**

(ii) Given that $x = \frac{1}{5}\pi$ and $x = \frac{2}{5}\pi$ are the two smallest positive solutions of $\sin(ax) = k$, where k is a positive constant, find the values of a and k. **[3]**

(iii) Given instead that $\sin(ax) = \sqrt{3}\cos(ax)$, find the two smallest positive solutions for x, giving your answers in an exact form in terms of a. **[4]**

END OF QUESTION PAPER