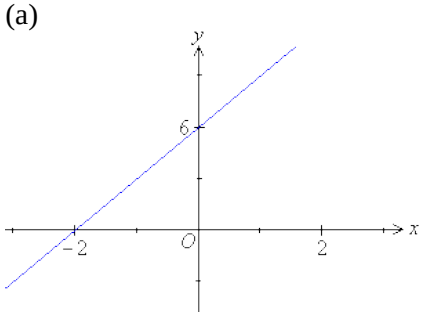
Differentiation (Year 12)

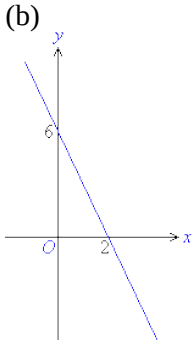
Exercise A

1. Sketch a possible graph for the gradient of each of the following functions.

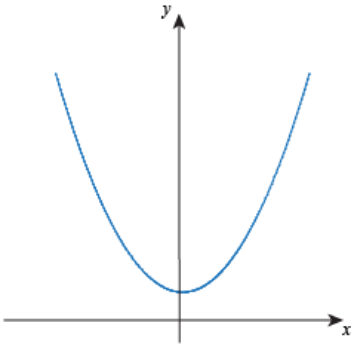
(a)



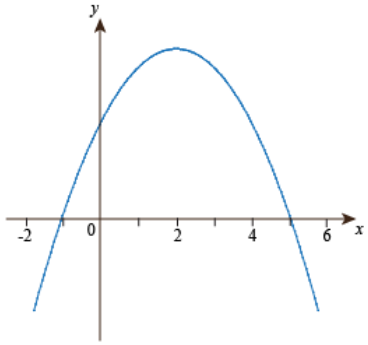
(b)



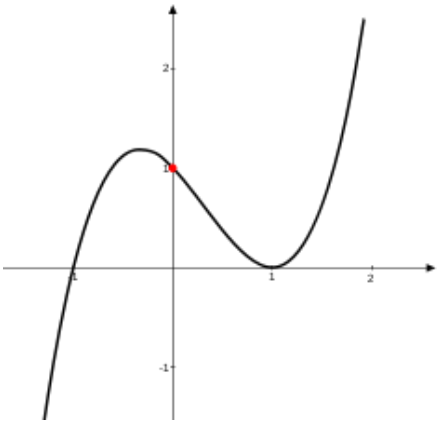
(c)



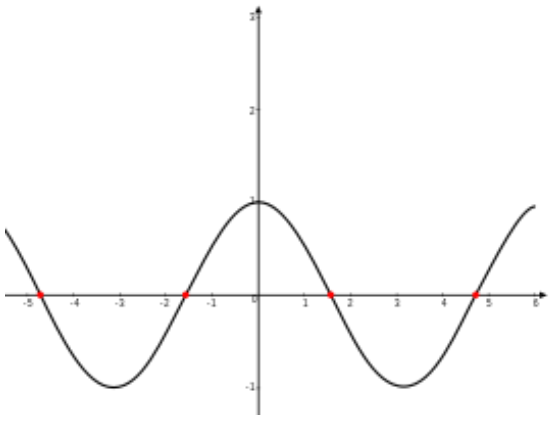
(d)



(e)



(f)



2.

F is the point with co-ordinates $(3, 9)$ on the curve with equation $y = x^2$.

a Find the gradients of the chords joining the point F to the points with coordinates:

i $(4, 16)$ **ii** $(3.5, 12.25)$ **iii** $(3.1, 9.61)$

iv $(3.01, 9.0601)$ **v** $(3 + h, (3 + h)^2)$

b What do you deduce about the gradient of the tangent at the point $(3, 9)$?

3.

G is the point with coordinates $(4, 16)$ on the curve with equation $y = x^2$.

a Find the gradients of the chords joining the point G to the points with coordinates:

i $(5, 25)$ **ii** $(4.5, 20.25)$ **iii** $(4.1, 16.81)$

iv $(4.01, 16.0801)$ **v** $(4 + h, (4 + h)^2)$

b What do you deduce about the gradient of the tangent at the point $(4, 16)$?

4. Use differentiation from first principles to find the gradient of each of the following functions at the given points.

(a) $y = x^2 + 3x$ at the point (2, 10)

(b) $y = 2x^2 - 5$ at the point (3, 13)

(c) $y = x^3$ at the point (-1, -1)

(d) $y = x^3 - 2x^2 + 5$ at the point (0, 5)

5. Find the derivatives of the following functions using differentiation from first principles.

(a) $y = x^2 + 3x$

(b) $y = 3x^2 - 4x + 5$

(c) $y = x^4$

(d) $y = 5x^3$

(e) $y = x^3 + 2x^2 + 4x - 6$

Exercise B

Differentiate each of the following with respect to x.

1 x^7	**2** x^8	**3** x^4
4 $x^{\frac{1}{3}}$	**5** $x^{\frac{1}{4}}$	**6** $\sqrt[3]{x}$
7 x^{-3}	**8** x^{-4}	**9** $\frac{1}{x^2}$
10 $\frac{1}{x^5}$	**11** $\frac{1}{\sqrt[3]{x}}$	**12** $\frac{1}{\sqrt{x}}$
13 $\frac{x^2}{x^4}$	**14** $\frac{x^3}{x^2}$	**15** $\frac{x^6}{x^3}$
16 $x^3 \times x^6$	**17** $x^2 \times x^3$	**18** $x \times x^2$

Exercise C

1 Find $\frac{dy}{dx}$ when y equals:

a $2x^2 - 6x + 3$ **b** $\frac{1}{2}x^2 + 12x$

c $4x^2 - 6$ **d** $8x^2 + 7x + 12$

e $5 + 4x - 5x^2$

2 Find the gradient of the curve whose equation is

a $y = 3x^2$ at the point $(2, 12)$ **b** $y = x^2 + 4x$ at the point $(1, 5)$

c $y = 2x^2 - x - 1$ at the point $(2, 5)$ **d** $y = \frac{1}{2}x^2 + \frac{3}{2}x$ at the point $(1, 2)$

e $y = 3 - x^2$ at the point $(1, 2)$ **f** $y = 4 - 2x^2$ at the point $(-1, 2)$

3 Find the y-coordinate and the value of the gradient at the point P with x-coordinate 1 on the curve with equation $y = 3 + 2x - x^2$.

4 Find the coordinates of the point on the curve with equation $y = x^2 + 5x - 4$ where the gradient is 3.

5 Find the gradients of the curve $y = x^2 - 5x + 10$ at the points A and B where the curve meets the line $y = 4$.

6 Find the gradients of the curve $y = 2x^2$ at the points C and D where the curve meets the line $y = x + 3$.

Exercise D

1 Use standard results to differentiate:

a $x^4 + x^{-1}$ **b** $\frac{1}{2}x^{-2}$ **c** $2x^{-\frac{1}{2}}$

2 Find the gradient of the curve with equation $y = f(x)$ at the point A where:

a $f(x) = x^3 - 3x + 2$ and A is at $(-1, 4)$ **b** $f(x) = 3x^2 + 2x^{-1}$ and A is at $(2, 13)$

3 Find the point or points on the curve with equation $y = f(x)$, where the gradient is zero:

a $f(x) = x^2 - 5x$ **b** $f(x) = x^3 - 9x^2 + 24x - 20$

c $f(x) = x^{\frac{3}{2}} - 6x + 1$ **d** $f(x) = x^{-1} + 4x$

Exercise E

1 Use standard results to differentiate:

a $2\sqrt{x}$ **b** $\frac{3}{x^2}$ **c** $\frac{1}{3x^3}$

d $\frac{1}{3}x^3(x - 2)$ **e** $\frac{2}{x^3} + \sqrt{x}$ **f** $\sqrt[3]{x} + \frac{1}{2x}$

g $\frac{2x + 3}{x}$ **h** $\frac{3x^2 - 6}{x}$ **i** $\frac{2x^3 + 3x}{\sqrt{x}}$

j $x(x^2 - x + 2)$ **k** $3x^2(x^2 + 2x)$ **l** $(3x - 2)\left(4x + \frac{1}{x}\right)$

2 Find the gradient of the curve with equation $y = f(x)$ at the point A where:

a $f(x) = x(x + 1)$ and A is at $(0, 0)$ **b** $f(x) = \frac{2x - 6}{x^2}$ and A is at $(3, 0)$

c $f(x) = \frac{1}{\sqrt{x}}$ and A is at $(\frac{1}{4}, 2)$ **d** $f(x) = 3x - \frac{4}{x^2}$ and A is at $(2, 5)$

Exercise F

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when y equals:

1 $12x^2 + 3x + 8$ **2** $15x + 6 + \frac{3}{x}$ **3** $9\sqrt{x} - \frac{3}{x^2}$

4 $(5x + 4)(3x - 2)$ **5** $\frac{3x + 8}{x^2}$

Exercise G

1 Find $\frac{d\theta}{dt}$ where $\theta = t^2 - 3t$

2 Find $\frac{dA}{dr}$ where $A = 2\pi r$

3 Find $\frac{dr}{dt}$ where $r = \frac{12}{t}$

4 Find $\frac{dv}{dt}$ where $v = 9.8t + 6$

5 Find $\frac{dR}{dr}$ where $R = r + \frac{5}{r}$

6 Find $\frac{dx}{dt}$ where $x = 3 - 12t + 4t^2$

7 Find $\frac{dA}{dx}$ where $A = x(10 - x)$

Exercise H

1 Find the equation of the tangent to the curve:

a $y = x^2 - 7x + 10$ at the point $(2, 0)$

b $y = x + \frac{1}{x}$ at the point $(2, 2\frac{1}{2})$

c $y = 4\sqrt{x}$ at the point $(9, 12)$

d $y = \frac{2x - 1}{x}$ at the point $(1, 1)$

e $y = 2x^3 + 6x + 10$ at the point $(-1, 2)$

f $y = x^2 + \frac{-7}{x^2}$ at the point $(1, -6)$

2 Find the equation of the normal to the curves:

a $y = x^2 - 5x$ at the point $(6, 6)$

b $y = x^2 - \frac{8}{\sqrt{x}}$ at the point $(4, 12)$

3 Find the coordinates of the point where the tangent to the curve $y = x^2 + 1$ at the point $(2, 5)$ meets the normal to the same curve at the point $(1, 2)$.

4 Find the equations of the normals to the curve $y = x + x^3$ at the points $(0, 0)$ and $(1, 2)$, and find the coordinates of the point where these normals meet.

5 For $f(x) = 12 - 4x + 2x^2$, find an equation of the tangent and normal at the point where $x = -1$ on the curve with equation $y = f(x)$.

E
