Answers - Coordinate Geometry – Circles - 2

1.

(a) $\left(\frac{5+13}{2}, \frac{-1+11}{2}\right), = \underline{\underline{(9,5)}}$	M1, A1 (2)
(b) $r^2 = (9-5)^2 + (5--1)^2 (= 52)$ or $r^2 = (13-9)^2 + (11-5)^2 (= 52)$ (or equiv.)	M1
Equation of circle: $(x-9)^2 + (y-5)^2 = 52$ (or equiv.)	M1 A1ft A1 (4)

2.

(a) Centre (5, 0) (or $x = 5, y = 0$)	B1 B1	(2)
(b) $(x \pm a)^2 \pm b \pm 9 + (y \pm c)^2 = 0 \Rightarrow r^2 = \ldots$ or $r = \ldots$, Radius = 4	M1, A1	(2)
(c) (1, 0), (9, 0) Allow just $x = 1$, $x = 9$	B1ft, B1ft	(2)
(d) Gradient of $AT = -\frac{2}{7}$	B1	
$y = -\frac{2}{7}(x-5)$	M1 A1ft	(3)

3.

(a)	Gradient of PQ is $-\frac{1}{3}$		B1	
	$y - 2 = -\frac{1}{3}(x-2)$ $(3y + x = 8)$		M1 A1	(3)
(b)	$y = 1$: $3 + x = 8$ $x = 5$	(*)	B1	(1)
(c)	$("5"-2)^2 + (1-2)^2$	M: Attempt PQ^2 or PQ	M1 A1	
	$(x-5)^2 + (y-1)^2 = 10$	M: $(x \pm a)^2 + (y \pm b)^2 = k$	M1 A1	(4)

4.

i	grad AC = $\frac{7-3}{3-1}$ or 4/2 o.e.[= 2]	M1	not from using – ½	
	so grad AT = – ½	M1	or ft their grad AC [for use of $m_1m_2 = -1$]	
	eqn of AT is $y - 7 = -½(x - 3)$	M1	or subst (3, 7) in $y = -½x + c$ or in $2y + x = 17$; allow ft from their grad of AT, except 2 (may be AC not AT)	
	one correct constructive step towards $x + 2y = 17$ [ans given]	M1	or working back from given line to $y = -½x + 8.5$ o.e.	4
ii	$x + 2(2x - 9) = 17$	M1	attempt at subst for x or y or elimination	
	$5x - 18 = 17$ or $5x = 35$ o.e. $x = 7$ and $y = 5$ [so (7, 5)]	A1 B1	allow $2.5x = 17.5$ etc graphically: allow M2 for both lines correct or showing (7, 5) fits both lines	3
iii	$(x-1)^2 + (2x-12)^2 = 20$ $5x^2 - 50x + 125[=0]$ $(x-5)^2 = 0$ equal roots so tangent	M1 M1 A1 B1	subst $2x - 9$ for y [oe for x] rearranging to 0; condone one error showing 5 is root and only root explicit statement of condition needed (may be obtained earlier in part) or showing line is perp. to radius at point of contact	
	(5, 1)	B1	condone $x = 5$, $y = 1$	
	<u>or</u>			
	$y - 3 = -½(x - 1)$ o.e. seen	M1	or if $y = 2x - 9$ is tgt then line through C with gradient –½ is radius	
	subst or elim. with $y = 2x - 9$ $x = 5$ (5,1) showing (5, 1) on circle	M1 A1 B1 B1	or showing distance between (1, 3) and (5, 1) = $\sqrt{20}$	5

5.

(a)	$(x-5)^2+(y--6)^2$	M1		one term correct
		A1		LHS correct with perhaps extra constant terms
	$(x-5)^2+(y+6)^2=20$	A1	3	equation completely correct
(b) (i)	$C(5,-6)$	B1√	1	correct or ft their **(a)**
(ii)	(radius =) $\sqrt{20}$	M1		correct or ft 'their' $\sqrt{k}$ provided RHS > 0
	$=2\sqrt{5}$	A1	2	must see $\sqrt{20}$ **first**
(c)	Grad $AC=\frac{-6--2}{5-3}$ $(=-2)$	M1		correct unsimplified, ft their coords of C
	Grad of tangent $=\frac{1}{2}$	B1√		ft their $-1/$ grad AC
	Equation of tangent is $(y--2)=\text{"}their\frac{1}{2}\text{"}(x-3)$	M1		clear attempt at **tangent** not normal through $(3,-2)$
	$y+2=\frac{1}{2}(x-3)$	A1		correct equation in any form but $y--2$ must be simplified to $y+2$
	$x-2y=7$	A1 cso	5	
(d)	$AB^2+(their\ \ r)^2=6^2$	M1		Pythagoras used with 6 as hypotenuse
	$d^2+20=36$ *or* $(AB^2)=36-20$	A1		values correct with $(2\sqrt{5})^2=20$ PI
	$AB^2=16$ Hence $AB=4$	A1cso	3	notation all correct

6.

(i)	Gradient DE = $-\frac{1}{2}$	B1	1	$-\frac{1}{2}$ (any working seen must be correct)
(ii)	$y-3=-\frac{1}{2}(x-2)$	M1		Correct equation for straight line, any gradient, passing through F
		A1		$y-3=-\frac{1}{2}(x-2)$ aef
	$x+2y-8=0$	A1	3	$x+2y-8=0$ (this form but can have fractional coefficients e.g. ½x + y – 4 = 0
(iii)	Gradient EF = $\frac{4}{2}$ =2	B1		Correct supporting working must be seen
	$-\frac{1}{2}\times 2=-1$	B1	2	Attempt to show that product of their gradients = - 1 o.e.

(iv)	DF= $\sqrt{4^2+3^2}=5$	M1 A1 2	$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ used 5
(v)	DF is a diameter as angle DEF is a right angle.	B1	Justification that DF is a diameter
	Mid-point of DF <u>or</u> centre of circle is $(0,1\frac{1}{2})$	B1	Mid-point of DF <u>or</u> centre of circle is $(0,1\frac{1}{2})$
	Radius = 2.5	B1	Radius = 2.5
	$x^2+(y-\left(\frac{3}{2}\right))^2=\left(\frac{5}{2}\right)^2$	B1 $\sqrt{}$	$x^2+(y-\left(\frac{3}{2}\right))^2=\left(\frac{5}{2}\right)^2$
	$x^2+y^2-3y+\frac{9}{4}=\frac{25}{4}$ $x^2+y^2-3y-4=0$	B1 5	$x^2+y^2-3y-4=0$ obtained correctly with at least one line of intermediate working. <u>SR</u> For working that only shows $x^2+y^2-3y-4=0$ is equation for a circle with centre $(0,1\frac{1}{2})$ B1 radius 2.5 B1