Integration (Year 13)

Exercise A

1 Integrate the following with respect to x.

a $3\sec^2 x + \frac{5}{x} + \frac{2}{x^2}$ **b** $5e^x - 4\sin x + 2x^3$

c $2(\sin x - \cos x + x)$ **d** $3\sec x \tan x - \frac{2}{x}$

e $5e^x + 4\cos x - \frac{2}{x^2}$ **f** $\frac{1}{2x} + 2\operatorname{cosec}^2 x$

g $\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}$ **h** $e^x + \sin x + \cos x$

i $2\operatorname{cosec} x \cot x - \sec^2 x$ **j** $e^x + \frac{1}{x} - \operatorname{cosec}^2 x$

2 Find the following integrals.

a $\int\left(\frac{1}{\cos^2 x} + \frac{1}{x^2}\right) dx$ **b** $\int\left(\frac{\sin x}{\cos^2 x} + 2e^x\right) dx$

c $\int\left(\frac{1 + \cos x}{\sin^2 x} + \frac{1 + x}{x^2}\right) dx$ **d** $\int\left(\frac{1}{\sin^2 x} + \frac{1}{x}\right) dx$

e $\int \sin x(1 + \sec^2 x)\, dx$ **f** $\int \cos x(1 + \operatorname{cosec}^2 x)\, dx$

g $\int \operatorname{cosec}^2 x(1 + \tan^2 x)\, dx$ **h** $\int \sec^2 x(1 - \cot^2 x)\, dx$

i $\int \sec^2 x(1 + e^x \cos^2 x)\, dx$ **j** $\int\left(\frac{1 + \sin x}{\cos^2 x} + \cos^2 x \sec x\right) dx$

3 Evaluate the following. Give your answers as exact values.

a $\int_3^7 2e^x\, dx$ **b** $\int_1^6 \frac{1 + x}{x^3}\, dx$

c $\int_{\frac{\pi}{2}}^{\pi} -5\sin x\, dx$ **d** $\int_{-\frac{\pi}{4}}^{0} \sec x(\sec x + \tan x)\, dx$

Watch out When applying limits to integrated trigonometric functions, always work in radians.

4 Given that a is a positive constant and $\int_a^{2a} \frac{3x - 1}{x}\, dx = 6 + \ln\left(\frac{1}{2}\right)$, find the exact value of a. **(4 marks)**

5 Given that a is a positive constant and $\int_{\ln 1}^{\ln a} e^x + e^{-x}\, dx = \frac{48}{7}$, find the exact value of a. **(4 marks)**

6 Given $\int_2^b (3e^x + 6e^{-2x})\, dx = 0$, find the value of b. **(4 marks)**

7 $f(x) = \frac{1}{8}x^{\frac{3}{2}} - \frac{4}{x}, x > 0$

a Solve the equation $f(x) = 0$. **(2 marks)**

b Find $\int f(x)\, dx$. **(2 marks)**

c Evaluate $\int_1^4 f(x)\, dx$, giving your answer in the form $p + q\ln r$, where p, q and r are rational numbers. **(3 marks)**

Exercise B

1 Integrate the following:

a $\sin(2x+1)$ **b** $3e^{2x}$

c $4e^{x+5}$ **d** $\cos(1-2x)$

e $\operatorname{cosec}^2 3x$ **f** $\sec 4x \tan 4x$

g $3\sin\left(\frac{1}{2}x+1\right)$ **h** $\sec^2(2-x)$ **i** $\operatorname{cosec} 2x \cot 2x$ **j** $\cos 3x - \sin 3x$

2 Find the following integrals.

a $\int\left(e^{2x} - \frac{1}{2}\sin(2x-1)\right)\mathrm{d}x$ **b** $\int(e^x+1)^2\,\mathrm{d}x$

c $\int \sec^2 2x(1+\sin 2x)\,\mathrm{d}x$ **d** $\int \frac{3-2\cos\frac{1}{2}x}{\sin^2\frac{1}{2}x}\,\mathrm{d}x$

e $\int\left(e^{3-x} + \sin(3-x) + \cos(3-x)\right)\mathrm{d}x$

3 Integrate the following:

a $\frac{1}{2x+1}$ **b** $\frac{1}{(2x+1)^2}$ **c** $(2x+1)^2$ **d** $\frac{3}{4x-1}$

e $\frac{3}{1-4x}$ **f** $\frac{3}{(1-4x)^2}$ **g** $(3x+2)^5$ **h** $\frac{3}{(1-2x)^3}$

4 Find the following integrals.

a $\int\left(3\sin(2x+1) + \frac{4}{2x+1}\right)\mathrm{d}x$ **b** $\int\left(e^{5x} + (1-x)^5\right)\mathrm{d}x$

c $\int\left(\frac{1}{\sin^2 2x} + \frac{1}{1+2x} + \frac{1}{(1+2x)^2}\right)\mathrm{d}x$ **d** $\int\left((3x+2)^2 + \frac{1}{(3x+2)^2}\right)\mathrm{d}x$

5 Evaluate:

a $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos(\pi - 2x)\,\mathrm{d}x$ **b** $\int_{\frac{1}{2}}^{1} \frac{12}{(3-2x)^4}\,\mathrm{d}x$ **c** $\int_{\frac{2\pi}{9}}^{\frac{5\pi}{18}} \sec^2(\pi - 3x)\,\mathrm{d}x$ **d** $\int_{2}^{3} \frac{5}{7-2x}\,\mathrm{d}x$

6 Given $\int_3^b (2x-6)^2\,\mathrm{d}x = 36$, find the value of b. **(4 marks)**

7 Given $\int_{e^3}^{e^8} \frac{1}{kx}\,\mathrm{d}x = \frac{1}{4}$, find the value of k. **(4 marks)**

8 Given $\int_{\frac{\pi}{4k}}^{\frac{\pi}{3k}} (1 - \pi \sin kx)\,\mathrm{d}x = \pi(7 - 6\sqrt{2})$,

find the exact value of k. **(7 marks)**

Exercise C

1 Use the substitutions given to find:

a $\int x\sqrt{1+x}\,dx;\ u = 1 + x$

b $\int \frac{1+\sin x}{\cos x}\,dx;\ u = \sin x$

c $\int \sin^3 x\,dx;\ u = \cos x$

d $\int \frac{2}{\sqrt{x}(x-4)}\,dx;\ u = \sqrt{x}$

e $\int \sec^2 x \tan x\sqrt{1+\tan x}\,dx;\ u^2 = 1 + \tan x$

f $\int \sec^4 x\,dx;\ u = \tan x$

2 Use the substitutions given to find the exact values of:

a $\int_0^5 x\sqrt{x+4}\,dx;\ u = x + 4$

b $\int_0^2 x(2+x)^3\,dx;\ u = 2 + x$

c $\int_0^{\frac{\pi}{2}} \sin x\sqrt{3\cos x+1}\,dx;\ u = \cos x$

d $\int_0^{\frac{\pi}{3}} \sec x \tan x\sqrt{\sec x+2}\,dx;\ u = \sec x$

e $\int_1^4 \frac{1}{\sqrt{x}(4x-1)}\,dx;\ u = \sqrt{x}$

3 By choosing a suitable substitution, find:

a $\int x(3+2x)^5\,dx$

b $\int \frac{x}{\sqrt{1+x}}\,dx$

c $\int \frac{\sqrt{x^2+4}}{x}\,dx$

4 By choosing a suitable substitution, find the exact values of:

a $\int_2^7 x\sqrt{2+x}\,dx$

b $\int_2^5 \frac{1}{1+\sqrt{x-1}}\,dx$

c $\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1+\cos\theta}\,d\theta$

5 Using the substitution $u^2 = 4x + 1$, or otherwise, find the exact value of $\int_6^{20} \frac{8x}{\sqrt{4x+1}}\,dx$. **(8 marks)**

6 Use the substitution $u^2 = e^x - 2$ to show that $\int_{\ln 3}^{\ln 4} \frac{e^{4x}}{e^x - 2}\,dx = \frac{a}{b} + c\ln d$, where a, b, c and d are integers to be found. **(7 marks)**

7 Prove that $-\int \frac{1}{\sqrt{1-x^2}}\,dx = \arccos x + c$. **(5 marks)**

8 Use the substitution $u = \cos x$ to show

$\int_0^{\frac{\pi}{3}} \sin^3 x\cos^2 x\,dx = \frac{47}{480}$ **(7 marks)**

Hint Use exact trigonometric values to change the limits in x to limits in u.

9 Using a suitable trigonometric substitution for x, find $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} x^2\sqrt{1-x^2}\,dx$. **(8 marks)**

Exercise D

1 Integrate the following functions.

a $\frac{x}{x^2+4}$

b $\frac{e^{2x}}{e^{2x}+1}$

c $\frac{x}{(x^2+4)^3}$

d $\frac{e^{2x}}{(e^{2x}+1)^3}$

e $\frac{\cos 2x}{3+\sin 2x}$

f $\frac{\sin 2x}{(3+\cos 2x)^3}$

g xe^{x^2}

h $\cos 2x(1+\sin 2x)^4$

i $\sec^2 x\tan^2 x$

j $\sec^2 x(1+\tan^2 x)$

2 Find the following integrals.

a $\int (x+1)(x^2+2x+3)^4\,\mathrm{d}x$

b $\int \operatorname{cosec}^2 2x \cot 2x\,\mathrm{d}x$

c $\int \sin^5 3x \cos 3x\,\mathrm{d}x$

d $\int \cos x\, \mathrm{e}^{\sin x}\,\mathrm{d}x$

e $\int \frac{\mathrm{e}^{2x}}{\mathrm{e}^{2x}+3}\,\mathrm{d}x$

f $\int x(x^2+1)^{\frac{3}{2}}\,\mathrm{d}x$

g $\int (2x+1)\sqrt{x^2+x+5}\,\mathrm{d}x$

h $\int \frac{2x+1}{\sqrt{x^2+x+5}}\,\mathrm{d}x$

i $\int \frac{\sin x \cos x}{\sqrt{\cos 2x+3}}\,\mathrm{d}x$

j $\int \frac{\sin x \cos x}{\cos 2x+3}\,\mathrm{d}x$

3 Find the exact value of each of the following:

a $\int_0^3 (3x^2+10x)\sqrt{x^3+5x^2+9}\,\mathrm{d}x$

b $\int_{\frac{\pi}{9}}^{\frac{2\pi}{9}} \frac{6\sin 3x}{1-\cos 3x}\,\mathrm{d}x$

c $\int_4^7 \frac{x}{x^2-1}\,\mathrm{d}x$

d $\int_0^{\frac{\pi}{4}} \sec^2 x\, \mathrm{e}^{4\tan x}\,\mathrm{d}x$

4 Given that $\int_0^k kx^2\mathrm{e}^{x^3}\,\mathrm{d}x = \frac{2}{3}(\mathrm{e}^8-1)$, find the value of k. **(3 marks)**

5 Given that $\int_0^\theta 4\sin 2x \cos^4 2x\,\mathrm{d}x = \frac{4}{5}$ where $0 < \theta < \pi$, find the exact value of θ.

6 **a** By writing $\cot x = \frac{\cos x}{\sin x}$, find $\int \cot x\,\mathrm{d}x$. **(2 marks)**

b Show that $\int \tan x\,\mathrm{d}x \equiv \ln|\sec x| + c$. **(3 marks)**

Exercise E

1 Find the following integrals.

a $\int x\sin x\,\mathrm{d}x$ **b** $\int x\mathrm{e}^x\,\mathrm{d}x$ **c** $\int x\sec^2 x\,\mathrm{d}x$

d $\int x\sec x\tan x\,\mathrm{d}x$ **e** $\int \frac{x}{\sin^2 x}\,\mathrm{d}x$

2 Find the following integrals.

a $\int 3\ln x\,\mathrm{d}x$ **b** $\int x\ln x\,\mathrm{d}x$ **c** $\int \frac{\ln x}{x^3}\,\mathrm{d}x$

d $\int (\ln x)^2\,\mathrm{d}x$ **e** $\int (x^2+1)\ln x\,\mathrm{d}x$

3 Find the following integrals.

a $\int x^2\mathrm{e}^{-x}\,\mathrm{d}x$ **b** $\int x^2\cos x\,\mathrm{d}x$ **c** $\int 12x^2(3+2x)^5\,\mathrm{d}x$ **d** $\int 2x^2\sin 2x\,\mathrm{d}x$ **e** $\int 2x^2\sec^2 x\tan x\,\mathrm{d}x$

4 Evaluate the following:

a $\int_0^{\ln 2} x\mathrm{e}^{2x}\,\mathrm{d}x$ **b** $\int_0^{\frac{\pi}{2}} x\sin x\,\mathrm{d}x$ **c** $\int_0^{\frac{\pi}{2}} x\cos x\,\mathrm{d}x$ **d** $\int_1^2 \frac{\ln x}{x^2}\,\mathrm{d}x$

e $\int_0^1 4x(1+x)^3\,\mathrm{d}x$ **f** $\int_0^\pi x\cos\frac{1}{4}x\,\mathrm{d}x$ **g** $\int_0^{\frac{\pi}{3}} \sin x\ln(\sec x)\,\mathrm{d}x$

5 **a** Use integration by parts to find $\int x\cos 4x\,\mathrm{d}x$. **(3 marks)**

b Use your answer to part **a** to find $\int x^2\sin 4x\,\mathrm{d}x$. **(3 marks)**

6 **a** Find $\int \sqrt{8 - x}\,\mathrm{d}x$. **(2 marks)**

b Using integration by parts, or otherwise, show that

$$\int (x - 2)\sqrt{8 - x}\,\mathrm{d}x = -\tfrac{2}{5}(8 - x)^{\frac{3}{2}}(x + 2) + c$$ **(6 marks)**

c Hence find $\int_4^7 (x - 2)\sqrt{8 - x}\,\mathrm{d}x$. **(2 marks)**

7 **a** Find $\int \sec^2 3x\,\mathrm{d}x$. **(3 marks)**

b Using integration by parts, or otherwise, find $\int x \sec^2 3x\,\mathrm{d}x$. **(6 marks)**

c Hence show that $\int_{\frac{\pi}{18}}^{\frac{\pi}{9}} x \sec^2 3x\,\mathrm{d}x = p\pi - q\ln 3$, finding the exact values of the constants p and q. **(4 marks)**

Exercise F

1 Use partial fractions to integrate the following:

a $\dfrac{3x + 5}{(x + 1)(x + 2)}$ **b** $\dfrac{3x - 1}{(2x + 1)(x - 2)}$ **c** $\dfrac{2x - 6}{(x + 3)(x - 1)}$ **d** $\dfrac{3}{(2 + x)(1 - x)}$

2 Find the following integrals.

a $\int \dfrac{2(x^2 + 3x - 1)}{(x + 1)(2x - 1)}\,\mathrm{d}x$ **b** $\int \dfrac{x^3 + 2x^2 + 2}{x(x + 1)}\,\mathrm{d}x$ **c** $\int \dfrac{x^2}{x^2 - 4}\,\mathrm{d}x$ **d** $\int \dfrac{x^2 + x + 2}{3 - 2x - x^2}\,\mathrm{d}x$

3 $\mathrm{f}(x) = \dfrac{4}{(2x + 1)(1 - 2x)}, x \neq \pm\dfrac{1}{2}$

a Given that $\mathrm{f}(x) = \dfrac{A}{2x + 1} + \dfrac{B}{1 - 2x}$, find the value of the constants A and B. **(3 marks)**

b Hence find $\int \mathrm{f}(x)\,\mathrm{d}x$, writing your answer as a single logarithm. **(4 marks)**

c Find $\int_1^2 \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in the form $\ln k$ where k is a rational constant. **(2 marks)**

4 $\mathrm{f}(x) = \dfrac{17 - 5x}{(3 + 2x)(2 - x)^2}, -\dfrac{3}{2} < x < 2$.

a Express $\mathrm{f}(x)$ in partial fractions. **(4 marks)**

b Hence find the exact value of $\int_0^1 \dfrac{17 - 5x}{(3 + 2x)(2 - x)^2}\,\mathrm{d}x$, writing your answer in the form $a + \ln b$, where a and b are constants to be found. **(5 marks)**

5 $\mathrm{f}(x) = \dfrac{9x^2 + 4}{9x^2 - 4}, x \neq \pm\dfrac{2}{3}$

a Given that $\mathrm{f}(x) = A + \dfrac{B}{3x - 2} + \dfrac{C}{3x + 2}$, find the values of the constants A, B and C. **(4 marks)**

b Hence find the exact value of $\int_{-\frac{1}{3}}^{\frac{1}{3}} \dfrac{9x^2 + 4}{9x^2 - 4}\,\mathrm{d}x$, writing your answer in the form $a + b\ln c$, where a, b and c are rational numbers to be found. **(5 marks)**

Problem-solving

Simplify the integral as much as possible before substituting your limits.

6 $f(x) = \dfrac{6 + 3x - x^2}{x^3 + 2x^2}$, $x > 0$

a Express $f(x)$ in partial fractions. **(4 marks)**

b Hence find the exact value of $\int_2^4 \dfrac{6 + 3x - x^2}{x^3 + 2x^2}\,dx$, writing your answer in the form $a + \ln b$, where a and b are rational numbers to be found. **(5 marks)**

7 $\dfrac{32x^2 + 4}{(4x + 1)(4x - 1)} \equiv A + \dfrac{B}{4x + 1} + \dfrac{C}{4x - 1}$

a Find the value of the constants A, B and C. **(4 marks)**

b Hence find the exact value of $\int_1^2 \dfrac{32x^2 + 4}{(4x + 1)(4x - 1)}\,dx$ writing your answer in the form $2 + k\ln m$, giving the values of the rational constants k and m. **(5 marks)**

Exercise G

1 Integrate the following:

a $\cot^2 x$ b $\cos^2 x$

c $\sin 2x \cos 2x$ d $(1 + \sin x)^2$

e $\tan^2 3x$ f $(\cot x - \operatorname{cosec} x)^2$

g $(\sin x + \cos x)^2$ h $\sin^2 x \cos^2 x$ i $\dfrac{1}{\sin^2 x \cos^2 x}$ j $(\cos 2x - 1)^2$

2 Find the following integrals.

a $\int \dfrac{1 - \sin x}{\cos^2 x}\,dx$ b $\int \dfrac{1 + \cos x}{\sin^2 x}\,dx$ c $\int \dfrac{\cos 2x}{\cos^2 x}\,dx$

d $\int \dfrac{\cos^2 x}{\sin^2 x}\,dx$ e $\int \dfrac{(1 + \cos x)^2}{\sin^2 x}\,dx$ f $\int (\cot x - \tan x)^2\,dx$

g $\int (\cos x - \sin x)^2\,dx$ h $\int (\cos x - \sec x)^2\,dx$ i $\int \dfrac{\cos 2x}{1 - \cos^2 2x}\,dx$

3 Show that $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\,dx = \dfrac{2 + \pi}{8}$ **(4 marks)**

4 Find the exact value of each of the following:

a $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \dfrac{1}{\sin^2 x \cos^2 x}\,dx$ b $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\sin x - \operatorname{cosec} x)^2\,dx$ c $\int_0^{\frac{\pi}{4}} \dfrac{(1 + \sin x)^2}{\cos^2 x}\,dx$ d $\int_{\frac{3\pi}{8}}^{\frac{\pi}{2}} \dfrac{\sin 2x}{1 - \sin^2 2x}\,dx$

5 a By expanding $\sin(3x + 2x)$ and $\sin(3x - 2x)$ using the double-angle formulae, or otherwise, show that $\sin 5x + \sin x \equiv 2\sin 3x \cos 2x$. **(4 marks)**

b Hence find $\int \sin 3x \cos 2x\,dx$ **(3 marks)**

6 $f(x) = 5\sin^2 x + 7\cos^2 x$

a Show that $f(x) = \cos 2x + 6$. **(3 marks)**

b Hence, find the exact value of $\int_0^{\frac{\pi}{4}} f(x)\,dx$. **(4 marks)**

7 a Show that $\cos^4 x \equiv \frac{1}{8}\cos 4x + \frac{1}{2}\cos 2x + \frac{3}{8}$ **(4 marks)**

b Hence find $\int \cos^4 x\,dx$. **(4 marks)**

Exercise H

1 Find the area of the finite region R bounded by the curve with equation $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$.

a $f(x) = \frac{2}{1+x}$; $a = 0$, $b = 1$ b $f(x) = \sec x$; $a = 0$, $b = \frac{\pi}{3}$ c $f(x) = \ln x$; $a = 1$, $b = 2$

d $f(x) = \sec x \tan x$; $a = 0$, $b = \frac{\pi}{4}$ e $f(x) = x\sqrt{4 - x^2}$; $a = 0$, $b = 2$

2 Find the exact area of the finite region bounded by the curve $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ where:

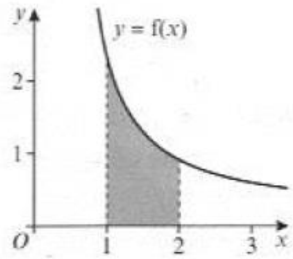
a $f(x) = \frac{4x-1}{(x+2)(2x+1)}$; $a = 0$, $b = 2$ b $f(x) = \frac{x}{(x+1)^2}$; $a = 0$, $b = 2$

c $f(x) = x \sin x$; $a = 0$, $b = \frac{\pi}{2}$ d $f(x) = \cos x\sqrt{2\sin x + 1}$; $a = 0$, $b = \frac{\pi}{6}$

e $f(x) = xe^{-x}$; $a = 0$, $b = \ln 2$

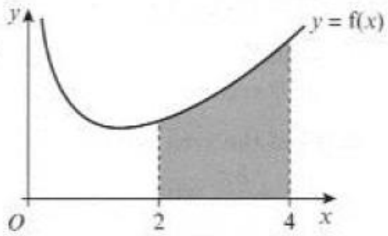
3 The diagram shows a sketch of the curve with equation, $y = f(x)$, where $f(x) = \frac{4x+3}{(x+2)(2x-1)}$, $x > \frac{1}{2}$

Find the area of the shaded region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$. **(7 marks)**



4 The diagram shows a sketch of the curve with equation $y = f(x)$, where $f(x) = e^{0.5x} + \frac{1}{x}$, $x > 0$.

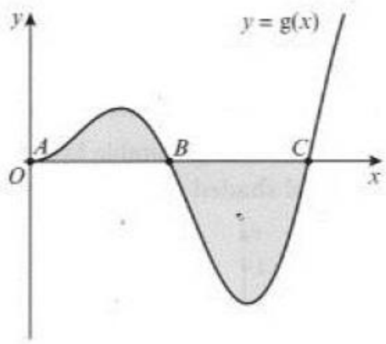
Find the area of the shaded region bounded by the curve, the x-axis and the lines $x = 2$ and $x = 4$. **(7 marks)**



5 The diagram shows a sketch of the curve with equation $y = g(x)$, where $g(x) = x \sin x$.

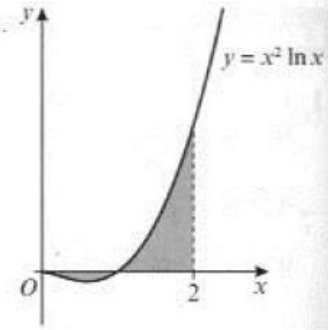
a Write down the coordinates of points A, B and C.

b Find the area of the shaded region.



Watch out Find the area of each region separately and then add the answers. Remember areas cannot be negative, so take the absolute value of any negative area.

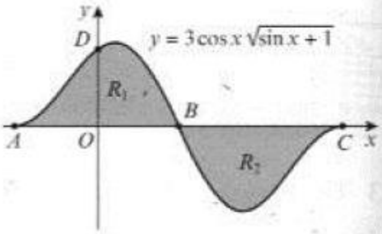
6 The diagram shows a sketch of the curve with equation $y = x^2 \ln x$. The shaded region is bounded by the curve, the x-axis and the line $x = 2$.



a Use integration by parts to find $\int x^2 \ln x \, dx$. **(3 marks)**

b Hence find the exact area of the shaded region, giving your answer in the form $\frac{2}{3}(a \ln 2 + b)$, where a and b are integers. **(5 marks)**

7 The diagram shows a sketch of the curve with equation $y = 3\cos x\sqrt{\sin x + 1}$.



a Find the coordinates of the points A, B, C and D. **(3 marks)**

b Use a suitable substitution to find

$\int 3\cos x\sqrt{\sin x + 1}\, dx$ **(5 marks)**

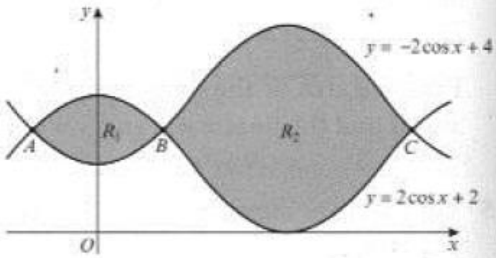
c Show that the regions R_1 and R_2 have the same area, and find the exact value of this area in the form $\sqrt{a}$, where a is a positive integer to be found. **(3 marks)**

8 $f(x) = x^2$ and $g(x) = 3x - x^2$

a On the same axes, sketch the graphs of $y = f(x)$ and $y = g(x)$, and find the coordinates of any points of intersection of the two curves.

b Find the area of the finite region bounded by the two curves.

9 The diagram shows a sketch of part of the curves with equations $y = 2\cos x + 2$ and $y = -2\cos x + 4$.



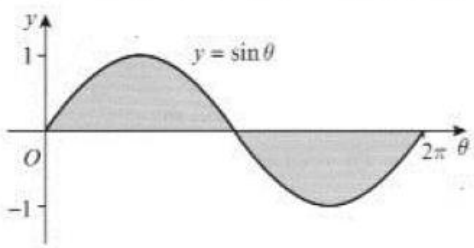
a Find the coordinates of the points A, B and C. **(2 marks)**

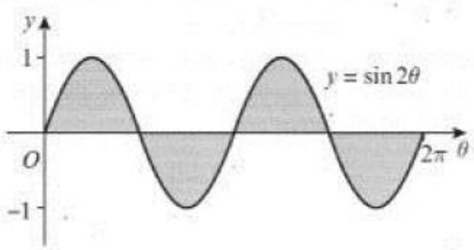
b Find the area of region R_1 in the form $a\sqrt{3} + \frac{b\pi}{c}$, where a, b and c are integers to be found. **(4 marks)**

c Show that the ratio of $R_2 : R_1$ can be expressed as $(3\sqrt{3} + 2\pi) : (3\sqrt{3} - \pi)$. **(5 marks)**

10 The diagrams show the curves $y = \sin\theta$, $0 \leqslant \theta \leqslant 2\pi$ and $y = \sin 2\theta$, $0 \leqslant \theta \leqslant 2\pi$.

By choosing suitable limits, show that the total shaded area in the first diagram is equal to the total shaded area in the second diagram, and state the exact value of this shaded area.





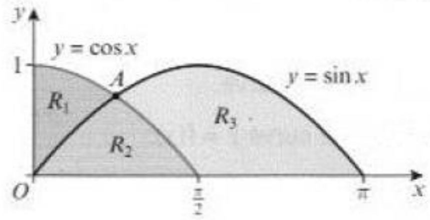
11 The diagram shows parts of the graphs of $y = \sin x$ and $y = \cos x$.

a Find the coordinates of point A.

b Find the areas of:

i R_1 **ii** R_2 **iii** R_3

c Show that the ratio of areas $R_1 : R_2$ can be written as $\sqrt{2} : 2$.

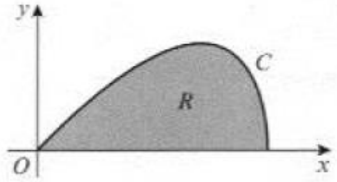


12 The curve C has parametric equations $x = t^3$, $y = t^2$, $t \geqslant 0$. Show that the exact area of the region bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$ is $k\sqrt[3]{2}$, where k is a rational constant to be found.

13 The curve C has parametric equations

$$x = \sin t,\ y = \sin 2t,\ 0 \leqslant t \leqslant \frac{\pi}{2}$$

The finite region R is bounded by the curve and the x-axis. Find the exact area of R. **(6 marks)**



14 This graph shows part of the curve C with parametric equations $x = (t + 1)^2$, $y = \frac{1}{2}t^3 + 3$, $t \geqslant -1$

P is the point on the curve where $t = 2$.

The line S is the normal to C at P.

a Find an equation of S. **(5 marks)**

The shaded region R is bounded by C, S, the x-axis and the line with equation $x = 1$.

b Using integration, find the area of R. **(5 marks)**