Parametric Equations - Answers

1.

(i)	$\frac{dy}{dt} = -2\sin 2t + 2\cos t$ soi	B1	NB $\frac{dx}{dt} = 2\cos t$	if B0M0A0 SC3 for $\frac{dy}{dx} = 1 - x$ from correct Cartesian equation seen in part (i) or part (ii) B1 for substitution of $x = 2\sin t$
	$\frac{dy}{dx} = \text{their } \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ oe	M1		
	$\frac{-2\sin 2t + 2\cos t}{2\cos t}$ soi	A1		
	$\frac{-4\sin t\cos t + 2\cos t}{2\cos t}$ or $\frac{2\cos t(-2\sin t + 1)}{2\cos t}$ and completion to $1 - 2\sin t$ www	A1	or equivalent intermediate step	
	(1, 1½)	B1 **[5]**	NB $t = \frac{\pi}{6}$	from $1 - 2\sin t = 0$
(ii)	$(y =)\ 1 - 2\sin^2 t + 2\sin t$	B1	may be awarded after correct substitution for x eg $(y =)\ 1 - {}^{x^2}/_4 - \sin^2 t + 2\sin t$	or $(y =)\ x + \cos 2t$
	substitution of $\sin t = ½x$ to eliminate t	M1		substitution of $t = \sin^{-1}({}^x/_2)$ to eliminate t
	$y = 1 + x - ½x^2$ oe isw	A1 **[3]**	or B3 www	$y = x + \cos 2(\sin^{-1}({}^x/_2))$ oe isw

(iii)		$-2 \le x \le 2$ **or** $x \ge -2$ (and) $x \le 2$ or $\lvert x \rvert \le 2$	B1	cao	
		sketch of negative quadratic with endpoints in 1st and 3rd quadrants	M1	RH point must be to the right of the maximum	
		positive *y*-intercept and one distinguishing feature isw	A1		one from: endpoints (–2, –3) and (2, 1), vertex at (1, 1½), *y* – intercept is (0, 1), *x*-intercept is (1 –√3, 0)
			[3]		

2.

(i)		$x = \sec\theta,\ y = 2\tan\theta$		
		$\Rightarrow \quad \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2\sec^2\theta}{\sec\theta\tan\theta}$	M1A1	M1 for their $(dy/d\theta) \div \sec\theta\tan\theta$ in terms of θ A1 cao (oe) allow for unsimplified form even if subsequently cancelled incorrectly ie can isw
		$= \frac{2\sec\theta}{\tan\theta} = \frac{2}{\cos\theta}.\frac{\cos\theta}{\sin\theta} = \frac{2}{\sin\theta} = 2\operatorname{cosec}\theta$ *	A1	cao www (NB **AG**) – must be at least one intermediate step between $\frac{2\sec\theta}{\tan\theta}$ (oe) and either $\frac{2}{\sin\theta}$ or $2\operatorname{cosec}\theta$
			[3]	
(ii)		$x^2 = \sec^2\theta = 1 + \tan^2\theta = 1 + \frac{1}{4}y^2$	M1	$\sec^2\theta = 1 + \tan^2\theta$ (oe) used
		$\Rightarrow \quad y^2 = 4(x^2 - 1) = 4x^2 - 4$ *	A1	www NB **AG**
			[2]	
	OR	$4\tan^2\theta = 4\sec^2\theta - 4$	B1*	Correct substitution of *x* and *y* into the given answer
		$\Rightarrow 1 + \tan^2\theta = \sec^2\theta$ which is true	B1dep*	Dependent on previous mark – must simplify/remove the factor of 4 from each term and state that the correctly derived trig identity is true

3.

(i)		$\frac{dy}{dt} = 2(+) - \frac{2}{t^3}$; $\frac{dx}{dt} = -\frac{1}{t^2}$ oe soi ISW $\frac{2}{t} - 2t^2$ or $-2\left(t^2 - \frac{1}{t}\right)$, $\frac{2t^3-2}{-t}$, $-t^2\left(2 - \frac{2}{t^3}\right)$ oe	B1, B1 B1 **[3]**	ISW. Must not involve (implied)'triple-deckers' e.g. fractions with neg powers…	… e.g. $\frac{2-2t^{-3}}{-t^2}$
(ii)		(Any of their expressions for $\frac{dy}{dx}$) = 0 or their $\frac{dy}{dt} = 0$ $t = 1 \rightarrow$ (stationary point) = (0, 3) Consider values of x on each side of their critical value of x which lead to finite values of $\frac{dy}{dx}$ Hence (0, 3) is a minimum point www	M1 A1 M1 A1 **[4]**	Not awarded if $\frac{dy}{dx}$ is wrong in (i) and used here BUT allow recovery if only explicitly considering $\frac{dy}{dt} = 0$ Totally satis; values of x must be close to 0 & not going below or equal to $x = -1$	
(iii)		Attempt to find t from $x = \frac{1}{t} - 1$ and substitute into the equation for y $y = \frac{2}{x+1} + (x+1)^2$ oe (can be unsimplified) ISW	M1 A1 **[2]**		

4.

(i)		EITHER $x = e^{3t}, y = te^{2t}$ $dy/dt = 2te^{2t} + e^{2t}$ $\Rightarrow \quad dy/dx = (2te^{2t} + e^{2t})/3e^{3t}$ when $t = 1$, $dy/dx = 3e^2/3e^3 = 1/e$	 B1 M1 A1 A1	 soi Their $dy/dt \div dx/dt$ in terms of t oe cao allow for unsimplified form even if subsequently cancelled incorrectly ie can isw cao www must be simplified to 1/e oe
		OR		
		$3t = \ln x,\ y = \dfrac{\ln x}{3} e^{2/3 \ln x} = \dfrac{x^{2/3} \ln x}{3}$ $dy/dx = \dfrac{1}{3} x^{2/3} \dfrac{1}{x} + \ln x \dfrac{2}{9} x^{-1/3}$ $= \dfrac{1}{3e^t} + \dfrac{2t}{3e^t}$ $dy/dx = 1/3e + 2/3e = 1/e$	B1 M1 A1 A1 **[4]**	Any equivalent form of y in terms of x only Differentiating their y provided not eased ie need a product including $\ln kx$ and x^p and subst $x = e^{3t}$ to obtain dy/dx in terms of t oe cao www cao **exact only** must be simplified to 1/e or e^{-1}
(ii)		$3t = \ln x \Rightarrow t = (\ln x)/3$ $y = (\ln x)/3e^{(2\ln x)/3}$ $y = \dfrac{1}{3} x^{\frac{2}{3}} \ln x$	B1 M1 A1 **[3]**	Finding t correctly in terms of x Subst in y using their t Required form $ax^b \ln x$ only NB If this work was already done in 5(i), marks can only be scored in 5(ii) if candidate specifically refers in this part to their part (i).

5.

(a)		$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$ $\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$ Substitute $(-1,-1)$ for (x, y) & attempt to solve for $\frac{dy}{dx}$ Obtain $\frac{dy}{dx} = -1$ WWW	B1 B1 M1 A1 **[4]**	 or solve then substitute
(b)	(i)	Tangent parallel y-axis $\rightarrow \frac{dx}{dt} = 0$ or $\frac{dy}{dx} \rightarrow \infty$ or $\frac{dy}{dx} = \infty$ Obtain $t = 0$ $(-1,0)$ with no other possibilities	M1 A1 A1 **[3]**	Accept clear intention Accept $x = -1, y = 0$
(b)	(ii)	State or imply or use $\frac{dy}{dt} = \frac{dx}{dt}$ Produce $3t^2 + 1 = 4t$ oe $t = \frac{1}{3}$ or 1	M1 A1 A1 **[3]**	

6.

Cartesian equation may be used in parts (i) - (iii) and corresponding marks awarded

(i)	Sub parametric eqns into $y = 3x$ & produce $t = -2$		
	<u>OR</u> sub $t = -2$ into para eqs, obtain $(-1,-3)$ & state $y = 3x$		
	<u>OR</u> other similar methods producing (or verifying) $t = -2$	B1	
	Value of t at other point is 2	B1 **2**	$t = \pm 2$ is sufficient for B1+B1
(ii)	Use (not just quote) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1	
	$= -(t+1)^2$	A1	or $\frac{-1}{x^2}$ or $\frac{-(2+y)}{x}$
	Attempt to use $-\frac{1}{\frac{dy}{dx}}$ for gradient of normal	M1	
	Gradient normal = 1 cao	A1	
	Subst $t = -2$ into the parametric eqns.	M1	to find pt at which normal is drawn
	Produce $y = x - 2$ as equation of the normal <u>WWW</u>	A1 **6**	'A' marks in **(ii)** are dep on prev 'A'
(iii)	Substitute the parametric values into their eqn of normal	M1	
	Produce $t = 0$ as final answer cao	A1 **2**	This is dep on final A1 in (ii)
	N.B. If $y = x - 2$ is found fortuitously in (ii) (& $\therefore$ given A0 in (ii)), you must award A0 here in (iii).		
(iv)	Attempt to eliminate t from the parametric equations	M1	
	Produce <u>any</u> correct equation	A1	e.g. $x = \frac{1}{y+2}$
	Produce $y = \frac{1}{x} - 2$ or $y = \frac{1-2x}{x}$ ISW	A1 **3**	Must be seen in (iv)
	{N.B. Candidate producing only $y = \frac{1}{x} - 2$ is awarded both A1 marks.}		

7.

(a)	At A, $x=-1+8=7$ & $y=(-1)^2=1 \Rightarrow A(7,1)$	$A(7,1)$	B1	
				(1)
(b)	$x=t^3-8t$, $y=t^2$,			
	$\frac{dx}{dt}=3t^2-8$, $\frac{dy}{dt}=2t$			
	$\therefore \frac{dy}{dx}=\frac{2t}{3t^2-8}$	Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$	M1	
		Correct $\frac{dy}{dx}$	A1	
	At A, $m(\mathbf{T})=\underline{\frac{2(-1)}{3(-1)^2-8}}=\underline{\frac{-2}{3-8}}=\underline{\frac{-2}{-5}}=\underline{\frac{2}{5}}$	Substitutes for t to give any of the four underlined oe:		
	$\mathbf{T}$: $y-(\text{their } 1)=m_T(x-(\text{their } 7))$ or $1=\frac{2}{5}(7)+c \Rightarrow c=1-\frac{14}{5}=-\frac{9}{5}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y=(\text{their gradient})x+"c"$.	dM1	
	Hence $\mathbf{T}$: $y=\frac{2}{5}x-\frac{9}{5}$			
	gives $\mathbf{T}$: $\underline{2x-5y-9=0}$ **AG**	$\underline{2x-5y-9=0}$	A1	**cso**
				(5)

(c)	$2(t^3-8t)-5t^2-9=0$	Substitution of both $x=t^3-8t$ and $y=t^2$ into **T**	M1	
	$2t^3-5t^2-16t-9=0$			
	$(t+1)\{(2t^2-7t-9)=0\}$ $(t+1)\{(t+1)(2t-9)=0\}$	A realisation that $(t+1)$ is a factor.	dM1	
	$\{t=-1 \text{ (at } A)\}$ $t=\frac{9}{2}$ at B	$t=\frac{9}{2}$	A1	
	$x=\left(\frac{9}{2}\right)^2-8\left(\frac{9}{2}\right)=\frac{729}{8}-36=\frac{441}{8}$ = 55.125 or awrt 55.1	Candidate uses their value of t to find either the x or y coordinate	ddM1	
	$y=\left(\frac{9}{2}\right)^2=\frac{81}{4}=20.25$ or awrt 20.3	One of either x or y correct.	A1	
		Both x and y correct.	A1	
	Hence $B\left(\frac{441}{8},\frac{81}{4}\right)$	awrt		(6)

8.

(a)		$\frac{dx}{dt}=-4\sin 2t$, $\frac{dy}{dt}=6\cos t$	B1, B1	
		$\frac{dy}{dx}=-\frac{6\cos t}{4\sin 2t}$ $\left(=-\frac{3}{4\sin t}\right)$	M1	
	At $t=\frac{\pi}{3}$,	$m=-\frac{3}{4\times\frac{\sqrt{3}}{2}}=-\frac{\sqrt{3}}{2}$ accept equivalents, awrt -0.87	A1	(4)

(b)	Use of $\cos 2t = 1 - 2\sin^2 t$	M1	
	$\cos 2t = \frac{x}{2}$, $\sin t = \frac{y}{6}$		
	$\frac{x}{2} = 1 - 2\left(\frac{y}{6}\right)^2$	M1	
	Leading to $y = \sqrt{(18-9x)}$ $\left(= 3\sqrt{(2-x)}\right)$ cao	A1	
	$-2 \le x \le 2$ $k = 2$	B1	(4)
(c)	$0 \le f(x) \le 6$ either $0 \le f(x)$ or $f(x) \le 6$	B1	
	Fully correct. Accept $0 \le y \le 6$, $[0, 6]$	B1	(2)

9.

(a)	$\frac{dx}{dt} = 3$, $\frac{dy}{dt} = 6t^{-2}$		
	$\frac{dy}{dx} = \frac{6t^{-2}}{3}$ $\left\{= \frac{6}{3t^2} = 2t^{-2} = \frac{2}{t^2}\right\}$	their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ to give $\frac{dy}{dx}$ in terms of t or their $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$ to give $\frac{dy}{dx}$ in terms of t	M1
		$\frac{6t^{-2}}{3}$, simplified or un-simplified, in terms of t. **See note.**	A1 isw
	Award **Special Case 1st M1 if both** $\frac{dx}{dt}$ **and** $\frac{dy}{dt}$ are stated **correctly** and **explicitly**.		**[2]**

(b)	$\left\{t=\frac{1}{2} \Rightarrow\right\} P\left(-\frac{5}{2}, -7\right)$	$x=-\frac{5}{2}, y=-7$ **or** $P\left(-\frac{5}{2}, -7\right)$ seen or implied.	B1
	$\frac{dy}{dx}=\frac{2}{\left(\frac{1}{2}\right)^2}$ **and** either • $y-"-7"="8"\left(x-"-\frac{5}{2}"\right)$ • $"-7"=("8")("-\frac{5}{2}")+c$ So, $y=(\text{their } m_T)x+"c"$	**Some** attempt to substitute $t=0.5$ into their $\frac{dy}{dx}$ which contains t in order to find m_T **and either** applies $y-(\text{their } y_P)=(\text{their } m_T)(x-\text{their } x_P)$ **or** finds c from $(\text{their } y_P)=(\text{their } m_T)(\text{their } x_P)+c$ and uses their numerical c in $y=(\text{their } m_T)x+c$	M1
	T: $y=8x+13$	$y=8x+13$ or $y=13+8x$	A1 cso
	Note: their x_P, their y_P and their m_T must be numerical values in order to award M1		[3]

(c)

$\left\{t=\frac{x+4}{3} \Rightarrow\right\} y=5-\frac{6}{\left(\frac{x+4}{3}\right)}$	An attempt to eliminate t. **See notes.**	M1
	Achieves a correct equation in x and y only	A1 o.e.
$\Rightarrow y=5-\frac{18}{x+4} \Rightarrow y=\frac{5(x+4)-18}{x+4}$		
So, $y=\frac{5x+2}{x+4}$, $\{x>-4\}$	$y=\frac{5x+2}{x+4}$ (or implied equation)	A1 cso
		[3]

10.
(a)

$\frac{dx}{dt} = 4\sec^2 t,\ \frac{dy}{dt} = 10\sqrt{3}\cos 2t$ $\Rightarrow \frac{dy}{dx} = \frac{10\sqrt{3}\cos 2t}{4\sec^2 t}\ \left\{= \frac{5}{2}\sqrt{3}\cos 2t\cos^2 t\right\}$	**Either both** x and y are differentiated correctly with respect to t **or their** $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **or** applies $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$	M1
	Correct $\frac{dy}{dx}$ (Can be implied)	A1 oe
$\left\{\text{At } P\left(4\sqrt{3}, \frac{15}{2}\right),\ t = \frac{\pi}{3}\right\}$		
$\frac{dy}{dx} = \frac{10\sqrt{3}\cos\left(\frac{2\pi}{3}\right)}{4\sec^2\left(\frac{\pi}{3}\right)}$	**dependent on the previous M mark** ***Some evidence*** of substituting $t = \frac{\pi}{3}$ or $t = 60^\circ$ into their $\frac{dy}{dx}$	dM1
$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 cso
		[4]

(b)

$\left\{10\sqrt{3}\cos 2t = 0 \Rightarrow t = \frac{\pi}{4}\right\}$		
So $x = 4\tan\left(\frac{\pi}{4}\right),\ y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$	At least one of either $x = 4\tan\left(\frac{\pi}{4}\right)$ **or** $y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$ **or** $x = 4$ **or** $y = 5\sqrt{3}$ **or** $y = $ awrt 8.7	M1
Coordinates are $\left(4, 5\sqrt{3}\right)$	$\left(4, 5\sqrt{3}\right)$ **or** $x = 4,\ y = 5\sqrt{3}$	A1
		[2]