Deformation of Materials

Elastic Object:

- When a distorting force is applied to an object and then removed, if the object regains its original shape, then it is an elastic object.

Plastic object (or Inelastic object):

- When a distorting force is applied to an object and then removed, if the object is permanently distorted, then it is a plastic object (or inelastic object).

Force - Extension Graphs

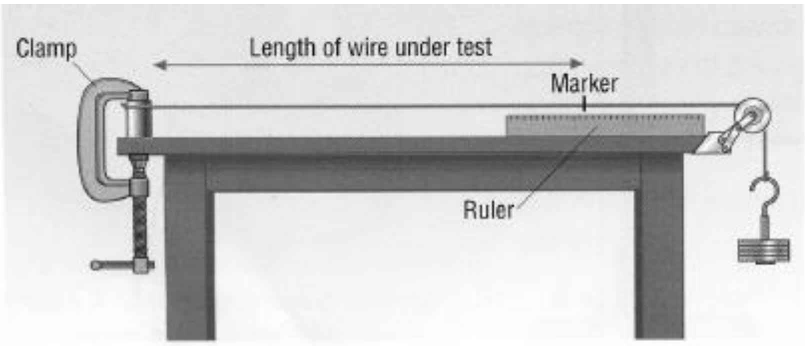
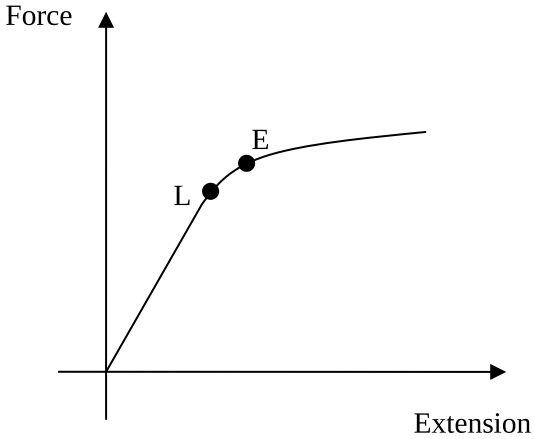
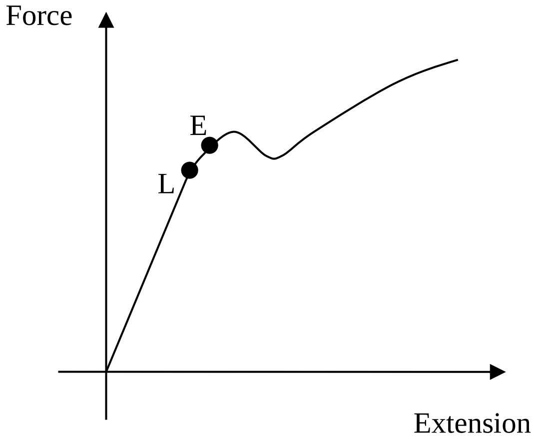


Figure 1 Experiment to stretch a wire



Force-Extension Graph for copper wire



Force-Extension Graph for mild steel wire

L - Limit of Proportionality

E - Elastic Limit

Limit of Proportionality

- This is the point up to which the Extension is proportional to the Force applied.

Elastic Limit

- This is the point up to which, the object shows elastic behavior. That is, up to this point, when the applied force is removed, the object returns to its original length.

Hooke's Law

- The extension of an elastic body is proportional to the force that causes it.
- The law applies up to the limit of proportionality on a Force – Extension graph.
- When the Hooke's law applies,

$$F = kx$$

where, F – Force on the elastic body
k – **Force Constant**
x – Extension, if the force is a tensile force and Compression, if it is a compressive force.

- When Hooke's law is applied to springs, k is often called the **spring constant**.

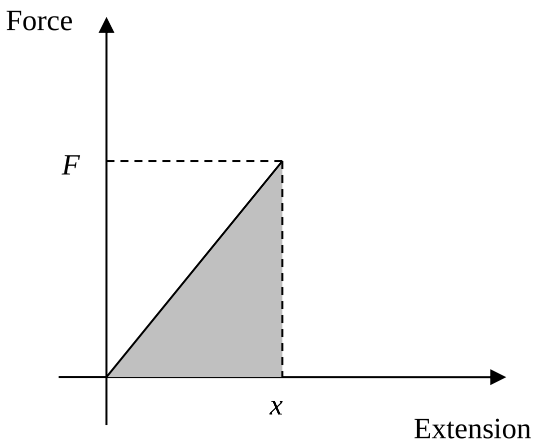
The work done to deform a solid

- We need to apply a force to deform a solid.
- When the force is applied, the point where the force is applied moves as the object stretches or compresses. This means there is a work done as there is a force causing a movement over a distance. However, since the force is not constant over the distance of movement, the formula,

Work Done = Force X Distance Moved

cannot be applied to find the work done. Hence the Force-Extension graph is used to calculate the work done.

- It can be proved that the area under a force-extension graph is equal to work done.



Work Done = Area under the Force-Extension Graph

- Work Done = Area under the graph

$$= \frac{1}{2}Fx$$
$$= \frac{1}{2}kx \times x$$
$$= \frac{1}{2}kx^2$$

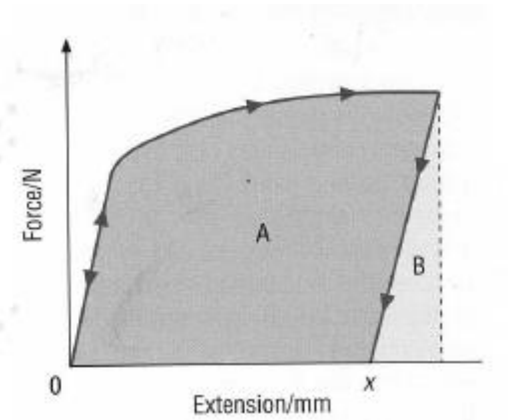
Work Done, $E = \frac{1}{2}Fx$	Work Done, $E = \frac{1}{2}kx^2$

- The *Work Done* when an object undergoes elastic deformation is stored in the object as **Elastic Potential Energy**.

Energy absorbed and released in elastic and plastic deformations

- When a force is applied within the elastic limit and then removed, all the energy absorbed by the object during the application of the force will be recovered.

- However, if the force is beyond the elastic limit and causes a plastic deformation, not all the energy will be recovered when the force is removed. Some of the energy will be dissipated as heat. For a given force, the extension while unloading will be longer than the extension while loading. This phenomenon is called **hysteresis**. The energy absorbed while loading, the energy recovered while unloading and the energy lost as heat can all be calculated using the areas on the graph.



- The diagram above shows the loading and unloading parts of the force-extension graph for a wire that is stretched beyond its elastic limit.

  Work done or energy absorbed when the wire is stretched = Area A + Area B

  Energy recovered when the force is removed = Area B

  Energy dissipated as heat = Area A

Example:

Consider a spring-loaded toy gun. When you load the bullet, you compress the spring. Let's just assume for this example that the loading process goes beyond the elastic limit. The work you do when compressing the spring to load the bullet will be given by Area A + Area B in the above graph. It is referred to as the energy absorbed while deforming the spring. When you fire the bullet, the spring transfers only some of that energy to the bullet as kinetic energy (ignore any friction between the spring and the internal surface of the gun). This is what is referred to as the recovered energy in the above graph. This recovered energy will be given by Area B in the graph. The difference between the work done by you in compressing the spring and the energy recovered (K.E of the bullet) will be the energy dissipated as heat in the spring. It is represented by Area A in the graph.

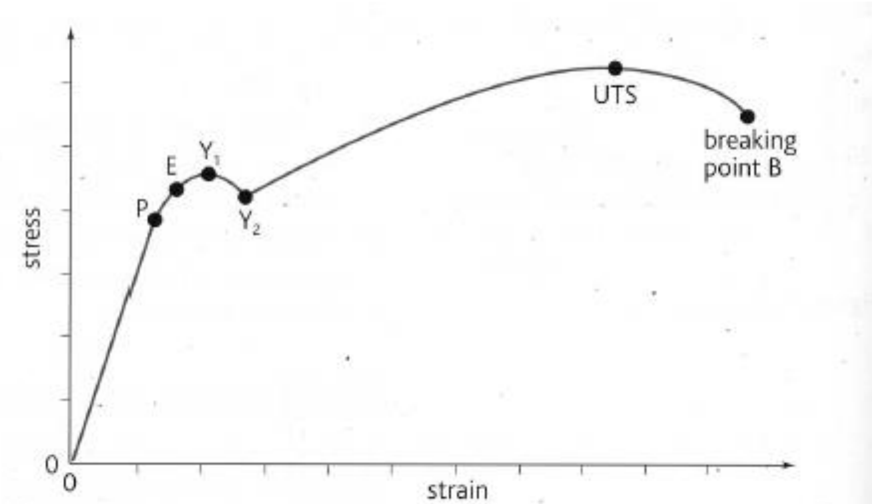
Stress, Strain and The Young Modulus

- **Stress**:
  - Stress is force per unit area.
  - The unit of Stress is Nm^{-2} or Pa.

- **Strain:**
  - Strain is extension per unit length. Strain does not have a unit as it is the ratio of two lengths.

Stress Vs Strain Graphs

- The shape of the stress-strain graph will be different for different type of materials. However, many of those stress-strain graphs have some common features. The following graph is for mild steel wire.



P – This is the **Limit of Proportionality**

E – This is the **Elastic Limit.** If the wire is stretched beyond this point, it will suffer a permanent plastic deformation.

Y_1 – This is the **Upper Yield Point.** This is where the wire weakens temporarily.

Y_2 – This is the **Lower Yield Point**. Beyond this point the material undergoes a plastic flow and therefore a small increase in stress causes a large increase in strain.

UTS – This is the **Ultimate Tensile Stress**, which means the maximum stress. At this point the wire is at its weakest point, it becomes narrower, stretches further and breaks at point B.

Young Modulus

- The Young modulus is the ratio between Stress and Strain within the limit of proportionality.

$$YoungModulus = \frac{Stress}{Strain}$$

- The gradient of a stress-strain graph within the limit of proportionality (the straight-line portion of the graph) will be equal to the Young Modulus.

Some common types of Materials and their Stress-Strain Behaviors

Ductile Materials

- A material is ductile, if it can be drawn out into a wire. E.g: Copper, Aluminium, some types of steel, Gold, etc.
- Most of the metals are ductile.
- In order for a material to be ductile, it should have a large plastic region in its stress-strain graph.
- Once the maximum tensile stress is applied to a ductile material, the material continues to stretch and eventually it breaks.

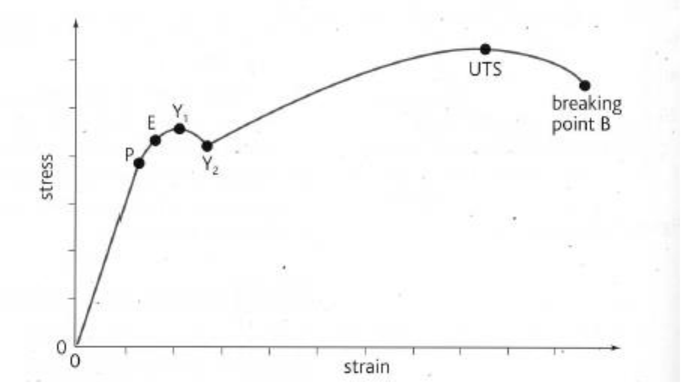


Figure: Stress-Strain graph for a metal with a large plastic region

Brittle Materials

Brittle materials deform very little, do not deform plastically and snap (shatter) suddenly when a large enough stress is applied.

Examples: Glass, Concrete, Cast Iron.

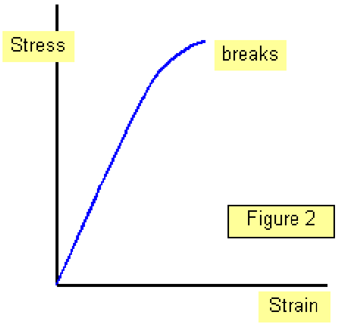


Figure: Stress-Strain graph for a brittle material

Polymeric Materials

- Polymeric materials can undergo very large strains compared to most of the other materials.

- When a stress is applied, most of the materials extend by only a tiny percentage of their original length. However polymeric materials can extend even up to 200% to 300% of their original length when stresses are applied.

- Polymeric materials consist of very long chains of polymer molecules. These chains are tangled together in a random way. When a stress is applied these tangled chains are straightened easily, hence we get a long stretch. Beyond a certain point of stretching, polymeric materials become stiffer (harder to stretch). This is because, once the chains are straightened, they will be as straight as possible. In order to stretch them further, we need to stretch the atomic bonds in the polymer chains. It would be harder to stretch these atomic bonds, hence polymeric materials initially stretch easily and beyond a certain point they become harder to stretch.

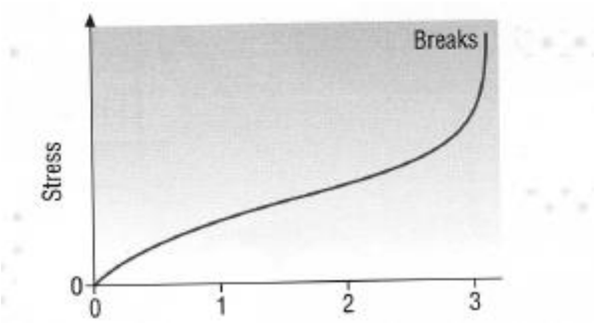


Figure: Stress-Strain graph for a polymeric material

Stress-Strain Graphs and Energy Stored

Area under the stress-strain graph = Energy stored per unit volume

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