Year 12 Mathematics

Test 4

Time Allowed: 2 Hours

Total Marks: 100

06 March 2021

Calculator Allowed

Full Name of Student: ……………………………………………………………

Section A: Pure Mathematics

1.

The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 + x^2 - 10x + 8$$

(a) (i) Using the factor theorem, show that $x - 2$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(ii) Hence express $\mathrm{p}(x)$ as the product of three linear factors. *(3 marks)*

(b) Sketch the curve with equation $y = x^3 + x^2 - 10x + 8$, showing the coordinates of the points where the curve cuts the axes.

(You are not required to calculate the coordinates of the stationary points.) *(4 marks)*

[Total for Question 1 = 9 marks]

2.

A rectangular garden is to have width x metres and length $(x + 4)$ metres.

(a) The perimeter of the garden needs to be greater than 30 metres.

Show that $2x > 11$. *(1 mark)*

(b) The area of the garden needs to be less than 96 square metres.

Show that $x^2 + 4x - 96 < 0$. *(1 mark)*

(c) Solve the inequality $x^2 + 4x - 96 < 0$. *(4 marks)*

(d) Hence determine the possible values of the width of the garden. *(1 mark)*

[Total for Question 2 = 7 marks]

3.

(a) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$\left(1 + \frac{x}{4}\right)^8$$

giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places.

(3)

[Total for Question 3 = 7 marks]

4.

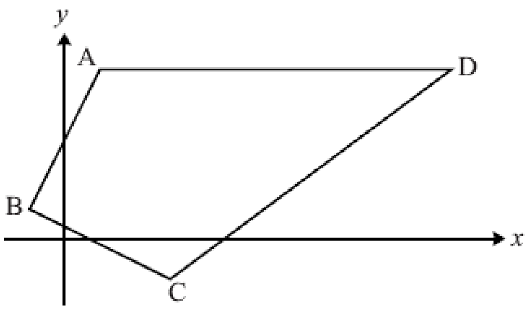


Fig. 10

Fig. 10 is a sketch of quadrilateral ABCD with vertices A (1, 5), B (–1, 1), C (3, –1) and D (11, 5).

(i) Show that AB = BC. **[3]**

(ii) Show that the diagonals AC and BD are perpendicular. **[3]**

(iii) Find the midpoint of AC. Show that BD bisects AC but AC does not bisect BD. **[5]**

[Total for Question 4 = 11 marks]

5.

The point A has x-coordinate 5 and lies on the curve $y = x^2 - 4x + 3$.

(i) Sketch the curve. **[2]**

(ii) Use calculus to find the equation of the tangent to the curve at A. **[4]**

(iii) Show that the equation of the normal to the curve at A is $x + 6y = 53$. Find also, using an algebraic method, the x-coordinate of the point at which this normal crosses the curve again. **[6]**

[Total for Question 5 = 12 marks]

(Question 6 is on the next page)

6.

The circle C has equation

$$x^2 + y^2 - 20x - 24y + 195 = 0$$

The centre of C is at the point M.

(a) Find

(i) the coordinates of the point M,

(ii) the radius of the circle C.

(5)

N is the point with coordinates $(25, 32)$.

(b) Find the length of the line MN.

(2)

The tangent to C at a point P on the circle passes through point N.

(c) Find the length of the line NP.

(2)

[Total for Question 6 = 9 marks]

7.

(a) Given that $2\sin\theta = 7\cos\theta$, find the value of $\tan\theta$. *(2 marks)*

(b) (i) Use an appropriate identity to show that the equation

$$6\sin^2 x = 4 + \cos x$$

can be written as

$$6\cos^2 x + \cos x - 2 = 0$$ *(2 marks)*

(ii) Hence solve the equation $6\sin^2 x = 4 + \cos x$ in the interval $0° < x < 360°$, giving your answers to the nearest degree. *(6 marks)*

[Total for Question 7 = 10 marks]

8.

A curve has equation $y = (x + 2)(x^2 - 3x + 5)$.

(i) Find the coordinates of the minimum point, justifying that it is a minimum. **[8]**

(ii) Calculate the discriminant of $x^2 - 3x + 5$. **[2]**

(iii) Explain why $(x + 2)(x^2 - 3x + 5)$ is always positive for $x > -2$. **[2]**

[Total for Question 8 = 12 marks]

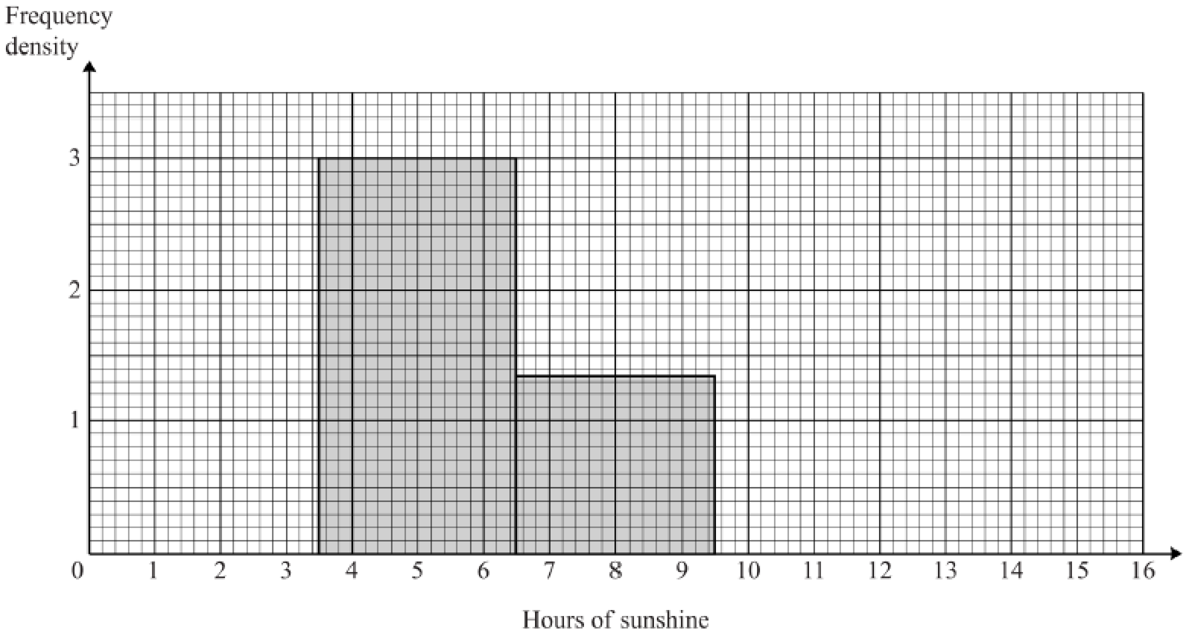
Section B: Mechanics and Statistics

9.

At a certain resort the number of hours of sunshine, measured to the nearest hour, was recorded on each of 21 days. The results are summarised in the table.

Hours of sunshine	0	1 – 3	4 – 6	7 – 9	10 – 15
Number of days	0	6	9	4	2

The diagram shows part of a histogram to illustrate the data. The scale on the frequency density axis is 2 cm to 1 unit.



(i) **(a)** Calculate the frequency density of the 1 – 3 class.

(b) Fred wishes to draw the block for the 10 – 15 class on the same diagram. Calculate the height, in centimetres, of this block.

[4]

(ii) A cumulative frequency graph is to be drawn. Write down the coordinates of the first two points that should be plotted. You are not asked to draw the graph. **[2]**

(iii) (a) Calculate estimates of the mean and standard deviation of the number of hours of sunshine. **[5]**

(b) Explain why your answers are only estimates. **[1]**

[Total for Question 9 = 12 marks]

10.

A stone is projected vertically upwards from a point A with speed $u\,\text{m}\,\text{s}^{-1}$. After projection the stone moves freely under gravity until it returns to A. The time between the instant that the stone is projected and the instant that it returns to A is $3\frac{4}{7}$ seconds.

Modelling the stone as a particle,

(a) show that $u = 17\frac{1}{2}$,

(3)

(b) find the greatest height above A reached by the stone,

(2)

(c) find the length of time for which the stone is at least $6\frac{3}{5}$ m above A.

(6)

[Total for Question 10 = 11 marks]

- End of Test -