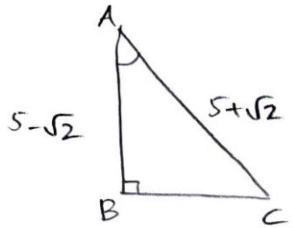
Answers – Surds - Extension

1.



$$\cos B\hat{A}C = \frac{5-\sqrt{2}}{5+\sqrt{2}}$$

$$= \frac{(5-\sqrt{2})(5-\sqrt{2})}{(5+\sqrt{2})(5-\sqrt{2})}$$

$$= \frac{25 - 10\sqrt{2} + 2}{25 - 2}$$

$$= \frac{27 - 10\sqrt{2}}{23}$$

2.

$$x\sqrt{5} + 2y = 3 \quad \text{—} \ (1)$$

$$x + y = 1 \quad \text{—} \ (2)$$

$$(2) \times 2 \Rightarrow \quad 2x + 2y = 2 \quad \text{—} \ (3)$$

$$(1) \Rightarrow \quad x\sqrt{5} + 2y = 3 \quad \text{—} \ (1)$$

$$(3) - (1) \Rightarrow \quad 2x - x\sqrt{5} = -1$$

$$x(2-\sqrt{5}) = -1$$

$$x = \frac{-1}{2-\sqrt{5}}$$

$$x = \frac{-1(2+\sqrt{5})}{(2-\sqrt{5})(2+\sqrt{5})}$$

$$x = \frac{-1(2+\sqrt{5})}{4-5}$$

$$= \frac{-1(2+\sqrt{5})}{-1}$$

$$\therefore \quad x = 2+\sqrt{5}$$

Sub $x = 2+\sqrt{5}$ into eq (2)

$$x + y = 1$$

$$(2+\sqrt{5}) + y = 1$$

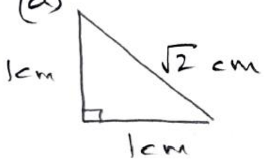
$$y = 1 - 2 - \sqrt{5}$$

$$y = -1 - \sqrt{5}$$

$$\therefore \quad x = 2+\sqrt{5} \quad \text{and} \quad y = -1-\sqrt{5}$$

3.

(a)



$$p = 1 + 1 + \sqrt{2}$$

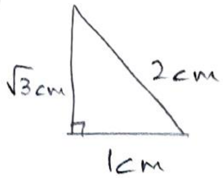
$$p = (2+\sqrt{2})\,\text{cm}$$

$$\Delta = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 1 \times 1$$
$$= \frac{1}{2}\ \text{cm}^2$$

$$r = \frac{2\Delta}{p}$$
$$r = \frac{2 \times \frac{1}{2}}{2+\sqrt{2}}$$
$$r = \frac{1}{2+\sqrt{2}}$$
$$= \frac{1\,(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})}$$
$$= \frac{2-\sqrt{2}}{4-2}$$

$$r = \frac{2-\sqrt{2}}{2}\ \text{cm}$$

b)

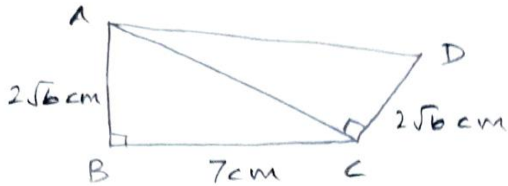


2cm must be the hypotenuse because $2 > \sqrt{3} > 1$.

$$p = 1 + 2 + \sqrt{3}$$
$$= (3+\sqrt{3})\ \text{cm}$$
$$\Delta = \frac{1}{2} \times 1 \times \sqrt{3}$$
$$= \frac{\sqrt{3}}{2}\ \text{cm}^2$$

$$r = \frac{2\Delta}{p}$$
$$= \frac{2 \times \sqrt{3}/2}{3+\sqrt{3}}$$
$$= \frac{\sqrt{3}}{3+\sqrt{3}}$$
$$= \frac{\sqrt{3}\,(3-\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})}$$
$$= \frac{\sqrt{3}\,(3-\sqrt{3})}{9-3}$$
$$= \frac{3\sqrt{3}-3}{6}$$
$$= \frac{\cancel{3}^{1}(\sqrt{3}-1)}{\cancel{6}_{2}}$$
$$r = \frac{\sqrt{3}-1}{2}\ \text{cm}$$

4.



$AC^2 = (2\sqrt{6})^2 + 7^2$

$= 24 + 49$

$= 73$

$AD^2 = AC^2 + CD^2$

$= 73 + (2\sqrt{6})^2$

$= 73 + 24$

$AD^2 = 97$

$AD = \sqrt{97}$ cm

$(4\sqrt{6})^2 = 96$

$(7\sqrt{2})^2 = 98$

We know that,

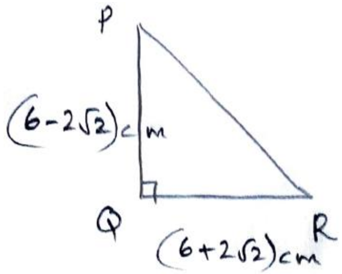
$96 < 97 < 98$

(97 is between 96 and 98).

$\therefore \quad 4\sqrt{6} < \sqrt{97} < 7\sqrt{2}$

$4\sqrt{6}$ cm $< AD < 7\sqrt{2}$ cm.

5.



(a) Area $= \frac{1}{2}(6+2\sqrt{2})(6-2\sqrt{2})$

$= \frac{1}{2}(36 - 8)$

$= \underline{14 \text{ cm}^2}$

(b) $PR^2 = (6+2\sqrt{2})^2 + (6-2\sqrt{2})^2$

$= (36 + 24\sqrt{2} + 8) + (36 - 24\sqrt{2} + 8)$

$= 36 + 24\sqrt{2} + 8 + 36 - 24\sqrt{2} + 8$

$= 88$

$\therefore \ PR = \sqrt{88}$

$= \underline{2\sqrt{22} \text{ cm}}$

6.

$\tan 75^\circ = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

$= \dfrac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \dfrac{3 + 2\sqrt{3} + 1}{3 - 1}$

$$= \frac{4 + 2\sqrt{3}}{2}$$

$$= \frac{\cancel{2}(2+\sqrt{3})}{\cancel{2}} = \underline{\underline{2 + \sqrt{3}}}$$

7.

$$5x - 3y = 41 \quad \text{— } ①$$

$$7\sqrt{2}\,x + 4\sqrt{2}\,y = 82 \quad \text{— } ②$$

$$① \times 4\sqrt{2} \Rightarrow \quad 20\sqrt{2}\,x - 12\sqrt{2}\,y = 164\sqrt{2} \quad \text{— } ③$$

$$② \times 3 \Rightarrow \quad 21\sqrt{2}\,x + 12\sqrt{2}\,y = 246 \quad \text{— } ④$$

$$③ + ④ \Rightarrow \quad 41\sqrt{2}\,x = 246 + 164\sqrt{2}$$

$$(\div 41) \qquad\qquad (\div 41)$$

$$\sqrt{2}\,x = 6 + 4\sqrt{2}$$

$$x = \frac{6 + 4\sqrt{2}}{\sqrt{2}}$$

$$x = \frac{(6 + 4\sqrt{2})\sqrt{2}}{\sqrt{2} \times \sqrt{2}}$$

$$x = \frac{6\sqrt{2} + 8}{2}$$

$$x = 3\sqrt{2} + 4$$

Sub $x = 3\sqrt{2} + 4$ into eq ①.

$$5(3\sqrt{2} + 4) - 3y = 41$$

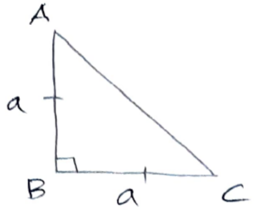
$$15\sqrt{2} + 20 - 3y = 41$$

$$3y = 15\sqrt{2} - 21$$

$$y = 5\sqrt{2} - 7$$

$$\therefore \quad \underline{\underline{x = 3\sqrt{2} + 4 \text{ and } y = 5\sqrt{2} - 7}}$$

8.



$$AC^2 = a^2 + a^2$$

$$AC^2 = 2a^2$$

$$AC = \sqrt{2}\,a$$

$$\text{Perimeter} = a + a + \sqrt{2}\,a$$

$$= 2a + \sqrt{2}\,a$$

$$= \underline{\underline{(2 + \sqrt{2})a}}$$

Perimeter $= 10$ m

$$\therefore \quad (2 + \sqrt{2})a = 10$$

$$a = \frac{10}{2 + \sqrt{2}}$$

$$= \frac{10(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})}$$

$$= \frac{10(2-\sqrt{2})}{4-2}$$

$$a = \underline{\underline{5(2-\sqrt{2})}}\ m.$$

$$AC = \sqrt{2}\ a$$
$$= 5(2-\sqrt{2}) \times \sqrt{2}$$
$$= 5(2\sqrt{2}-2)$$
$$= 10(\sqrt{2}-1).$$

$\therefore$ The lengths are,

$5(2-\sqrt{2})\ m$, $\underline{\underline{5(2-\sqrt{2})}}\ m$ and $10(\sqrt{2}-1)\ m$.