Trigonometry Worksheet - Answers

Exercise C

(1)(a) $4\cos^2\theta = 1$, $0 \leq \theta \leq 360^\circ$

$\cos^2\theta = \frac{1}{4}$

$\cos\theta = \pm\sqrt{\frac{1}{4}}$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = -\frac{1}{2}$

$\theta = \cos^{-1}\left(\frac{1}{2}\right)$ $\quad$ $\theta = \cos^{-1}\left(-\frac{1}{2}\right)$

$\theta = 60^\circ, (360-60^\circ)$ $\quad$ $\theta = 120^\circ, (360-120^\circ)$

↑ 1st ↑ 2nd $\quad$ ↑ 1st ↑ 2nd

$\theta = 60^\circ, 300^\circ$ or $\theta = 120^\circ, 240^\circ$

$\therefore$ $\boxed{\theta = 60^\circ, 120^\circ, 240 \text{ or } 300^\circ}$

(b) $2\sin^2\theta - 1 = 0$, $0 \leq \theta \leq 360^\circ$

$\sin^2\theta = \frac{1}{2}$

$\sin\theta = \pm\frac{1}{\sqrt{2}}$

$\sin\theta = \frac{1}{\sqrt{2}}$ or $\sin\theta = -\frac{1}{\sqrt{2}}$

↓

Continue as we did in Q1(a).

(c) $3\sin^2\theta + \sin\theta = 0$, $0 \leq \theta \leq 360°$

Let $x = \sin\theta$

$3x^2 + x = 0$

$x(3x+1) = 0$

$x = 0$ or $x = -1/3$

$\sin\theta = 0$ or $\sin\theta = -1/3$

⇓

continue as we did in Q1(a).

(d) $\tan^2\theta - 2\tan\theta - 10 = 0$, $0 \leq \theta \leq 360°$

Let $x = \tan\theta$

$x^2 - 2x - 10 = 0$

$x = \frac{2 \pm \sqrt{4 - 4(1)(-10)}}{2}$

$x = \frac{2 \pm \sqrt{44}}{2}$

$x = 1 \pm \sqrt{11}$

$\therefore \tan\theta = 1 + \sqrt{11}$ or $\tan\theta = 1 - \sqrt{11}$

⇓

Continue as in Q1(a).

(e) $2\cos^2\theta - 5\cos\theta + 2 = 0$, $0 \leq \theta \leq 360°$

Let $x = \cos\theta$ and continue as in Q1(d).

(f) $\sin^2\theta - 2\sin\theta - 1 = 0$, $0 \leq \theta \leq 360°$

Let $x = \sin\theta$ and continue as in Q1(d)

(g) $\tan^2 2\theta = 3$, $0 \le \theta \le 360^\circ$

$\tan 2\theta = \pm\sqrt{3}$

$\tan 2\theta = \sqrt{3}$ or $-\sqrt{3}$

Range of 2θ :

$0 \le \theta \le 360^\circ$

$0 \le 2\theta \le 720^\circ$

$\tan 2\theta = \sqrt{3}$ or $\tan 2\theta = -\sqrt{3}$

$2\theta = 60^\circ, (180+60^\circ)$ or $2\theta = -60^\circ, (180+-6$

1st 2nd 1st 2nd

$2\theta = 60^\circ, 240^\circ$ or $2\theta = -60^\circ, 120^\circ$

$+360^\circ$ $+360^\circ$ $+360^\circ$ $+360^\circ$

420° 600° 300° 480°

$+360^\circ$

660°

$\therefore 2\theta = 60^\circ, 240^\circ, 420^\circ, 600^\circ, 120^\circ, 300^\circ, 660^\circ$ or 480

$\therefore \boxed{\theta = 30^\circ, 120^\circ, 210^\circ, 300^\circ, 60^\circ, 150^\circ, 330^\circ \text{ or } 240}$

(h) $4\sin\theta = \tan\theta$, $0 \le \theta \le 360^\circ$

$4\sin\theta = \dfrac{\sin\theta}{\cos\theta}$

*** (Do not divide by $\sin\theta$ both sides)

$4\sin\theta\cos\theta = \sin\theta$

$4\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(4\cos\theta - 1) = 0$

$\sin\theta = 0$ or $\cos\theta = \dfrac{1}{4}$

⇓

continue.

(i) $\sin\theta + 2\cos^2\theta + 1 = 0, \quad 0 \le \theta \le 360°$

$\sin\theta + 2(1-\sin^2\theta) + 1 = 0$

$\sin\theta + 2 - 2\sin^2\theta + 1 = 0$

$-2\sin^2\theta + \sin\theta + 3 = 0$

Continue as in Q1(f).

(j) $\tan^2(\theta - 45°) = 1, \quad 0 \le \theta \le 360°$

$\tan(\theta - 45°) = \pm\sqrt{1}$

$\tan(\theta - 45°) = 1$ or $\tan(\theta - 45) = -1$

Continue each of this as in Ex D/Q2(a).

(k) $3\sin^2\theta = \sin\theta\cos\theta, \quad 0 \le \theta \le 360°$

$3\sin^2\theta - \sin\theta\cos\theta = 0$ } → [Do not divide both sides by $\sin\theta$ look at notes.]

$\sin\theta(3\sin\theta - \cos\theta) = 0$

$\sin\theta = 0$ or $3\sin\theta - \cos\theta = 0$

$\sin\theta = 0$ or $3\sin\theta = \cos\theta$ → (Now divide by $\sin\theta$ - look at notes)

$(\div \sin\theta) \quad (\div \sin\theta)$

$\sin\theta = 0$ or $\frac{3\sin\theta}{\cos\theta} = 1$

$3\tan\theta = 1$

$\tan\theta = \frac{1}{3}$

Continue as in previous questions.