Pure Mathematics

Date: ……………..

Differentiation 2

Exercise A

1.

(i) Given that $y = \frac{1}{3}x^3 - 9x$, find $\frac{dy}{dx}$. [2]

(ii) Find the coordinates of the stationary points on the curve $y = \frac{1}{3}x^3 - 9x$. [3]

(iii) Determine whether each stationary point is a maximum point or a minimum point. [3]

2.

(i) Find the coordinates of the stationary points on the curve $y = x^3 - 3x^2 + 4$. [6]

(ii) Determine whether each stationary point is a maximum point or a minimum point. [3]

3.

(i) Solve the equation $x^4 - 10x^2 + 25 = 0$. [4]

(ii) Given that $y = \frac{2}{5}x^5 - \frac{20}{3}x^3 + 50x + 3$, find $\frac{dy}{dx}$. [2]

(iii) Hence find the number of stationary points on the curve $y = \frac{2}{5}x^5 - \frac{20}{3}x^3 + 50x + 3$. [2]

4.

By considering the gradient on either side of the stationary point on the curve $y = x^3 - 3x^2 + 3x$, show that this point is a point of inflexion.
Sketch the curve $y = x^3 - 3x^2 + 3x$.

5.

A cuboid has a volume of $8\ \text{m}^3$. The base of the cuboid is square with sides of length x metres. The surface area of the cuboid is $A\ \text{m}^2$.

(i) Show that $A = 2x^2 + \frac{32}{x}$. [3]

(ii) Find $\frac{dA}{dx}$. [3]

(iii) Find the value of x which gives the smallest surface area of the cuboid, justifying your answer. [4]

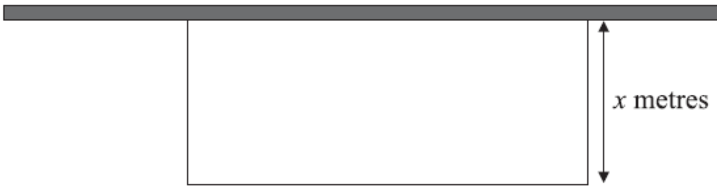
6.

(i) Find the coordinates of the stationary points of the curve $y = 27 + 9x - 3x^2 - x^3$. [6]

(ii) Determine, in each case, whether the stationary point is a maximum or minimum point. [3]

(iii) Hence state the set of values of x for which $27 + 9x - 3x^2 - x^3$ is an increasing function. [2]

7.



The diagram shows a rectangular enclosure, with a wall forming one side. A rope, of length 20 metres, is used to form the remaining three sides. The width of the enclosure is x metres.

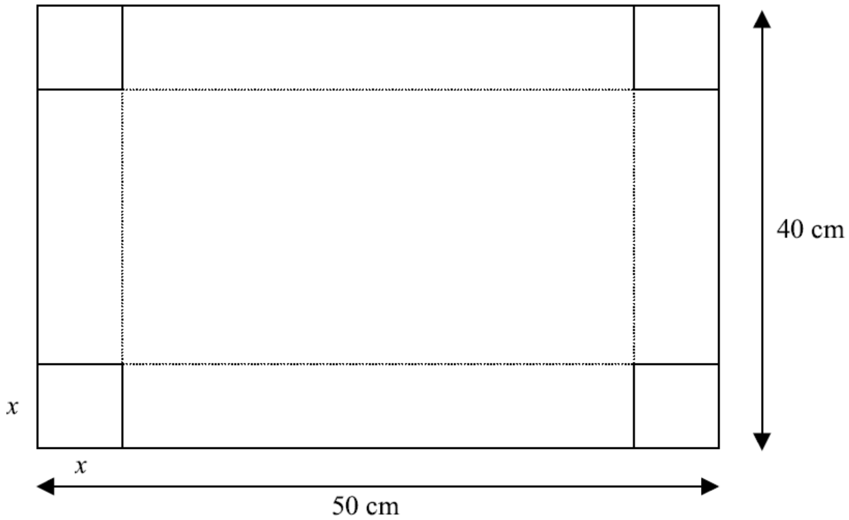
(i) Show that the enclosed area, A m^2, is given by

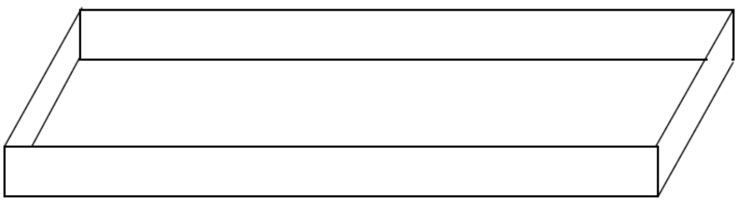
$$A = 20x - 2x^2.$$ [2]

(ii) Use differentiation to find the maximum value of A. [4]

8.

Figure 2





A rectangular sheet of metal measures 50 cm by 40 cm. Squares of side x cm are cut from each corner of the sheet and the remainder is folded along the dotted lines to make an open tray, as shown in Fig. 2.

(a) Show that the volume, V cm^3, of the tray is given by

$$V = 4x(x^2 - 45x + 500).$$

(3)

(b) State the range of possible values of x.

(1)

(c) Find the value of x for which V is a maximum.

(4)

(d) Hence find the maximum value of V.

(2)

(e) Justify that the value of V you found in part (d) is a maximum.

(2)

Exercise B

1 Find the values of x for which f(x) is an increasing function, given that f(x) equals:

a $3x^2 + 8x + 2$ **b** $4x - 3x^2$ **c** $5 - 8x - 2x^2$

d $2x^3 - 15x^2 + 36x$ **e** $3 + 3x - 3x^2 + x^3$ **f** $5x^3 + 12x$

g $x^4 + 2x^2$ **h** $x^4 - 8x^3$

2 Find the values of x for which f(x) is a decreasing function, given that f(x) equals:

a $x^2 - 9x$ **b** $5x - x^2$ **c** $4 - 2x - x^2$

d $2x^3 - 3x^2 - 12x$ **e** $1 - 27x + x^3$ **f** $x + \dfrac{25}{x}$

g $x^{\frac{1}{2}} + 9x^{-\frac{1}{2}}$ **h** $x^2(x + 3)$
