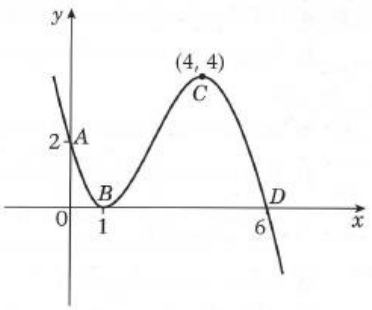
Transformations of Graphs

Exercise A

1 The following diagram shows a sketch of the curve with equation $y = f(x)$. The points $A(0, 2)$, $B(1, 0)$, $C(4, 4)$ and $D(6, 0)$ lie on the curve.



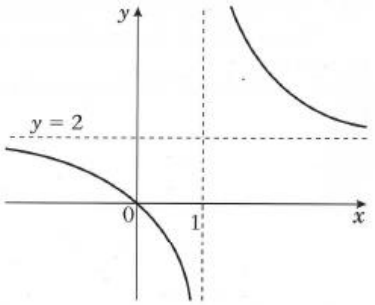
Sketch the following graphs and give the coordinates of the points A, B, C and D after each transformation:

a $f(x + 1)$ **b** $f(x) - 4$ **c** $f(x + 4)$

d $f(2x)$ **e** $3f(x)$ **f** $f(\frac{1}{2}x)$

g $\frac{1}{2}f(x)$ **h** $f(-x)$

2 The curve $y = f(x)$ passes through the origin and has horizontal asymptote $y = 2$ and vertical asymptote $x = 1$, as shown in the diagram.



Sketch the following graphs and give the equations of any asymptotes and, for all graphs except **a**, give coordinates of intersections with the axes after each transformation.

a $f(x) + 2$ **b** $f(x + 1)$ **c** $2f(x)$

d $f(x) - 2$ **e** $f(2x)$ **f** $f(\frac{1}{2}x)$

g $\frac{1}{2}f(x)$ **h** $-f(x)$

Exercise B

1.

(i) Sketch the curve $y = x^2(3-x)$ stating the coordinates of points of intersection with the axes. **[3]**

(ii) The curve $y = x^2(3-x)$ is translated by 2 units in the positive direction parallel to the x-axis. State the equation of the curve after it has been translated. **[2]**

(iii) Describe fully a transformation that transforms the curve $y = x^2(3-x)$ to $y = \frac{1}{2}x^2(3-x)$. **[2]**

2.

(i) Sketch the curve $y = -\frac{1}{x}$. **[2]**

(ii) The curve $y = -\frac{1}{x}$ is translated by 2 units parallel to the x-axis in the positive direction. State the equation of the transformed curve. **[2]**

(iii) Describe a transformation that transforms the curve $y = -\frac{1}{x}$ to the curve $y = -\frac{1}{3x}$. **[2]**

3.

The curve $y = f(x)$ passes through the point P with coordinates $(2, 5)$.

(i) State the coordinates of the point corresponding to P on the curve $y = f(x) + 2$. **[1]**

(ii) State the coordinates of the point corresponding to P on the curve $y = f(2x)$. **[1]**

(iii) Describe the transformation that transforms the curve $y = f(x)$ to the curve $y = f(x+4)$. **[2]**

4.

(i) Sketch the curve $y = \frac{2}{x^2}$. **[2]**

(ii) The curve $y = \frac{2}{x^2}$ is translated by 5 units in the negative x-direction. Find the equation of the curve after it has been translated. **[2]**

(iii) Describe a transformation that transforms the curve $y = \frac{2}{x^2}$ to the curve $y = \frac{1}{x^2}$. **[2]**

5.

(i) Sketch the curve $y = (1+x)(2-x)(3+x)$, giving the coordinates of all points of intersection with the axes. **[3]**

(ii) Describe the transformation that transforms the curve $y = (1+x)(2-x)(3+x)$ to the curve $y = (1-x)(2+x)(3-x)$. **[2]**

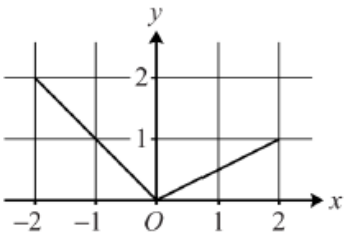
6.

(i) Sketch the curve $y = \sqrt{x}$. **[2]**

(ii) Describe the transformation that transforms the curve $y = \sqrt{x}$ to the curve $y = \sqrt{x-4}$. **[2]**

(iii) The curve $y = \sqrt{x}$ is stretched by a scale factor of 5 parallel to the x-axis. State the equation of the transformed curve. **[2]**

7.



The graph of $y = f(x)$ for $-2 \leqslant x \leqslant 2$ is shown above.

(i) Sketch the graph of $y = f(-x)$ for $-2 \leqslant x \leqslant 2$. **[2]**

(ii) Sketch the graph of $y = f(x) + 2$ for $-2 \leqslant x \leqslant 2$. **[2]**

8.

(i) Sketch the curve $y = -x^3$. **[2]**

(ii) The curve $y = -x^3$ is translated by 3 units in the positive x-direction. Find the equation of the curve after it has been translated. **[2]**

(iii) Describe a transformation that transforms the curve $y = -x^3$ to the curve $y = -5x^3$. **[2]**