Exponentials and Logarithms

Exercise A

1 Rewrite as a logarithm:

a $4^4 = 256$ **b** $3^{-2} = \frac{1}{9}$ **c** $10^6 = 1\,000\,000$

d $11^1 = 11$ **e** $(0.2)^3 = 0.008$

2 Rewrite using a power:

a $\log_2 16 = 4$ **b** $\log_5 25 = 2$ **c** $\log_9 3 = \frac{1}{2}$

d $\log_5 0.\dot{2} = -1$ **e** $\log_{10} 100\,000 = 5$

3 Find the value of:

a $\log_2 8$ **b** $\log_5 25$

c $\log_{10} 10\,000\,000$ **d** $\log_{12} 12$

e $\log_3 729$ **f** $\log_{10} \sqrt{10}$

g $\log_4 (0.25)$ **h** $\log_{0.25} 16$

i $\log_a (a^{10})$ **j** $\log_{(\frac{2}{3})} (\frac{9}{4})$

4 Find the value of x for which:

a $\log_5 x = 4$ **b** $\log_x 81 = 2$

c $\log_7 x = 1$ **d** $\log_x (2x) = 2$

Exercise B

Find from your calculator the value to 3 s.f. of:

1 $\log_{10} 20$ **2** $\log_{10} 4$

3 $\log_{10} 7000$ **4** $\log_{10} 0.786$

5 $\log_{10} 11$ **6** $\log_{10} 35.3$

7 $\log_{10} 0.3$ **8** $\log_{10} 999$

Exercise C

1 Write as a single logarithm:

a $\log_2 7 + \log_2 3$

b $\log_2 36 - \log_2 4$

c $3 \log_5 2 + \log_5 10$

d $2 \log_6 8 - 4 \log_6 3$

e $\log_{10} 5 + \log_{10} 6 - \log_{10} (\frac{1}{4})$

2 Write as a single logarithm, then simplify your answer:

a $\log_2 40 - \log_2 5$

b $\log_6 4 + \log_6 9$

c $2\log_{12} 3 + 4\log_{12} 2$

d $\log_8 25 + \log_8 10 - 3\log_8 5$

e $2\log_{10} 20 - (\log_{10} 5 + \log_{10} 8)$

3 Write in terms of $\log_a x$, $\log_a y$ and $\log_a z$:

a $\log_a (x^3y^4z)$

b $\log_a\left(\frac{x^5}{y^2}\right)$

c $\log_a (a^2x^2)$

d $\log_a\left(\frac{x\sqrt{y}}{z}\right)$

e $\log_a \sqrt{ax}$

Exercise D

1 Solve, giving your answer to 3 significant figures:

a $2^x = 75$

b $3^x = 10$

c $5^x = 2$

d $4^{2x} = 100$

e $9^{x+5} = 50$

f $7^{2x-1} = 23$

g $3^{x-1} = 8^{x+1}$

h $2^{2x+3} = 3^{3x+2}$

i $8^{3-x} = 10^x$

j $3^{4-3x} = 4^{x+5}$

2 Solve, giving your answer to 3 significant figures:

a $2^{2x} - 6(2^x) + 5 = 0$

b $3^{2x} - 15(3^x) + 44 = 0$

c $5^{2x} - 6(5^x) - 7 = 0$

d $3^{2x} + 3^{x+1} - 10 = 0$

e $7^{2x} + 12 = 7^{x+1}$

Hint for question 2d:
Note that
$3^{x+1} = 3^x \times 3^1 = 3(3^x)$

Exercise E

1.

Solve the equation

$$\log_5 (4x + 3) - \log_5 (x - 1) = 2. \quad \textbf{(4)}$$

2.

Solve,

$\log_2 (2x + 1) - \log_2 x = 2.$

(4)

3.

Solve

$$\log_2(16x) - \log_2\left(\frac{x^4}{2}\right) = \frac{1}{2},$$

giving your answer in its simplest surd form. **(4)**

4.

Given that

$$2\log_{10}\left(\frac{x}{y}\right) = 1 + \log_{10}(10x^2y),$$

find the value of y correct to 3 decimal places. [4]

5.

(i) Express $\log_3(4x + 7) - \log_3 x$ as a single logarithm. [1]

(ii) Hence solve the equation $\log_3(4x + 7) - \log_3 x = 2$. [3]
