Differentiation - Optimisation Problems

1.

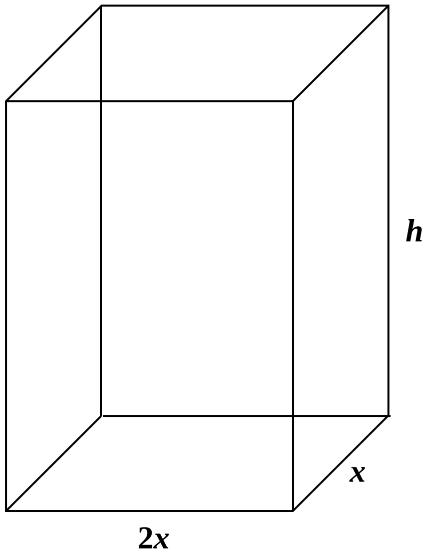

Fig. 4

A manufacturer produces cartons for fruit juice. Each carton is in the shape of a closed cuboid with base dimensions 2x cm by x cm and height *h* cm, as shown in Fig. 4.

Given that the capacity of a carton has to be 1030 cm^3,

(*a*) show that the surface area, *A* cm^2, of a carton is given by

$$A = 4x^2 + \frac{3090}{x}$$ **(3 marks)**

The manufacturer needs to minimise the surface area of a carton.

(b) Use calculus to find the value of x for which *A* is a minimum. **(5 marks)**

(c) Calculate the minimum value of *A*. **(2 marks)**

(*d*) Prove that this value of *A* is a minimum. **(2 marks)**

2.

Figure 3

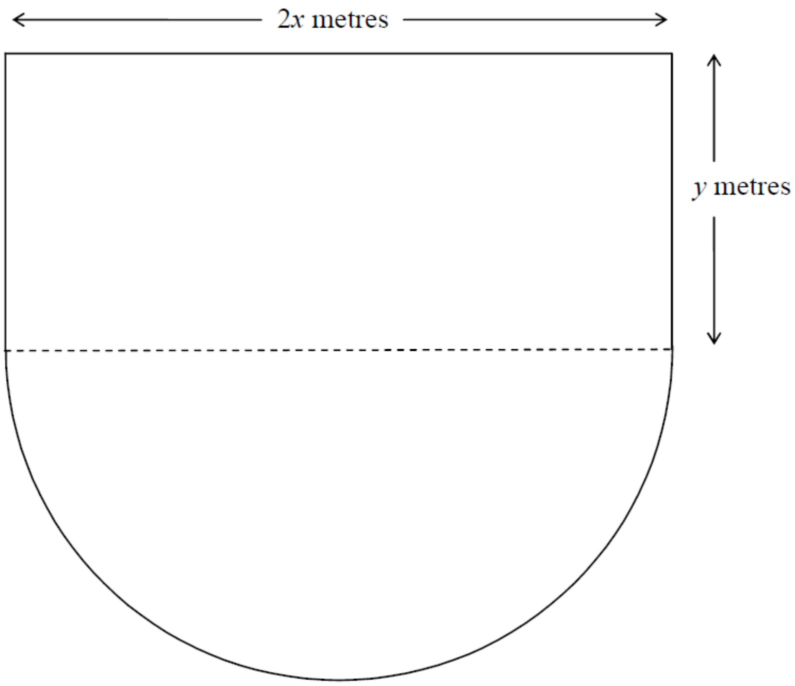

Figure 3 shows the plan of a stage in the shape of a rectangle joined to a semicircle. The length of the rectangular part is $2x$ metres and the width is y metres. The diameter of the semicircular part is $2x$ metres. The perimeter of the stage is 80 m.

(a) Show that the area, A m^2, of the stage is given by

$$A = 80x - \left(2 + \frac{\pi}{2}\right)x^2.$$

(4)

(b) Use calculus to find the value of x at which A has a stationary value.

(4)

(c) Prove that the value of x you found in part (b) gives the maximum value of A.

(2)

(d) Calculate, to the nearest m^2, the maximum area of the stage.

(2)

3.

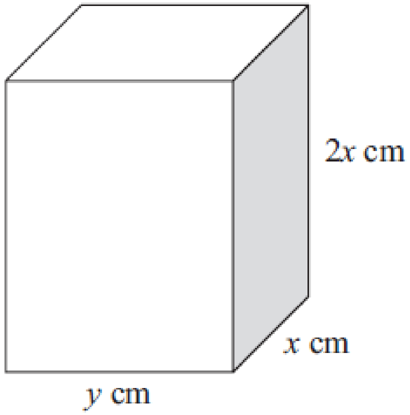

Figure 4

Figure 4 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2.

(a) Show that the volume, V cm^3, of the brick is given by

$$V = 200x - \frac{4x^3}{3}.$$

(4)

Given that x can vary,

(b) use calculus to find the maximum value of V, giving your answer to the nearest cm^3.
(5)

(c) Justify that the value of V you have found is a maximum.

(2)

4.

Figure 4

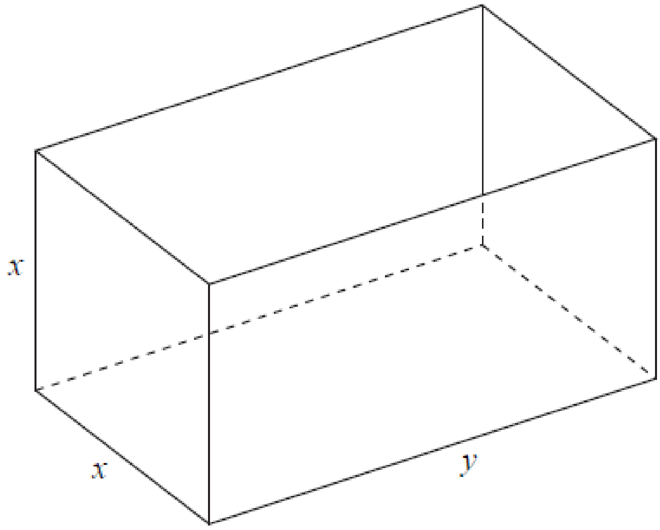

Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is $100\ \text{m}^3$.

(a) Show that the area $A\ \text{m}^2$ of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2.$$

(4)

(b) Use calculus to find the value of x for which A is stationary.

(4)

(c) Prove that this value of x gives a minimum value of A.

(2)

(d) Calculate the minimum area of sheet metal needed to make the tank.

(2)

5.

A solid right circular cylinder has radius r cm and height h cm.

The total surface area of the cylinder is 800 cm^2 .

(a) Show that the volume, V cm^3 , of the cylinder is given by

$$V = 400r - \pi r^3.$$

(4)

Given that r varies,

(b) use calculus to find the maximum value of V, to the nearest cm^3.

(6)

(c) Justify that the value of V you have found is a maximum.

(2)